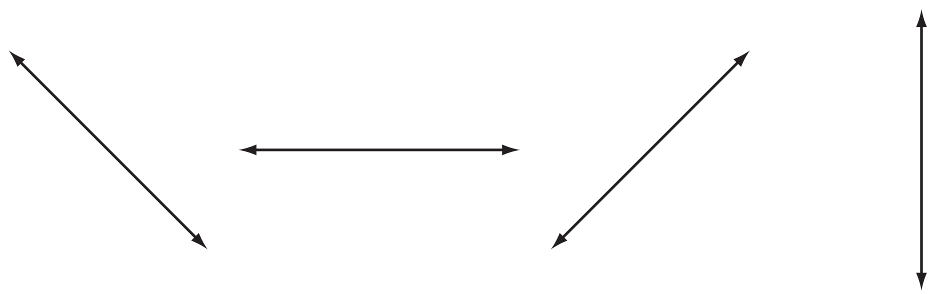
4.1 *Slope*

The slope of a linear equation describes the steepness and direction of a line. As a graph is traced from left to right, the vertical change relative to the horizontal change is the slope of a line.



Negative Slope Zero Slope Positive Slope Undefined Slope

Finding Slope from a Graph

To find the slope of a line or line segment from a graph, divide the vertical change between two points by the horizontal change of the same two points. An equation with a positive slope increases as its graph is traced from left to right. An equation with a negative slope decreases as its graph is traced from left to right.

Slope (*m*)

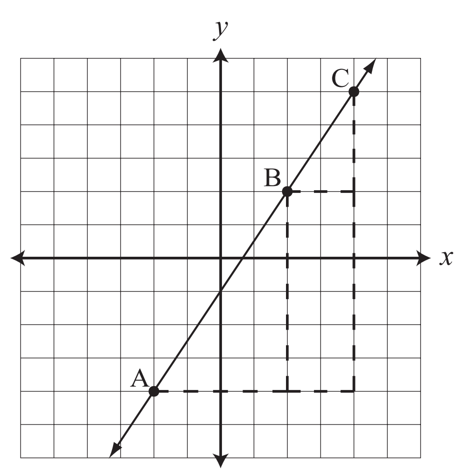
$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

Example of Positive Slope:

$$\text{Slope of segment AB} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{6}{4} = \frac{3}{2}$$

$$\text{Slope of segment AC} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{9}{6} = \frac{3}{2}$$

$$\text{Slope of segment BC} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{3}{2}$$

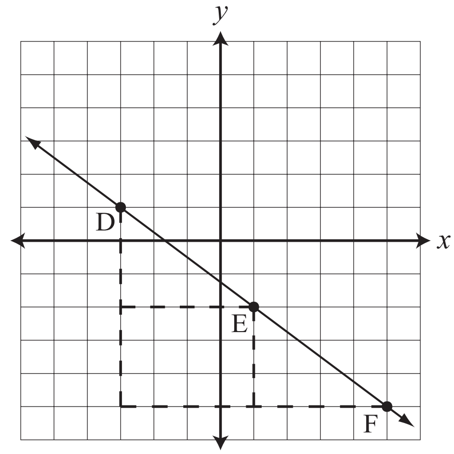


Example of Negative Slope:

Slope of segment DE $= \dfrac{\text{vertical change}}{\text{horizontal change}} = -\dfrac{3}{4}$

Slope of segment DF $= \dfrac{\text{vertical change}}{\text{horizontal change}} = -\dfrac{6}{8} = -\dfrac{3}{4}$

Slope of segment EF $= \dfrac{\text{vertical change}}{\text{horizontal change}} = -\dfrac{3}{4}$



Finding Slope from Ordered Pairs

In the previous chapter, it was shown that every point in a coordinate system is an ordered pair, therefore a slope can be defined in terms of ordered pairs.

Slope Formula

The slope, m, of a line segment between two ordered pairs (x_1, y_1) and (x_2, y_2) is:

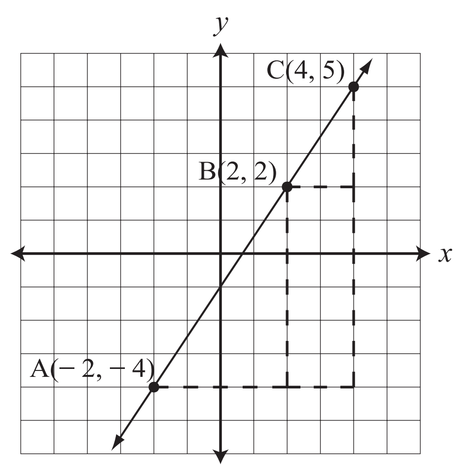
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example of Positive Slope:

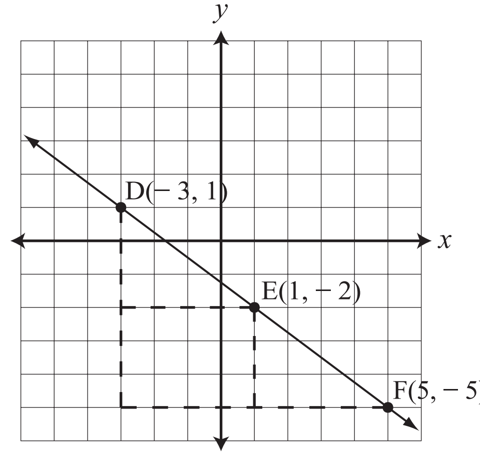
Slope of segment AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4-2}{-2-2} = \dfrac{-6}{-4} = \dfrac{3}{2}$

Slope of segment AC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4-5}{-2-4} = \dfrac{-9}{-6} = \dfrac{3}{2}$

Slope of segment BC $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-5}{2-4} = \dfrac{-3}{-2} = \dfrac{3}{2}$



Example of Negative Slope:



Slope of segment DE $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-(-2)}{-3-1} = \dfrac{3}{-4} = -\dfrac{3}{4}$

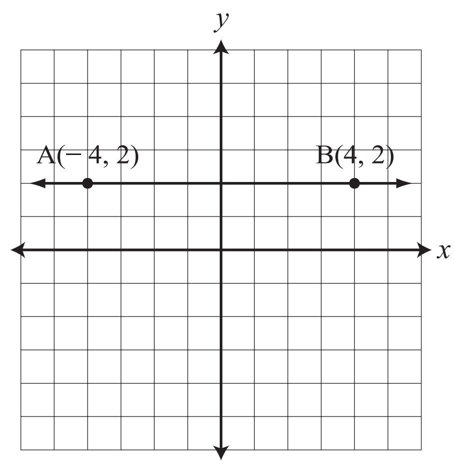
Slope of segment DF $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-(-5)}{-3-5} = \dfrac{6}{-8} = -\dfrac{3}{4}$

Slope of segment EF $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-(-5)}{1-5} = \dfrac{3}{-4} = -\dfrac{3}{4}$

Note: The slope does not depend on the order of the points.

Lines With Zero Slope and Undefined Slope

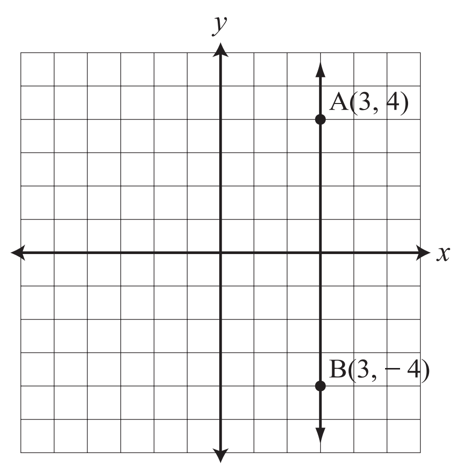
If two different points have the same *y*-value, the line (or line segment) joining the two points is horizontal.



Example of Zero Slope:

Slope of segment AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-2}{4-(-4)} = \dfrac{0}{8} = 0$

If two different points have the same *x*-value, the line (or line segment) joining the two points is vertical.



Example of Undefined Slope:

Slope of segment AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-(-4)}{3-3} = \dfrac{8}{0} =$ undefined

4.1 Exercise Set

1. Fill in the blank with the appropriate word.

a) The run between two points on a coordinate system refers to change in the ________________ variable.

b) The rise between two points on a coordinate system refers to a change in the ________________ variable.

c) The letter ______ is used to indicate the slope of a line.

d) The formula for finding the slope of a line is ________________ .

e) The slope of a vertical line is ____________ .

f) The slope of a horizontal line is ____________ .

g) A ________________ has both the x-coordinate and y-coordinate increasing.

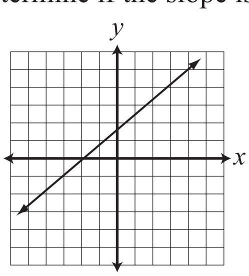
h) A ________________ has both the x-coordinate and y-coordinate decreasing.

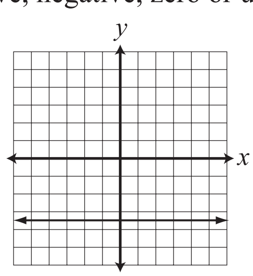
i) ________________ represents a rate of change.

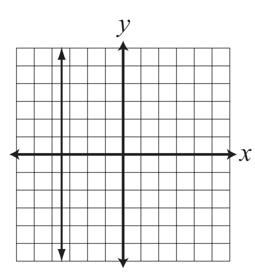
2. Match the column on the left with the column on the right.

