5.1 Different Forms of Linear Equations

Standard Form of a Linear Equation

If A, B and C are real numbers, the equation $Ax + By = C$ is called the **standard form** of the equation of a line. Whenever possible, it is best to write the equation with A, B and C as integers, and $A \geq 0$.

For example: $-3x + y = 4$ can be expressed as $3x - y = -4$ ← *multiply each term by (− 1)*

$\frac{2}{3}x + 2y = 3$ can be expressed as $2x + 6y = 9$ ← *multiply each term by 3*

Slope - Intercept Form of a Linear Equation

The equation $y = mx + b$ is the **slope-intercept form** of the equation of a line. The y-intercept of the line is $(0, b)$, and the slope of the line is m.

The standard form of an equation of a line can be re-written in slope-intercept form as follows:

$$Ax + By = C \quad \rightarrow \quad By = -Ax + C \quad \rightarrow \quad y = -\frac{A}{B}x + \frac{C}{B}$$

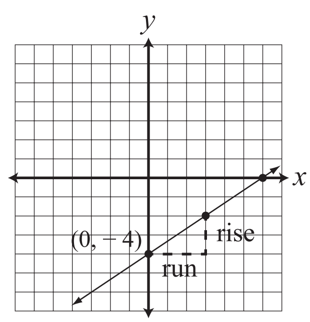
The slope of $Ax + By = C$ is $-\frac{A}{B}$

The y-intercept of $Ax + By = C$ is $\frac{C}{B}$, and the point at which the graph crosses the y-axis is $\left(0, \frac{C}{B}\right)$.

For example, consider the linear equation $2x - 3y = 12$. The slope intercept form of the line can be found in two ways:

$$\begin{aligned} 2x - 3y &= 12 \\ -3y &= -2x + 12 \\ y &= \frac{2}{3}x - 4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} m &= -\frac{A}{B} = -\frac{2}{-3} = \frac{2}{3} \\ y\text{-intercept} &= \frac{C}{B} = \frac{12}{-3} = -4 \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

The slope of the line is $\frac{2}{3}$, and the y-intercept is $(0, -4)$.



Graphing a Line Using the Slope and *y*-Intercept

Step 1: Write the equation in slope-intercept form by solving for y.
Step 2: Identify the y-intercept $(0, b)$ and graph this point.
Step 3: Graph another point using the slope, counting from the y-intercept.
Step 4: Draw the line connecting the two points to obtain the graph.

Example 1 Graph $3x + 2y = 12$ by using the slope and y-intercept.

► *Solution*: Step 1:

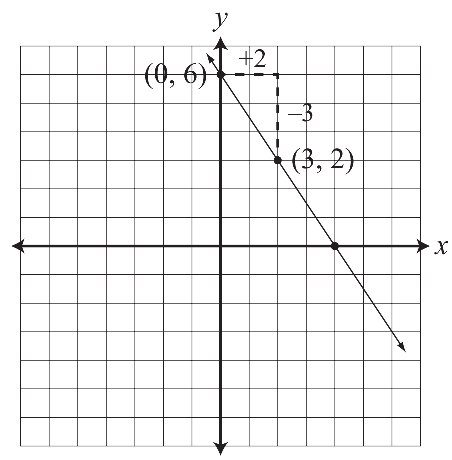
$$\begin{aligned} 3x + 2y &= 12 \\ 2y &= -3x + 12 \\ y &= -\frac{3}{2}x + 6 \end{aligned}$$

Step 2: The y-intercept is $(0, 6)$: mark this point.

Step 3: The slope is: $m = \dfrac{\text{rise}}{\text{run}} = -\dfrac{3}{2}$.

From $(0, 6)$, go to the right 2 units, and go down 3 units, to obtain the point $(3, 2)$.

Step 4: Draw the line through the points $(0, 6)$ and $(3, 2)$.

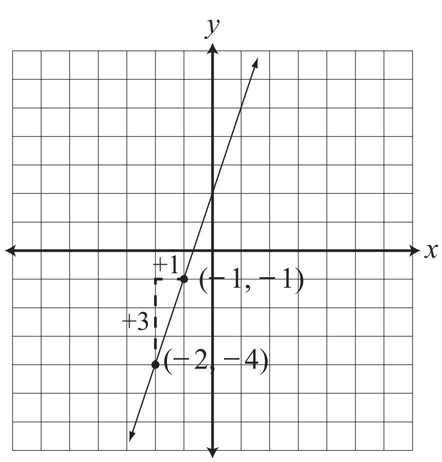


Graphing a Line Using the Slope and a Point

Step 1: Locate and graph the given point.
Step 2: Graph another point using the slope, counting from the first point.
Step 3: Draw a line connecting the two points to obtain the graph.