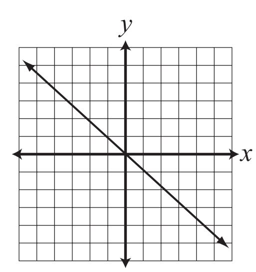
a) rise	________	*i*)	$x = 3$
b) run	________	*ii*)	difference in x
c) slope	________	*iii*)	$\dfrac{\text{difference in } y}{\text{difference in } x}$
d) vertical line	________	*iv*)	difference in y
e) horizontal line	________	*v*)	$y = -1$

3. Determine if the slope is positive, negative, zero or undefined.

a)


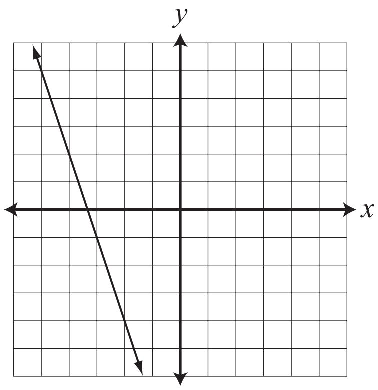
b)


c)


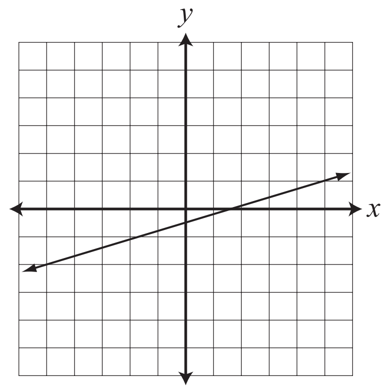
d)


4. Determine the slope of the line.

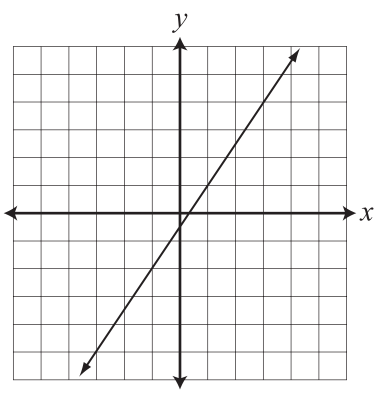
a)



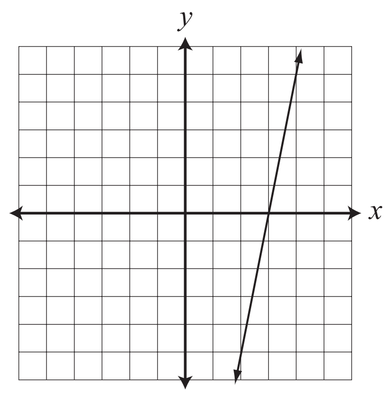
b)



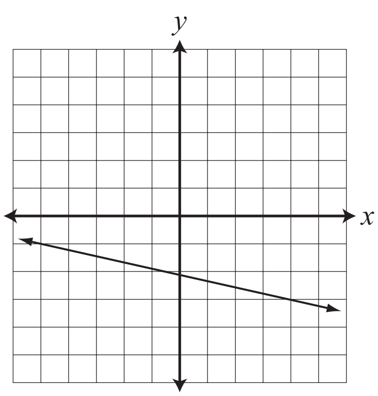
c)



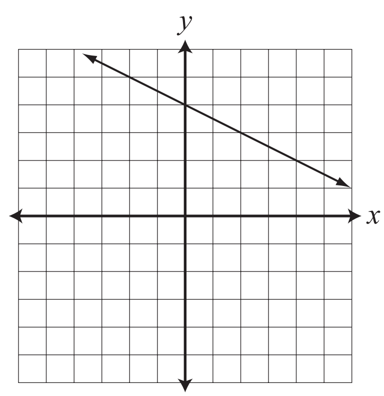
d)



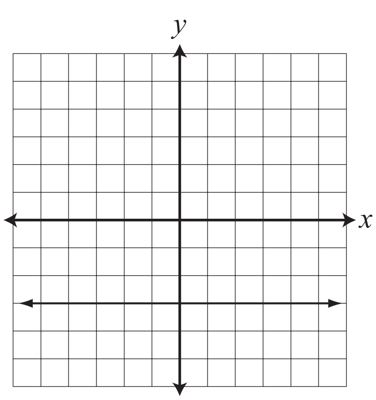
e)



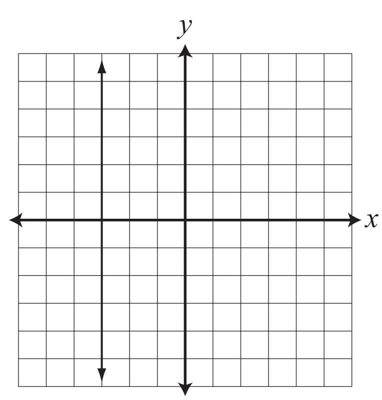
f)



g)



h)



5. Find the slope of the line containing each pair of points.

a) (2, 3) and (6, 9) ________

b) (3, 2) and (7, 10) ________

c) (− 1, 5) and (4, 1) ________

d) (2, 2) and (2, − 2) ________

e) (− 3, − 3) and (3, − 3) ________

f) (− 3, 3) and (3, − 2) ________

g) (− 3, 1) and (6, 8) ________

h) (2, − 1) and (− 5, − 1) ________

i) (− 4, 0) and (− 4, 2) ________

j) (0, 8) and (− 4, 0) ________

k) $\left(\frac{1}{2}, \frac{1}{3}\right)$ and $\left(\frac{1}{3}, \frac{1}{5}\right)$ ________

l) $(0.26, 0.45)$ and $(0.38, 0.20)$ ________

m) $\left(-\frac{1}{3}, \frac{1}{5}\right)$ and $\left(\frac{1}{2}, -\frac{1}{3}\right)$ ________

n) $(1.23, 3.57)$ and $(1.31, 3.39)$ ________

6. Arrange the slopes from flattest to steepest.

a) $-1\,,\ \frac{1}{2}\,,\ -\frac{2}{3}\,,\ \frac{3}{4}$

b) $-1\,,\ \frac{4}{5}\,,\ -\frac{5}{4}\,,\ \frac{5}{6}$

c) $-2\,,\ \frac{1}{2}\,,\ -\frac{2}{3}\,,\ \frac{3}{2}$

d) $-\frac{3}{2}\,,\ 0\,,\ \frac{4}{3}$, undefined

7. If one line has a slope of -4 and another line has a slope of 3, which line is steeper? Why?

8. In calculating the slope between (7, 3) and (1, 2), one student calculated $(3 - 2)$ divided by $(7 - 1)$. Another student calculated $(2 - 3)$ divided by $(1 - 7)$. Are they both correct? Why?

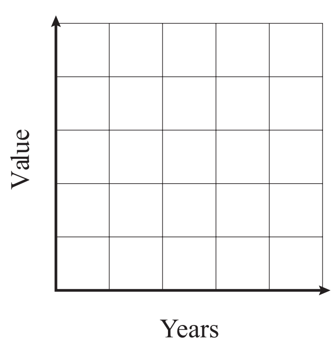
9. Explain how you could show that the point A(1, 3), B(1, 9) and C(4, 3) are the vertices of a right triangle.

10. Determine whether the three points A(-2, -1), B(0, 4) and C(2, 9) all lie on the same line.

11. A car was purchased for \$40 000. The value of the car over a four year period is shown.

year x	value y
0	40 000
1	32 000
2	27 000
3	23 000
4	20 000

a) Draw a graph of this data.



b) What is the slope of the line segment from:

i) year 0 to year 1 ? ______

ii) year 1 to year 2 ? ______

iii) year 2 to year 3 ? ______

iv) year 3 to year 4 ? ______

12. This problem is based on a concept from this section. Find the mistake.

Let $x = y = 1$

Then:

$$\begin{array}{rcll}
x &=& y & \textit{given} \\
x^2 &=& xy & \textit{multiply by } x \\
x^2 - y^2 &=& xy - y^2 & \textit{subtract } y^2 \\
(x-y)(x+y) &=& y(x-y) & \textit{factor} \\
(x+y) &=& y & \textit{divide by } (x-y) \\
1+1 &=& 1 & \textit{substitute variables} \\
2 &=& 1 & \textit{add}
\end{array}$$

4.2 Rate of Change

The greek letter Delta (Δ) is used to represent change. Rates of change are special ratios for comparing quantities with different units. The formula $\Delta y / \Delta x$ is the change of *y* over the change of *x,* or the average change of *y* per unit *x*. Mathematically, this formula is calculated exactly the same as slope.

Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Rates of Change in Fraction Notation:

1. kilometres per hour $\rightarrow \frac{\text{km}}{\text{h}}$ or km/h
2. miles per gallon $\rightarrow \frac{\text{mi}}{\text{gal}}$ or mi/gal
3. dollars per hour $\rightarrow \frac{\text{dollars}}{\text{hr}}$ or dollars/hr

Rates of Change:

1. If the city of Surrey grew by 120 000 people over a five year period, it has a rate of change of $\frac{120\,000 \text{ people}}{5 \text{ years}} = 24\,000$ people per year.
2. If a person runs the 400 metre race in 56 seconds, he is running at a rate of $\frac{400 \text{ m}}{56 \text{ sec}} = 7.14$ metres per second.

Example 1 Carly rents a car with the gas tank full. The odometer registered 86 347 km. Carly used it for three days. When the car was returned the odometer reading was 86 721 km and it needed 63 litres to fill up. The cost of renting the car was \$96 plus gas which cost 90¢ per litre.

a) Determine the rate of gas consumption for the car.
b) Determine the average rate of travel per day.
c) Determine the cost of renting the car per day.

► *Solution*:

a) $\frac{\Delta y}{\Delta x} = \frac{(86\,721 - 86\,347) \text{ km}}{(63 - 0) \text{ litres}} = 5.94$ km per litre

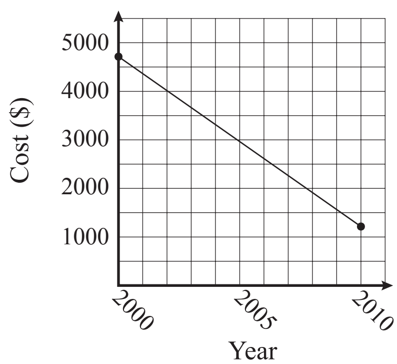
b) $\frac{\Delta y}{\Delta x} = \frac{(86\,721 - 86\,347) \text{ km}}{(3 - 0) \text{ days}} = 124.7$ km per day

c) $\frac{\Delta y}{\Delta x} = \frac{(96 - 0)\,\$}{(3 - 0) \text{ days}} + \frac{63(0.90)\,\$}{3 \text{ days}} = \$50.90$ per day

Rates of change can be visualized by using a graph. As a general rule, the quantity in the denominator is placed on the horizontal axis, and the value in the numerator on the vertical axis.

Example 2 Between 2000 and 2010, the cost of a 42″ LCD TV dropped from \$4600 to \$1200. Graph this result, and determine the average drop in price per year.

► *Solution*:



$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{4600 - 1200}{2000 - 2010} = -340$$

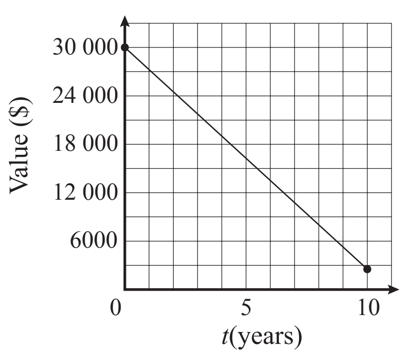
LCD TVs have been dropping at an average rate of \$340 per year.

Example 3 Most cars depreciate as they age. A car costing \$30 000 will have a value of \$2500 at the end of 10 years.

a) Write a formula for its value V, when it is t years old. $0 \le t \le 10$
b) Draw a graph of this linear function.
c) Determine the car's value after 4.5 years.
d) When is the car's value between \$12 000 and \$15 000?
e) How much value does the car lose every 2.5 years?
f) What is the rate of change of the car's value with respect to time?

► *Solution*: **a)** $\text{Slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{30\,000 - 2500}{0 - 10} = -2750$. Therefore $V = 30\,000 - 2750\,t$

b)



c) $V = 30\,000 - 2750\,(4.5) = \$17\,625$

d) $V = 30\,000 - 2750\,t = 12\,000 \;\rightarrow\; 2750\,t = 18\,000 \;\rightarrow\; t = 6.5$ years
$V = 30\,000 - 2750\,t = 15\,000 \;\rightarrow\; 2750\,t = 15\,000 \;\rightarrow\; t = 5.5$ years

e) Since it loses \$2750 per year, $\$2750 \times 2.5$ years $= \$6875$ lost every 2.5 years.

f) The rate of change of value is the slope $= -\$2750$ per year.

Example 4 Georgia sells computers. She is paid a basic monthly salary of \$1500, plus \$400 for every five computers she sells.

a) Write a formula for Georgia's monthly wage.
b) How many computers must be sold for Georgia to make at least \$3440 in one month?
c) Determine Georgia's wage in a month when she sells 60 computers.
d) What is the rate of change of Georgia's wage with respect to the number of computers sold?

► *Solution*: **a)** $\dfrac{\Delta y}{\Delta x} = \dfrac{400 - 0}{5 - 0} = 80$

$W = 1500 + 80x$

b) $W = 1500 + 80x = 3440 \rightarrow 80x = 1940 \rightarrow x = 24.25$
Georgia must sell 25 computers.

c) $W = 1500 + 80(60) = \$6300$

d) The rate of change is \$80 per computer.

Example 5 In the morning, Anna typed nine pages in 45 minutes. After lunch, she typed 18 pages in 1 hour, 20 minutes. If the pages typed were approximately the same length, did she type faster in the morning or after lunch?

► *Solution*: Morning rate: $\dfrac{9 \text{ pages}}{45 \text{ min}} = 0.2 \text{ pages/min}$

After lunch rate: 1 hour, 20 minutes = 80 minutes

$$\frac{15 \text{ pages}}{80 \text{ min}} = 0.1875 \text{ pages/min}$$

Therefore, Anna typed faster in the morning.

4.2 Exercise Set

1. Determine if the slope is positive, negative, zero or undefined.

a) A line only going through the quadrants I, II, and III.

b) A line only going through the quadrants I, II, and IV.

c) A line only going through the quadrants II, III, and IV.

d) A line only going through the quadrants I, III, and IV.

e) A line only going through the quadrants II, and IV.

f) A line only going through the quadrants I and III

g) A line only going through the quadrants III, and IV.

h) A line only going through the quadrants II, and III.

2. State whether sentence describes a positive, negative or zero rate of change.

a) The rate of growth in a baby's height.

b) The rate that Lake Superior has risen in the last 10 years.

c) The number of fans seated when the hockey game ends.

d) The area of the polar ice caps in the last 100 years.

e) The population of Canada since Confederation.

f) The population of Ireland during the potato famine of 1848.

g) Driving at a steady speed of 100 km/h.

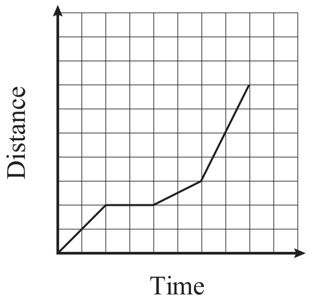
h) The acceleration of a falling rock on the moon.

i) The average number of sockeye salmon that have spawned on the Fraser River over the last 25 years.

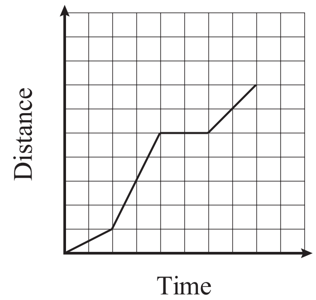
j) The height of a person from age 25 to age 40.

3. In order to get in shape, Josie decides to swim, run, bicycle and stretch. Josie can swim at a speed of 5 km/h, run at a speed of 10 km/h, and bicycle at a speed of 20 km/h. Match the graph with the description given below.

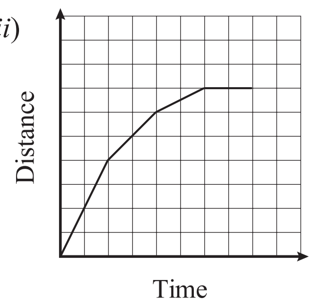
i)



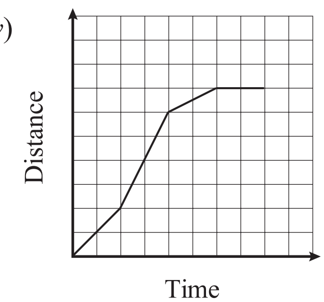
ii)



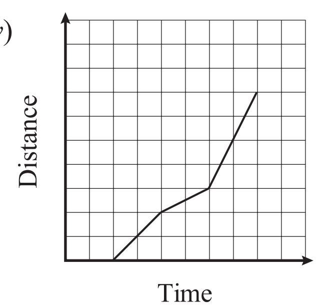
iii)



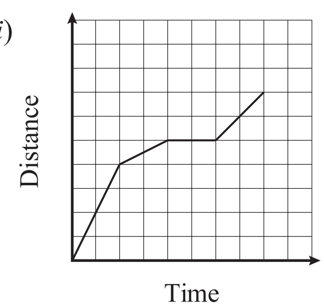
iv)



v)



vi)



a) runs, bicycles, swims, then stretches _____

b) stretches, runs, swims, then bicycles _____

c) runs, stretches, swims then bicycles _____

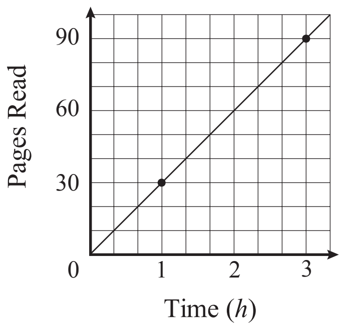
d) bicycles, runs, swims then stretches _____

e) bicycles, swims, stretches, then runs _____

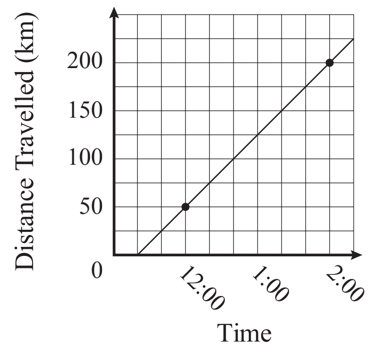
f) swims, bicycles, stretches, then runs _____