Example 2 Graph the line through $(-2, -4)$ with slope 3.

► *Solution*: The slope is 3, therefore, from the point $(-2, -4)$, go up 3 units, and to the right 1 unit to obtain the point $(-1, -1)$.



Writing an Equation of a Line Using a Slope and a Point

By substituting given values for a slope and point of a line into $y = mx + b$, the line's equation can be found.

Example 3 Write the equation of the line with slope 2 that runs through (− 4, 1) in slope intercept-form.

► *Solution*: The point (− 4, 1) gives a x-value of -4 and a y-value of 1.

$$y = mx + b \quad \rightarrow \quad 1 = 2(-4) + b$$
$$1 = -8 + b$$
$$b = 9$$

Therefore, the equation of the line is $y = 2x + 9$.

Point - Slope Form of a Linear Equation

The equation $y - y_1 = m(x - x_1)$ is the **point-slope** equation of a line. The given point is $(x_1,\ y_1)$ and the slope of the line is m. This formula comes from re-arranging the definition of slope, $m = \dfrac{y - y_1}{x - x_1}$.

Example 4 Write the equation of a line with slope 2 that passes through (− 4, 1) in slope intercept form.

► *Solution*: Substituting the given point and slope into the point-slope equation gives:

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 1 = 2(x - (-4))$$
$$y - 1 = 2(x + 4)$$
$$y - 1 = 2x + 8$$
$$y = 2x + 9$$

Example 5 Write the equation of a line with slope $\frac{4}{5}$ that passes through (3, − 2) in standard form.

► *Solution*: Substituting the given point and slope into the point-slope equation gives:

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - (-2) = \frac{4}{5}(x - 3)$$
$$y + 2 = \frac{4}{5}(x - 3)$$
$$5(y + 2) = 4(x - 3)$$
$$5y + 10 = 4x - 12$$
$$4x - 5y = 22$$

5.1 Exercise Set

1. Complete each statement.

a) The formula for the point-slope form of a line is ______________.

b) In the equation $y = mx + b$, (0, b) is called the ______________.

c) The equation $y = mx + b$ is called the ______________ form of the equation of a line.

d) The standard form of the equation of a line is ______________.

e) The slope of $Ax + By = C$ is ______________.

f) The y-intercept of $Ax + By = C$ is ______________.