4. Determine the rate of change.

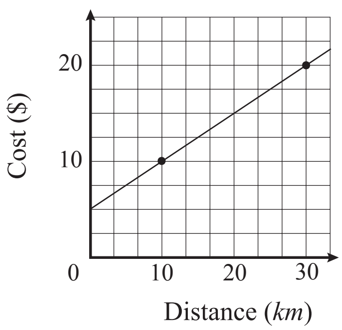
a) Reading a Novel



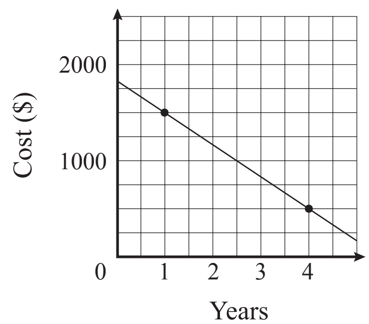
b) Highway travel



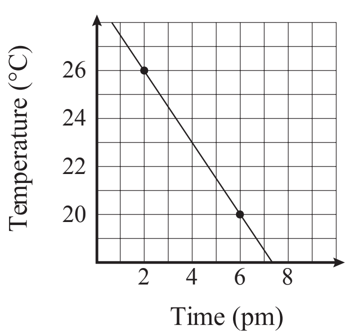
c) Taxi Fare



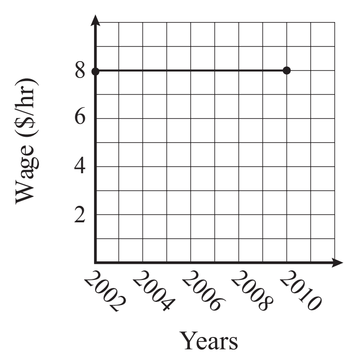
d) Computer Value



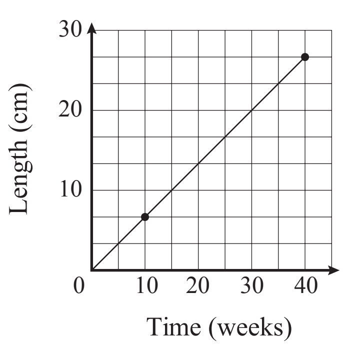
e) Temperature Change



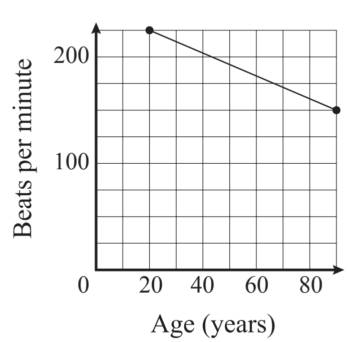
f) Minimum Wage in BC



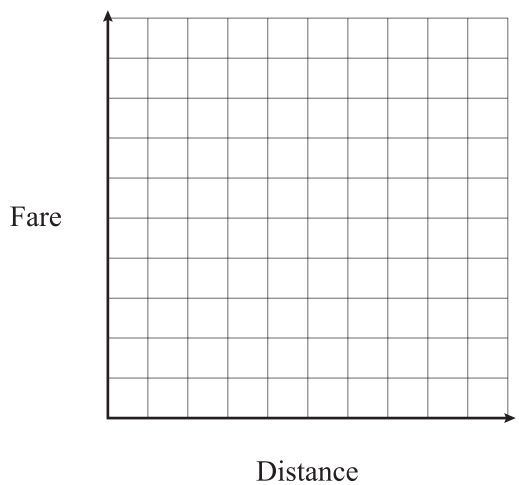
g) Length of Unborn Child



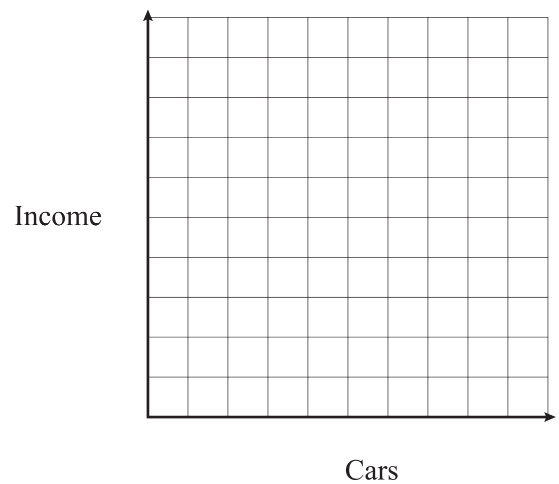
h) Maximum Heart Rate



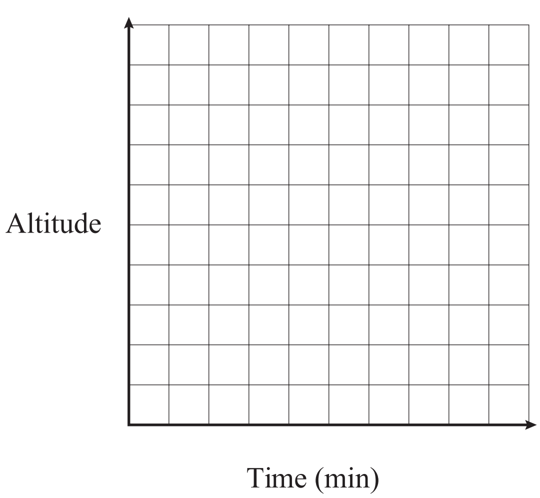
5. Draw a graph to describe the fare charged by a taxi with an initial cost of $3.00 plus $1.20 per km traveled.



6. Draw a graph to describe the income of a car sales person who earns $600 per month plus $250 for every car sold.



7. Draw a graph to describe a jet whose altitude climbs from sea level to 9600 metres at a rate of 1600 metres per minute, then cruises at a steady altitude for five minutes, then descends back to sea level at a rate of 800 metres per minute.



8. George ran at a steady rate from the 3 km mark to the end of a 10 km race. At the 3 km mark, his time was 12 minutes, and at the 8 km mark, his time was 27 minutes. Determine the time it took for George to complete the 10 km race.

9. A long distance runner passes the 24 km mark of a race in 1 hr 20 min, and passes the 42 km mark 1 hour later. Assuming a constant rate, find the speed of the long distance runner in km/hr.

10. A plane at an altitude of 20 000 feet starts to descend for landing after flying for six hours. The entire flight time was 6 hours 40 minutes. Determine the average rate of descent of the plane.

11. As a window washer begins work on a high rise, one-third of the windows were already clean. Eight hours later, three-quarters of all the windows are clean. Calculate the window washer's cleaning rate.

12. A five foot long treadmill rises six inches to make an incline for running uphill. What is the slope of the treadmill?

13. A highway cannot exceed a 4% grade because of safety reasons. How much change in elevation would be allowed on a 2500 m stretch of road?

14. The roof of a house rises 9 ft over a run of 12 ft. Determine the height of a vertical brace placed 8 ft from the low end of the roof.

15. A water tower holds 10 000 litres of water. When a valve is opened, the tower has 9700 litres after 30 seconds, and 9400 litres after 60 seconds. The water continues to drain at the same rate until the tower is empty.

a) Determine the rate of change of volume with respect to time.

b) How much water remains after 4 minutes?

c) How much time is needed to empty $\frac{3}{4}$ of the water in the tower?

d) How long does it take to empty the water tower?

16. Lulu hires a band to play at a wedding. The cost for the wedding was \$900 for the band, plus \$20 per guest for food. Beverages cost \$2 extra each.

a) Determine the cost per person if 120 guests attended the wedding, and averaged two drinks per person.

b) Determine the cost per person if 200 guests attended the wedding, and averaged three drinks per person.

17. At noon, Annette rented a bicycle to travel around Stanley Park. She returned the bicycle at 4:00 pm the same day after travelling 48 km. It cost \$22 to rent the bicycle.

a) Determine Annette's average speed in km/hr.

b) Determine the rental rate in dollars per hour.

c) Determine the rental rate in dollars per km.

18. Pearl was hired by Crescent Beach Publishing to type the manuscript for a mathematics book. She started work at 8:00 am, took an hour for lunch, and finished at 5:00 pm. Her pay for typing page 25 to the end of page 40 was \$180.

a) Determine Pearl's pay in dollars per hour.

b) Determine Pearl's typing rate in pages per hour.

c) Determine Pearl's rate of pay in dollars per page.

19. Liz rents a van to take a hockey team to a three day tournament. The odometer read 36 645 km when she picked up the van, and read 37 118 km when the van was returned. The total cost for the trip, including the 60 litres of gas used, was \$180.

a) Determine the rate of gas consumption per km.

b) Determine the average cost of renting the van in dollars per day

c) Determine the average distance travelled per day.

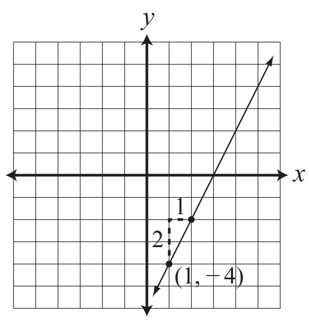
d) Determine the rental rate in cents per km.

4.3 Graphing Linear Functions

If the slope of a line, and a point on the line are known, it is possible to graph the line. With this information it is also possible to find other points on the line.

Example 1 Graph a line with slope 2, going through the point (1, − 4).

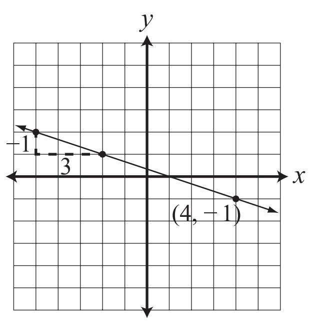
► *Solution*:



$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1}$$

Example 2 Find a point in quadrant IV on the line with slope $-\frac{1}{3}$ going through the point (− 5, 2).

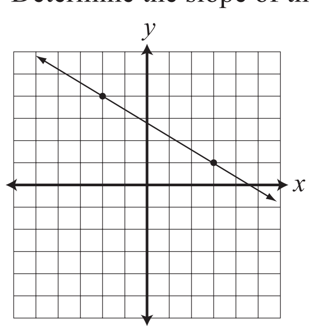
► *Solution*:



$$m = \frac{\text{rise}}{\text{run}} = -\frac{1}{3}$$

One of the infinite number of answers is (4, − 1)

Example 3 Determine the slope of the graph.



► *Solution*: Two points on the graph that have integer *x* and *y* coordinates are: (− 2, 4) and (3, 1)

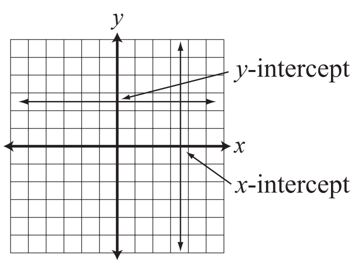
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{3 - (-2)} = -\frac{3}{5}$$

Intercepts

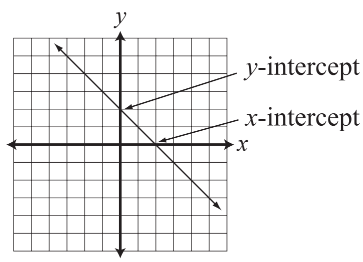
The point at which a graph crosses the y-axis is called the ***y*-intercept** and the point at which a graph crosses the x-axis is called the ***x*-intercept**. A linear relation can have one intercept, two intercepts or an infinite number of intercepts.

> ***x*-intercept and *y*-intercept**
>
> The x-intercept of a line is the point $(a, 0)$ where the line intersects the x-axis.
>
> The y-intercept of a line is the point $(0, b)$ where the line intersects the y-axis.

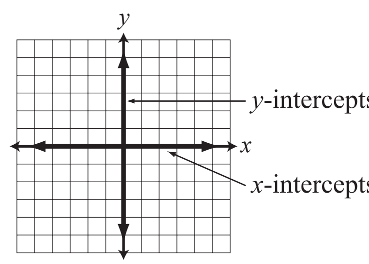
One Intercept



Two Intercepts

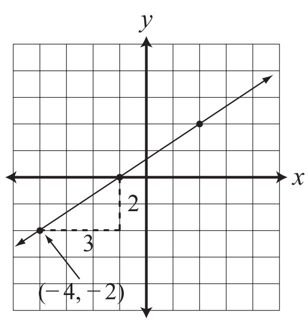


Infinite Intercepts



Example 4 Determine the x-intercept and y-intercept of the linear equation with slope $\frac{2}{3}$, going through $(-4, -2)$.

► *Solution*:



x-intercept is $(-1, 0)$

y-intercept is $\left(0, \frac{2}{3}\right)$

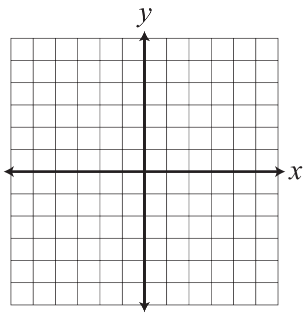
Example 5 Determine the slope of a line with x-intercept $(-3, 0)$ and y-intercept $(0, -4)$.

► *Solution*: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-4)}{-3 - 0} = -\frac{4}{3}$

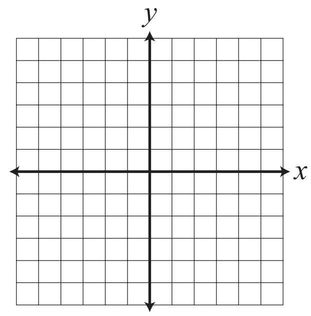
4.3 Exercise Set

1. Graph the line that passes through the given point and has the given slope.

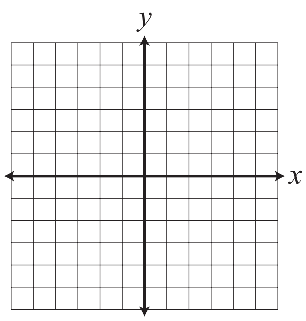
a) $(0, 2)$; $m = 1$



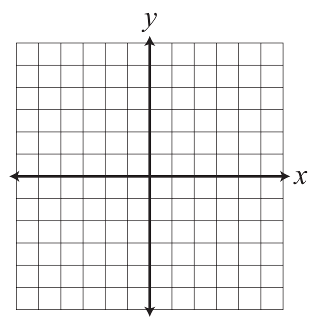
b) $(-3, 1)$; $m = 2$



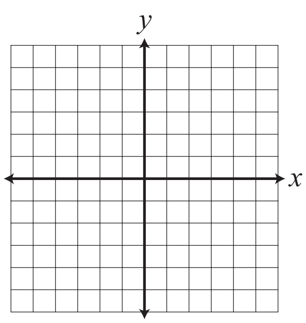
c) $(-2, -2)$; $m = -\frac{3}{4}$



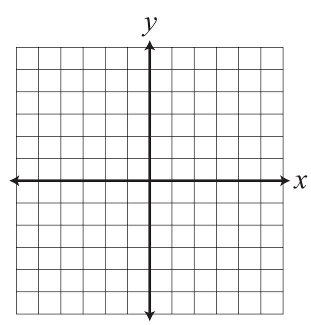
d) $(-3, -1)$; $m = \frac{3}{2}$



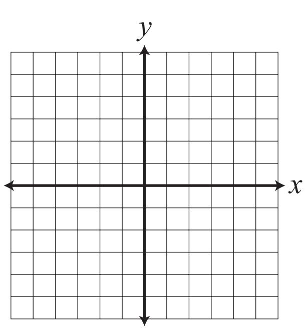
e) $(6, -4)$; $m = -\frac{3}{5}$



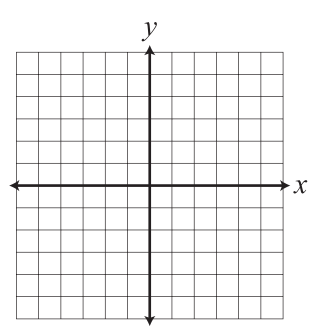
f) $(3, -5)$; $m = -\frac{2}{3}$



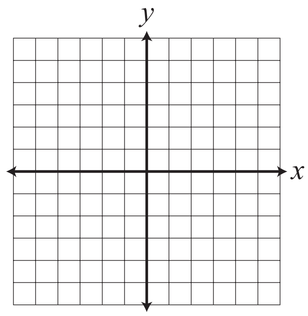
g) $(0, 0)$; $m = -3$



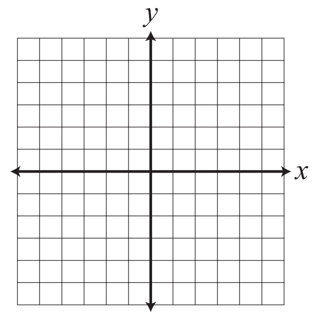
h) $(0, 0)$; $m = 4$



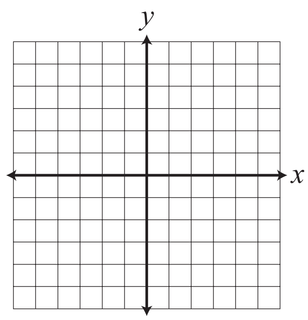
1. **i)** $(-5, -6)$; $m = \frac{7}{2}$



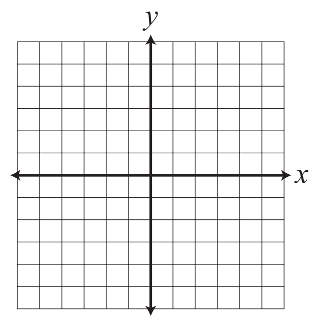
j) $(-4, 6)$; $m = -\frac{5}{2}$



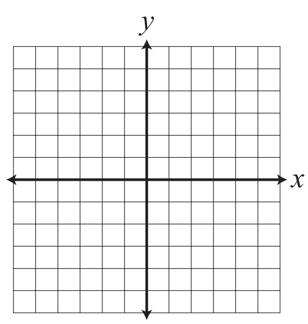
k) $(5, -5)$; $m = -3$



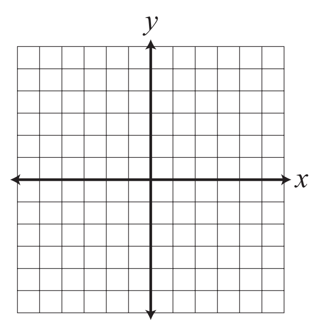
l) $(6, 3)$; $m = \frac{2}{3}$



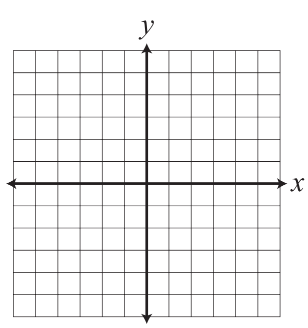
m) $(-4, 1)$; $m = 0$



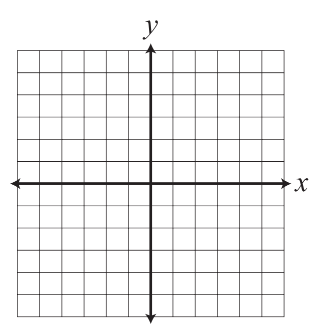
n) $(0, 2)$; $m =$ undefined



o) $(-2, -5)$; $m =$ undefined

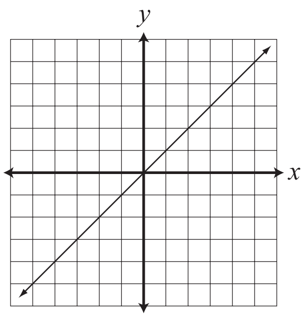


p) $(-4, -2)$; $m = 0$



2. Find the slope, and x and y intercepts, of each line.

a)

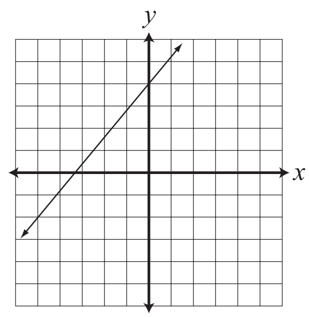


m: ______

y int: ______

x int: ______

b)

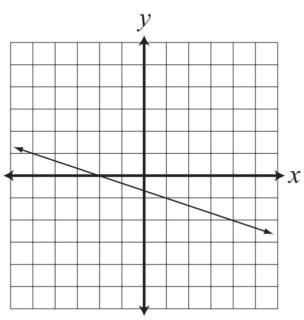


m: ______

y int: ______

x int: ______

c)

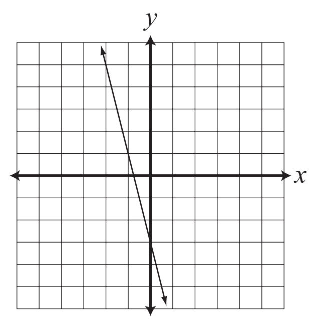


m: ______

y int: ______

x int: ______

d)

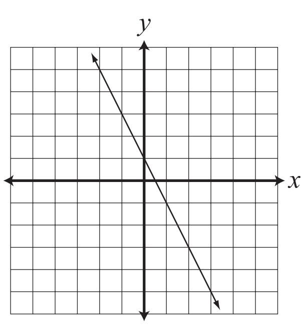


m: ______

y int: ______

x int: ______

e)

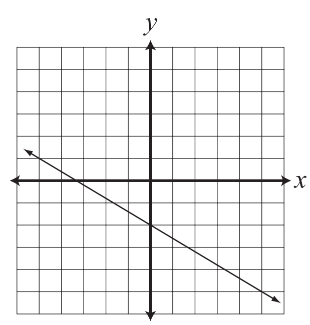


m: ______

y int: ______

x int: ______

f)

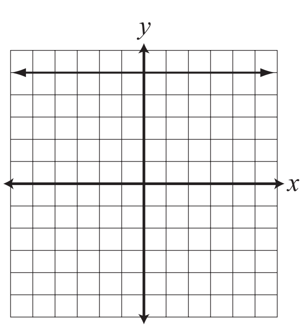


m: ______

y int: ______

x int: ______

g)

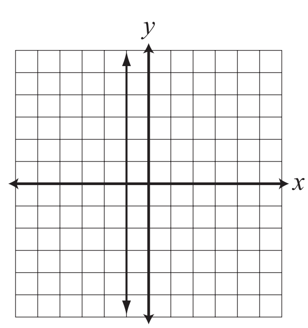


m: ______

y int: ______

x int: ______

h)



m: ______

y int: ______

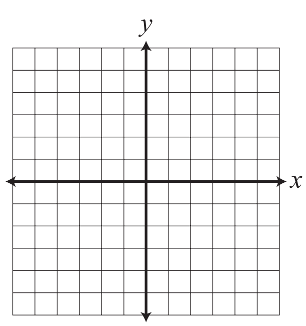
x int: ______

3. Match the phrase on the left with the appropriate choice from the column on the right.

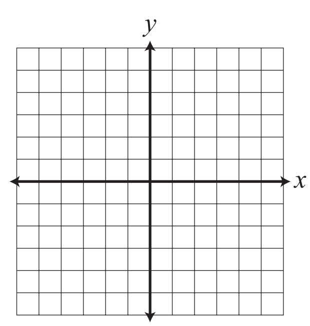
a)	The x-intercept of the graph $x - 3y = -6$	________	*i*) $\frac{1}{2}$
b)	The y-intercept of the graph of $x - 3y = -6$	________	*ii*) (0, 2)
c)	The slope of the graph of $x - 3y = -6$	________	*iii*) (0, − 2)
d)	The x-intercept of the graph of $x = 2y + 4$	________	*iv*) (− 6, 0)
e)	The y-intercept of the graph of $x = 2y + 4$	________	*v*) $\frac{1}{3}$
f)	The slope of the graph of $x = 2y + 4$	________	*vi*) (4, 0)
g)	The slope of the graph of $x = 4$	________	*vii*) zero
h)	The slope of the graph of $y - 2 = 0$	________	*viii*) $-\frac{3}{2}$
i)	The slope of x-intercept 2 and y-intercept -3	________	*ix*) undefined
j)	The slope of x-intercept -2 and y-intercept -3	________	*x*) $\frac{3}{2}$