2. Find the slope and y-intercept.

a) $3x - 2y = 6$ slope ________ y-intercept ________

b) $4x + 3y = 12$ slope ________ y-intercept ________

c) $2x - 5y = -7$ slope ________ y-intercept ________

d) $5x + 2y = 0$ slope ________ y-intercept ________

e) $x - 4y = -4$ slope ________ y-intercept ________

f) $6x - y = -3$ slope ________ y-intercept ________

3. Rewrite the standard form equation in slope-intercept form.

a) $2x + y = 6$

b) $3x - y = 4$

c) $4x + 3y = 12$

d) $2x - 3y = 6$

e) $5x + 4y = 3$

f) $6x - 3y = 4$

4. Rewrite the slope-intercept equation in standard form.

a) $y = -2x + 1$

b) $y = 3x - 1$

c) $y = 3x$

d) $y = -\frac{2}{3}x + 1$

e) $y = \frac{3}{4}x + 5$

f) $y = -\frac{2}{5}x + \frac{1}{2}$

5. Rewrite the point-slope equation in slope-intercept form.

a) $y - 2 = 3(x + 1)$

b) $y + 4 = -2(x - 1)$

c) $y - 1 = \frac{1}{3}(x + 2)$

d) $y + 4 = -\frac{2}{5}(x - 3)$

e) $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

f) $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

6. Rewrite the point-slope equation in standard form.

a) $y - 2 = 3(x + 1)$

b) $y + 4 = -2(x - 1)$

c) $y - 1 = \frac{1}{3}(x + 2)$

d) $y + 4 = -\frac{2}{5}(x - 3)$

e) $y - \frac{2}{3} = \frac{1}{4}(x - 8)$

f) $y - \frac{1}{4} = \frac{1}{2}\left(x + \frac{2}{3}\right)$

7. Match each description with an equation.

a) Slope $= -3$, passing through $(-1, 2)$ _________

b) Slope $= 3$, y-intercept $(0, -6)$ _________

c) Passing through $(0, 0)$ and $(3, -1)$ _________

d) Passing through $(0, 0)$ and $(-1, 3)$ _________

e) Passing through $(2, 0)$ and $(0, -6)$ _________

i) $y = 3x$

ii) $y = -\frac{1}{3}x$

iii) $y = -3x$

iv) $x - 3y = 6$

v) $3x - y = 6$

vi) $y - 2 = -3(x + 1)$

vii) $y + 2 = -3(x - 1)$

8. Match each equation with the graph it most closely resembles.

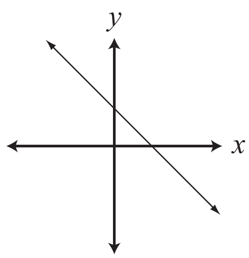
a) $y = x - 2$ _______

b) $y = -x - 2$ _______

c) $y = -x + 2$ _______

d) $y = x + 2$ _______

i)



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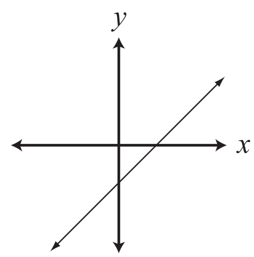

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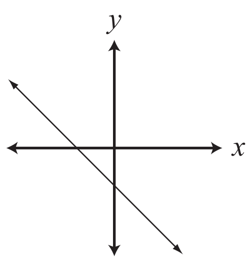

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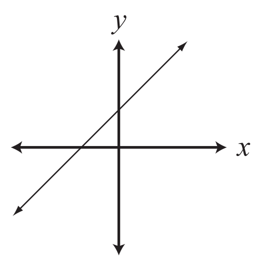