4. Draw a line with the given slope and intercept.

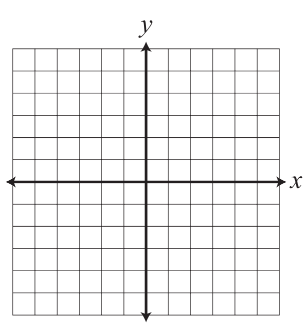
a) $m = 1$; x-intercept: $(-2, 0)$



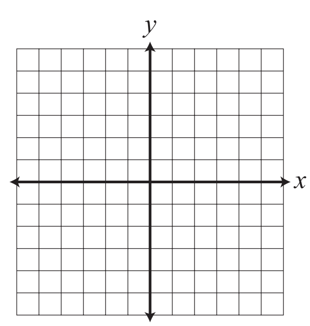
b) $m = -2$; y-intercept: $(0, -3)$



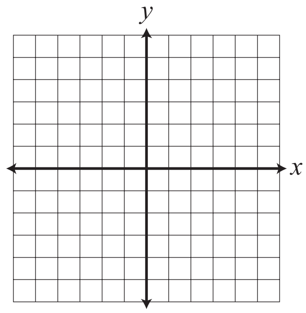
c) $m = -\frac{3}{4}$; x-intercept: $(3, 0)$



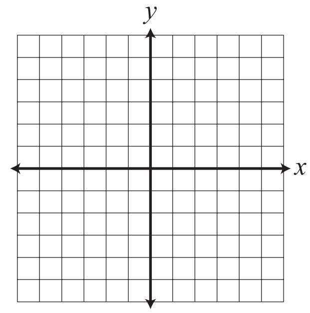
d) $m = \frac{3}{2}$; y-intercept: $(0, -1)$



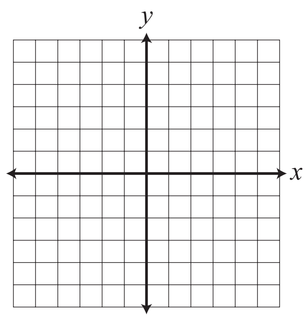
4. **e)** $m = \frac{3}{5}$; x-intercept: $(-4, 0)$



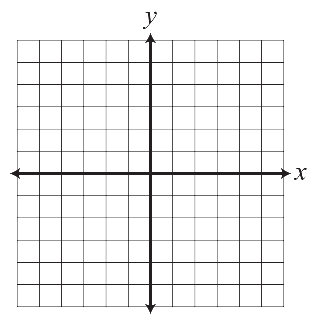
f) $m = -\frac{2}{3}$; y-intercept: $(0, 2)$



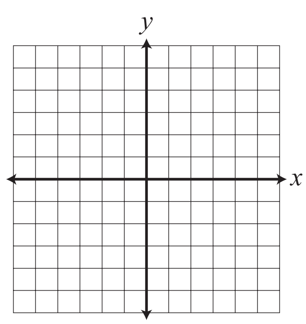
g) $m = -\frac{5}{3}$; x-intercept: $(1, 0)$



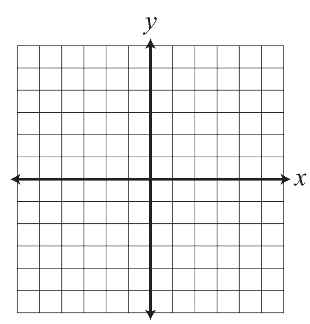
h) $m = \frac{3}{5}$; y-intercept: $(0, -2)$



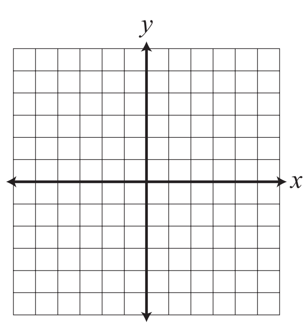
i) $m = -3$; x-intercept: $(0, 0)$



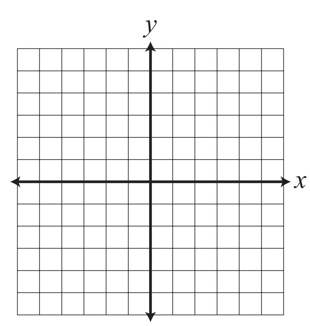
j) $m = 4$; y-intercept: $(0, 0)$



k) $m =$ undefined; x-intercept: $(-4, 0)$

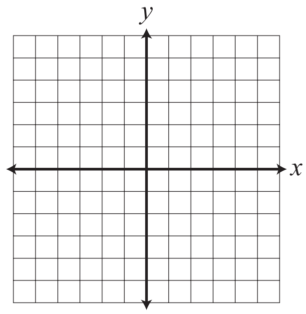


l) $m = 0$; y-intercept: $(0, -3)$

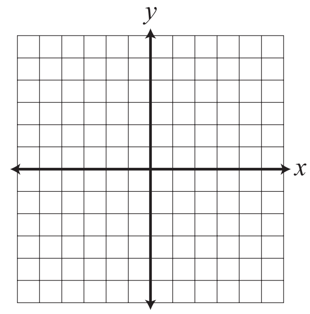


5. Determine (if they exist) the x and y-intercepts of the lines with the given slope, passing through the given point.

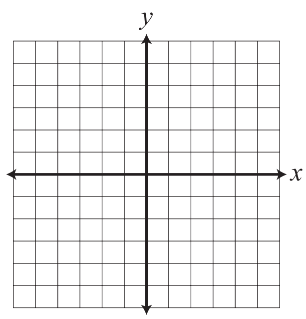
a) $m = 1;\ (0, 2)$



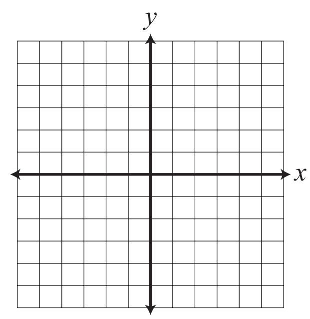
b) $m = 2;\ (-3, -1)$



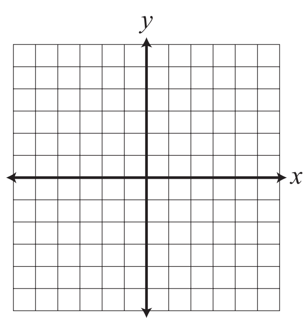
c) $m = \frac{3}{4};\ (4, 3)$



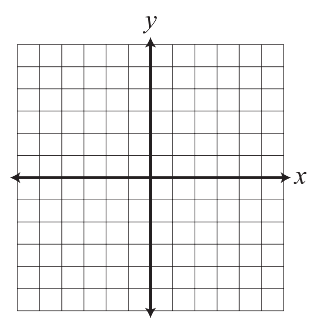
d) $m = -\frac{2}{3};\ (-3, 4)$



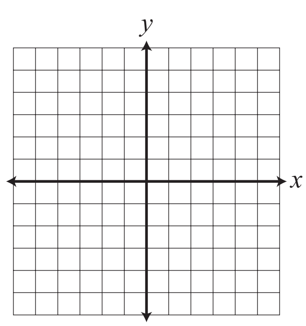
e) $m = -\frac{1}{2};\ (5, -4)$



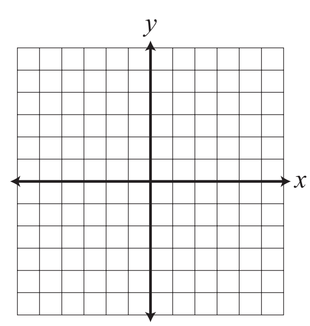
f) $m = \frac{2}{5};\ (-6, -4)$



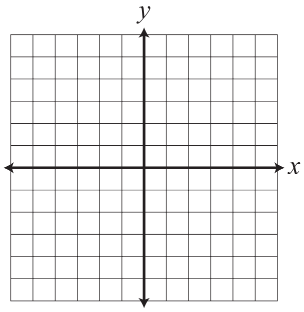
g) $m = -\frac{5}{3};\ (1, 2)$



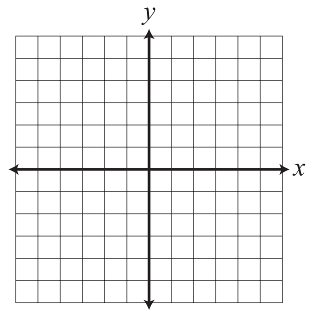
h) $m = \frac{5}{2};\ (-3, -4)$



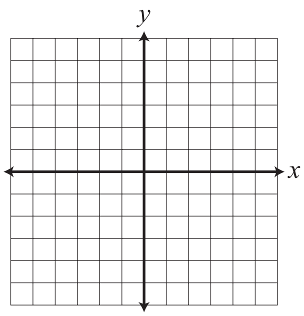
5. **i)** $m = -\frac{7}{3}$; $(-5, 6)$



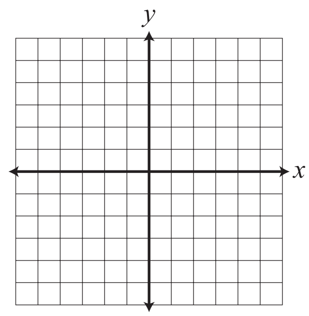
j) $m = \frac{7}{4}$; $(-3, -5)$



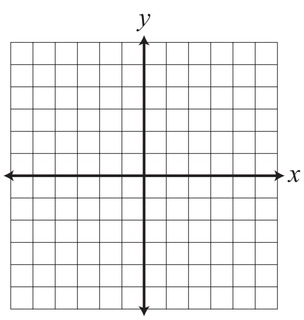
k) $m = 0$; $(-4, 1)$



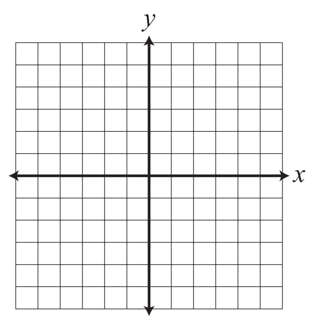
l) $m =$ undefined; $(-1, 2)$



m) $m =$ undefined; $(-2, -5)$



n) $m = 0$; $(-4, -2)$



6. Determine the slope of the line with the given x and y-intercept.

a) $(2, 0)$, $(0, 2)$ $m =$ ______

b) $(-2, 0)$, $(0, -2)$ $m =$ ______

c) $(2, 0)$, $(0, -2)$ $m =$ ______

d) $(-2, 0)$, $(0, 2)$ $m =$ ______

e) $(0, 3)$, $(0, 0)$ $m =$ ______

f) $(-4, 0)$, $(0, 0)$ $m =$ ______

7. Which is a greater slope: an incline that is 50%, or one with an incline of 100%?

8. A line passes through $(5, -6)$ and never enters the first quadrant. Between what values is the slope?

9 A line passes through $(-3, 4)$ and never enters the fourth quadrant. Between what values is the slope?

10. Find a number m so that the line passing through the point $(4, -2)$ and $(1, m)$ has slope -5.

11. Find a number n so that the line passing through the point $(n, 2)$ and $(-3, -1)$ has slope 4.

12. The line through the point $(x, 2)$ and $(-3, 6)$ has the same slope as a line through the point $(7, -3)$ and $(-1, 5)$. Determine x.

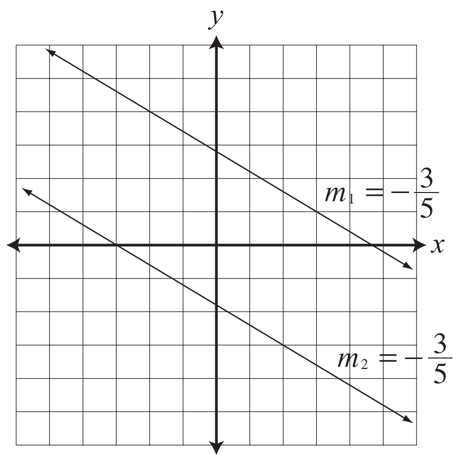
13. The line through the point $(5, -3)$ and $(4, y)$ has the same slope as a line through $(-2, 4)$ and $(1, 1)$. Determine y.

14. The line through (x, y) and $(x-2, 5)$ has a slope of -4. Determine y.

4.4 Parallel and Perpendicular Lines

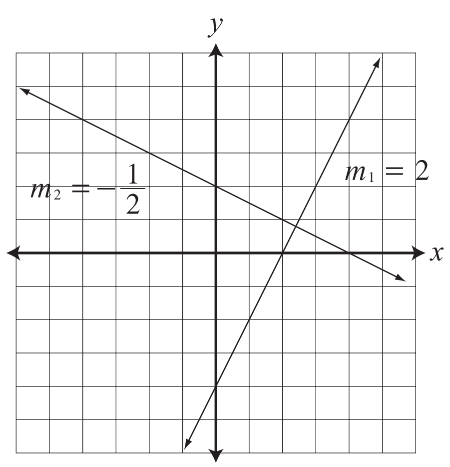
Parallel Lines

Parallel lines are lines in a coordinate system that never intersect. They have identical slopes because they rise or fall at the same rate.



Perpendicular Lines

Perpendicular lines are lines that form right angles when they intersect. If the slope of one line is $\frac{a}{b}$, the slope of a line that is perpendicular to it has slope $-\frac{b}{a}$. The product of the slopes of perpendicular lines is -1.



$$m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$$

Example 1 Determine if the line through the first pair of points is parallel to, perpendicular to, or neither parallel nor perpendicular to the line through the second pair of points.

a) $(-4, 1)$ and $(3, 5)$; $(1, -3)$ and $(15, -11)$
b) $(-4, 1)$ and $(3, 5)$; $(1, -11)$ and $(15, -3)$
c) $(-4, 1)$ and $(3, 5)$; $(-13, 10)$ and $(-9, 3)$

► *Solution*: **a)** $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-5}{-4-3} = \dfrac{4}{7}$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-(-11)}{1-15} = -\dfrac{4}{7}$

$m_1 \neq m_2$, therefore the lines are neither parallel nor perpendicular.

b) $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-5}{-4-3} = \dfrac{4}{7}$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-11-(-3)}{1-15} = \dfrac{4}{7}$

$m_1 = m_2$, therefore the lines are parallel.

c) $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-5}{-4-3} = \dfrac{4}{7}$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{10-3}{-13-(-9)} = -\dfrac{7}{4}$

$m_1 \times m_2 = -1$, therefore the lines are perpendicular.

Example 2 Determine if the line through the first pair of points is parallel to, perpendicular to, or neither parallel nor perpendicular to the line through the second pair of points.

a) $(x, 3)$ and $(3, x)$; $(7, x)$ and $(x, 7)$
b) $(-x, -2)$ and $(2, x)$; $(x, 8)$ and $(8, x)$

► *Solution*: **a)** $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{x-3}{3-x} = \dfrac{x-3}{-(x-3)} = -1$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-x}{x-7} = \dfrac{-(x-7)}{x-7} = -1$

$m_1 = m_2$, therefore the lines are parallel.

b) $m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{x-(-2)}{2-(-x)} = \dfrac{x+2}{2+x} = \dfrac{x+2}{x+2} = 1$

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{x-8}{8-x} = \dfrac{x-8}{-(x-8)} = -1$

$m_1 \times m_2 = -1$, therefore the lines are perpendicular.

4.4 Exercise Set

1. Determine if the slopes are parallel, perpendicular, or neither.

a) $m_1 = \frac{2}{3},\ m_2 = \frac{3}{2}$

b) $m_1 = 5,\ m_2 = -\frac{1}{5}$

c) $m_1 = \frac{6}{3},\ m_2 = 2$

d) $m_1 = 0,\ m_2 =$ undefined

e) $m_1 = -\frac{9}{6},\ m_2 = \frac{3}{2}$

f) $m_1 = 0,\ m_2 = 0$

g) $m_1 = \frac{15}{12},\ m_2 = \frac{4}{5}$

h) $m_1 = 4,\ m_2 = \frac{12}{3}$

i) $m_1 = \sqrt{8},\ m_2 = 2\sqrt{2}$

j) $m_1 = 0.125,\ m_2 = \frac{1}{8}$

k) $m_1 = -0.125,\ m_2 = 8$

l) $m_1 = ab^{-1},\quad m_2 = -a^{-1}b$

2. Complete the table.

Line l_1 has slope m_1, line l_2 has slope m_2, line l_3 has slope m_3, with $l_1 \parallel l_2$, and $l_1 \perp l_3$

m_1	m_2	m_3
$\frac{2}{3}$		
	$-\frac{3}{4}$	
		-4
undefined		
	undefined	
		undefined
0		
	0	
		0

3. Determine whether the line passing through the first pair of points is parallel, perpendicular, or neither to the line passing through the second pair of points.

a) $(3, 2)$ and $(1, 4)$; $(-1, -2)$ and $(-3, -4)$

b) $(5, 6)$ and $(7, 8)$; $(-5, -6)$ and $(-7, -8)$

c) $(0, 4)$ and $(-1, 2)$; $(-3, 5)$ and $(1, 7)$

d) $(2, 3)$ and $(3, 0)$; $(-2, -5)$ and $(1, -6)$

e) $(3, 5)$ and $(-2, 5)$; $(1, 4)$ and $(1, -2)$

f) $(4, -3)$ and $(-2, -1)$; $(10, -1)$ and $(1, -4)$

g) (a, b) and (b, a); (c, d) and (d, c)

h) (a, b) and (b, a); (c, d) and $\left(\frac{d}{2}, \frac{c}{2}\right)$

i) (a, b) and (b, a); $(-c, d)$ and $(-d, c)$

j) (a, b) and (b, a); (c, d) and $(-d, -c)$

4. Find the value of c so that the line passing through the points $(-2, c)$ and $(-c, 1)$ is parallel to the line passing through the points $(-5, c)$ and $(-c, 3)$.

5. Find the value of c so that the line through the points $(c, 1)$ and $(1, c)$ is perpendicular to the line through the points $(-2, c)$ and $(3, -4)$.

6. Show that the points A$(-3, 0)$, B$(1, 2)$ and C$(3, -2)$ are the vertices of a right triangle.

7. Show that the points A$(-3, 1)$, B$(-2, -2)$, C$(2, -1)$ and D$(1, 2)$ are the vertices of a parallelogram.

8. Show that the points A$(-1, -1)$, B$(3, 0)$, C$(2, 4)$ and D$(-2, 3)$ are the vertices of a square.

9. Show that the points A$(-1, -2)$, B$(4, 1)$, C$(3, 4)$ and D$(-3, 4)$ are the vertices of a trapezoid.

10. The line through $(2, y)$ and $(1, -4)$ is perpendicular to a line with slope $\frac{1}{2}$. Find y.

11. A line through $(3, y)$ and $(1, -2)$ is parallel to a line with slope -2. Find y.

12. The line through $(x, -6)$ and $(-2, -1)$ is perpendicular to a line with slope -2. Find x.

13. The line through $(x, 2)$ and $(3, 4)$ is parallel to a line with slope $-\frac{4}{3}$. Find x.

4.5 Applications of Linear Relations

This section will focus on problems involving intercepts, slope, and domain and range, of linear relations.

Example 1 A TV repair company charges a fixed amount, plus an hourly rate for a service call. A two hour service call is \$80, and a four hour service call is \$140.

a) Write the equation that shows how the total cost, T, depends on the number of hours, h, and the fixed cost, C. Use R for hourly rate.
b) Find the hourly rate.
c) Find the fixed amount cost.
d) Find the domain and range.

► *Solution*:

a) $T = Rh + C$

b) The hourly rate is the slope.

$$R = \frac{\Delta T}{\Delta h} = \frac{\text{change in total cost}}{\text{change in hours}} = \frac{T_4 - T_2}{h_4 - h_2} = \frac{(140 - 80)\$}{(4 - 2)\text{hr}} = \$30/\text{h}$$

c) The fixed cost will be when $h = 0$, total cost $T = 30h + C$.
For 2 hours, T, is \$80, therefore $C = T - 30h = 80 - 30(2) = 20$.
For 4 hours, T, is \$140, therefore $C = T - 30h = 140 - 30(4) = 20$.
The fixed cost is \$20.

d) The domain is {0, 1, 2, 3, ...}
The range is {20, 50, 80, 110, 140, ...}

Example 2 An antique dresser increases in value \$50 per year. The dresser is worth \$600 now.

a) Write the equation that shows how the current worth of the dresser, C, depends on the number of years, t.
b) What price was paid for the dresser if it was bought three years ago?
c) What will the value of the dresser be in five years?
d) Determine the domain and range.

► *Solution*:

a) $C = 600 + 50t$

b) $C = 600 + 50t = 600 + 50(-3) = 600 - 150 = 450$
The dresser initially cost \$450.

c) $C = 600 + 50t = 600 + 50(5) = 600 + 250 = 850$
The dresser will be worth \$850 in five years.

d) The domain is $\{-3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ ...\}$
The range is $\{450,\ 500,\ 550,\ 600,\ 650,\ ...\}$

4.5 Exercise Set

Assume all questions are linear relations.

1. An appliance repair shop charges an hourly rate plus a fixed amount. One hour costs \$60, and three hours costs \$140.

a) Determine the hourly rate.

b) Write the equation that shows how the total cost, C, depends on the number of hours, H, and the fixed cost, F.

c) Determine the fixed amount cost.

d) Determine the domain and range.

2. A four year old car is worth \$27 600, and will be worth \$4200 10 years from now.

a) Find the yearly depreciation of the car.

b) Write the equation that shows the value of the car, V, depends on the new cost of the car, N, and how many years old it is, Y.

c) Find the new price of the car.

d) Determine the domain and range.

3. A spring that is 24 inches long is compressed to 20 inches by a force of 16 pounds, and to 15 inches by a force of 36 pounds.

a) Determine the linear relation between the length of the spring, L, and the force, F.

b) What is the length of the spring if a force of 28 pounds is applied?

c) How much force is needed to compress the spring to 10 inches?

4. An apartment building is worth \$860 000. The owner expects the property to double its value over the next five years.

a) Find the yearly increase of the apartment.

b) What was the value of the apartment building three years ago, if the rate of increase is constant?

c) Find the domain and range.

5. A taxi driver charged a passenger \$21.50 to travel 15 km, and charged another passenger \$37.10 to travel 28 km.

a) Find the cost per km.

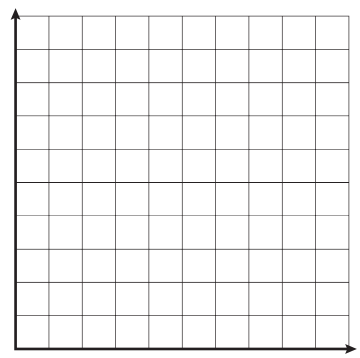
b) Write the equation that shows how the total cost, T, depends on the number of kilometres, K, plus a fixed amount.

c) How far can a person travel for \$53.90?

d) Determine the domain and range.

6. The cost of producing n toys is a fixed cost of \$600 plus \$12 per toy. The toys sell for \$15 each.

a) Graph the cost of producing each toy for $n = 0$ to 250. Also graph the selling price for $n = 0$ to 250 toys.



b) Find the minimum production of toys needed to break even.

7. Supply and demand is always related to price. Suppose demand for a video game is 4500 units when the price is \$30 per unit, and 3000 units when the price is \$40 per unit.

a) Determine a linear equation relating price to units sold.

b) Determine the demand if the price drops to \$25 per unit.

c) Determine the price per unit if 6000 units units are sold.

8. A manufacturer determines that the relationship between the income earned, I, and the number of items produced, p, is linear. Suppose the income is \$1600 on 50 items, and \$2500 on 65 items.

a) Find the income per each item.

b) Determine the fixed cost.

c) Find the profit on 100 items. (Profit = Income − Expense)

9. Nasir found that the relationship between the profit, P, which he made selling n paintings is linear. Suppose he made a profit of \$10 on five paintings, and \$90 on 15 paintings.

a) Find the profit on each painting.

b) Find the fixed cost of material.

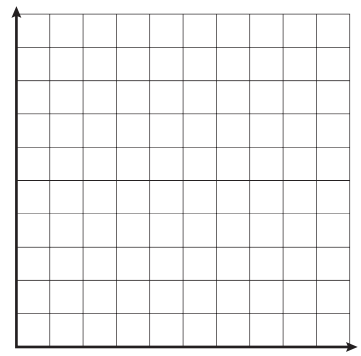
c) Find the profit on 35 paintings.

d) Find the domain and range.

10. Mel has two options when renting a car:

A: \$45 per day, no charge per kilometre.
B: \$36 per day, 12¢ per kilometre.

a) Graph rental options A and B.



b) How many kilometres must Mel travel to make option A the cheapest rate?

11. When the price to rent a movie was \$6, Sue rented two each month. When the price to rent a movie was reduced to \$4, she rented six movies each month.

a) If the number of movies Sue rents depends only on price, how many movies would she rent if the price to rent was lowered to \$2?

b) If Sue only rented one movie in a month because of the price, what would she have paid?

12. When dresses sell for \$80, five of them are sold each day. When the dresses are on sale for \$40, 13 dresses are sold each day.

a) For 10 dresses to be sold on a given day, what should be the cost per dress?

b) If the dresses were \$50 each, how many would the store expect to sell in a day?

4.6 Chapter Review

Section 4.1

1. Find the slope of the line containing each pair of points.

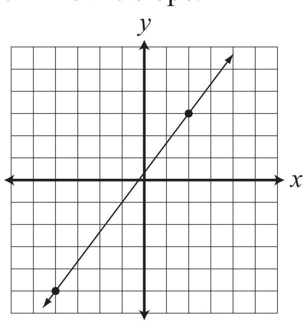
a) (− 2, 5) and (4, − 3) ________ **b)** (6, − 2) and (− 4, − 3) ________

c) (3, 1) and (− 4, 6) ________ **d)** (a, − b) and (− b, a) ________

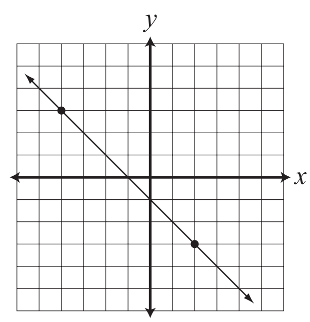
e) (− 3, 0) and (− 3, 4) ________ **f)** (4, − 1) and (− 2, − 1) ________

2. Determine the slope.

a)



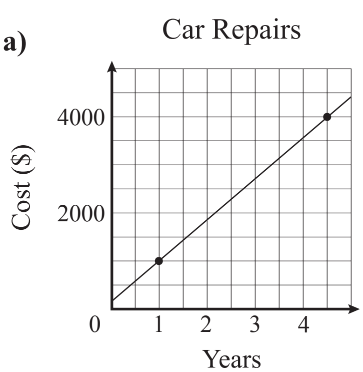
b)



Section 4.2

3. Determine the rate of change.

a)



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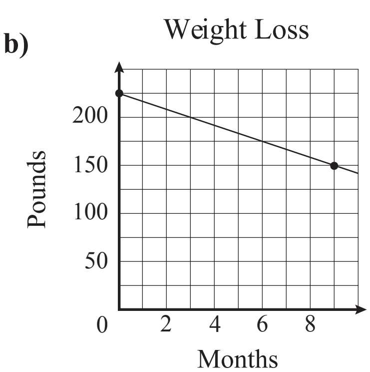


4. George rents a motor scooter for three hours to travel around Crescent Beach. It cost him \$36.00 for travelling 42 km.

   a) Determine George's average speed in km/h.

   b) Determine the rental rate in dollars per hour.

   c) Determine the rental rate in cents per km.

5. Marelee rented a stall at a craft market for four hours at a cost of \$120. She sold \$600 worth of pottery.

   a) Determine her rental cost per hour.

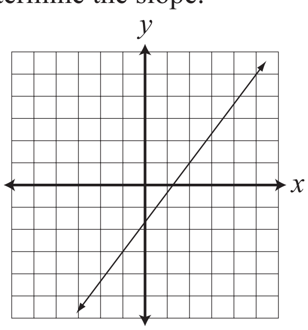
   b) Determine her average sales per hour.

   c) Determine her average profit per hour.

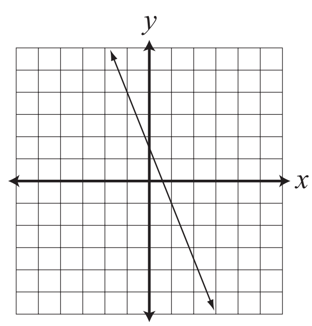
Section 4.3

6. Determine the slope.

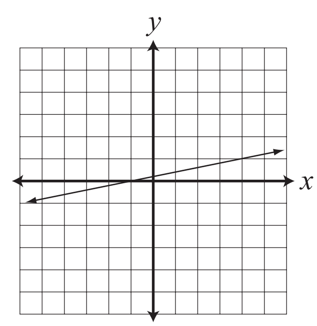
a)



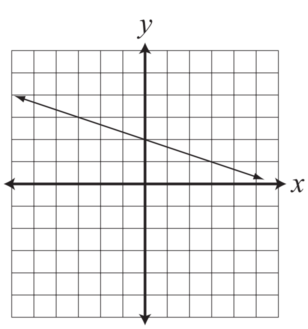
b)



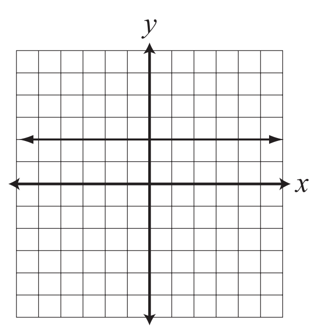
c)



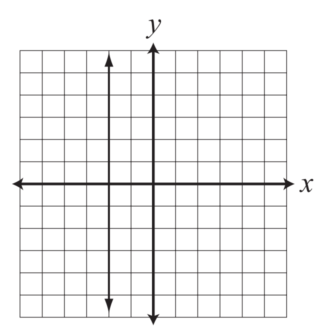
d)



e)



f)



7. Find the number n, so that the line passing through the point $(-3, 5)$ and $(-4, n)$ has slope 3.

8. The line through the point $(8, y)$ and $(2, -3)$ has a slope parallel to a line with x-intercept 3 and y-intercept -1. Determine y.

Section 4.4

9. Find the value of c so that the line through the points $(-2, -4)$ and $(-1, -1)$ is parallel to the line through the points $(6, -2)$ and $(3, c)$.

10. Find the value of c so that the line through the points $(-2, -4)$ and $(-1, -1)$ is perpendicular to the line through the points $(6, -2)$ and $(3, c)$.

11. Find the value of c so that the line through the points $(0, 3)$ and $(-1, 0)$ is parallel to the line through the points $(c, 1)$ and $(-2, 3)$.

12. Find the value of c so that the line through the points $(0, 3)$ and $(-1, 0)$ is perpendicular to the line through the points $(c, 1)$ and $(-2, 3)$.

Section 4.5

13. A television production company charges \$19 000 for seven hours of work and \$15 000 for five hours of work.

a) Find the hourly rate.

b) Find the fixed amount cost.

c) Find the domain and range.

14. Each semester at college, a student must pay tuition costs plus a student service fee. To take five courses in a semester costs \$3270, and to take four courses costs \$2640.

a) Find the cost per course.

b) Find the student service fee.

c) Find the domain and range.