9. Write the equation of each line in slope-intercept form.

a) $(0, 2)$, $m = 2$

b) $(0, -3)$, $m = \frac{1}{2}$

c) $(0, 3)$, $m = 0$

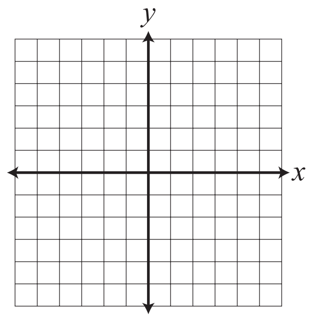
d) $(0, -2)$, $m = -\frac{2}{3}$

e) $\left(0, -\frac{1}{2}\right)$, $m = -\frac{3}{4}$

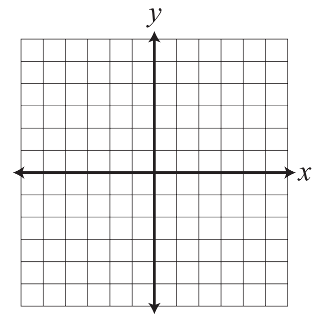
f) $(0, 2.3)$, $m = 0.4$

10. Graph the linear equation.

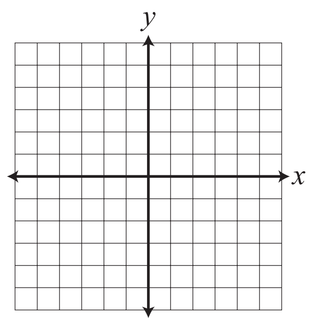
a) $4x - 3y = 12$



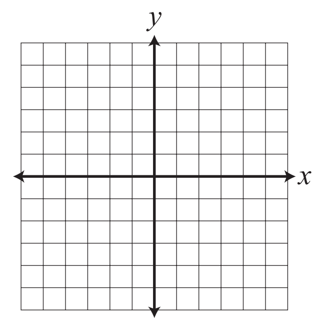
b) $y = -\frac{2}{3}x + 4$



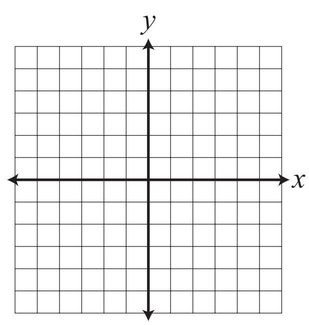
c) $y - 3 = \frac{1}{2}(x + 4)$



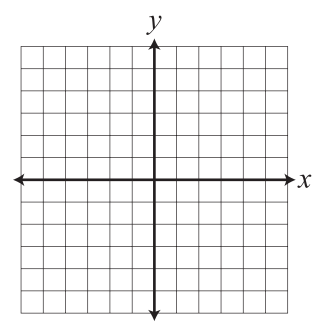
d) $2x + 3y = 10$



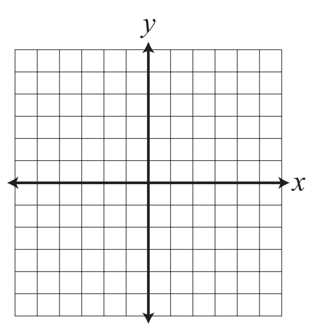
e) $y + 2 = -\frac{2}{3}(x + 5)$



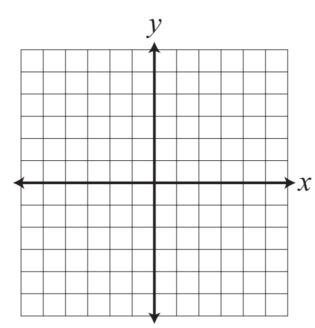
f) $5x - 2y = 0$



g) $y - \frac{5}{2} = -\frac{1}{2}\left(x + \frac{3}{2}\right)$

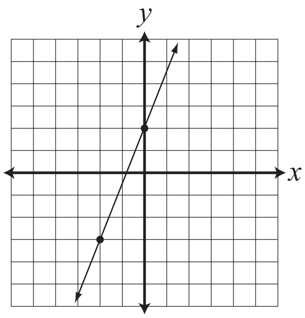


h) $y = \frac{5}{3}x - \frac{7}{2}$



11. Write the equation in standard form, slope-intercept form, and point-slope form.

a)

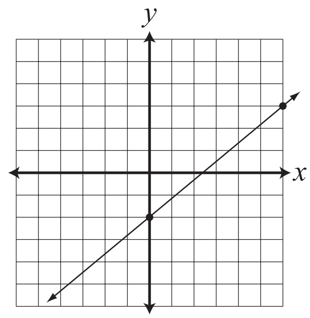


standard form ______________

slope-intercept form ______________

point-slope form ______________

b)

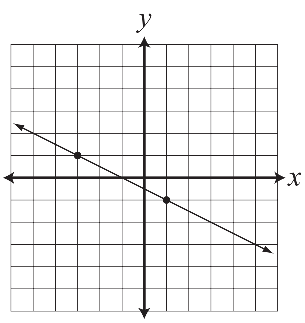


standard form ______________

slope-intercept form ______________

point-slope form ______________

c)

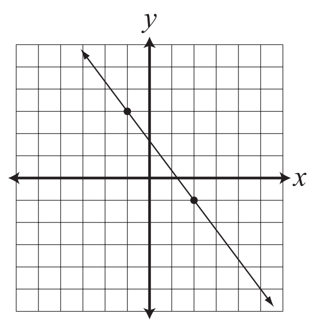


standard form ______________

slope-intercept form ______________

point-slope form ______________

d)

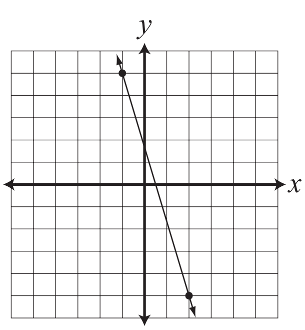


standard form ______________

slope-intercept form ______________

point-slope form ______________

e)

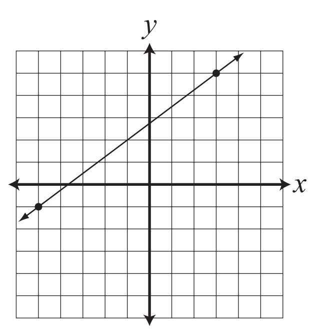


standard form ______________

slope-intercept form ______________

point-slope form ______________

f)



standard form ______________

slope-intercept form ______________

point-slope form ______________

5.2 Special Cases of Linear Equations

Horizontal Lines

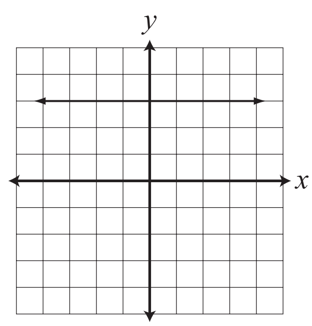
A horizontal line can be thought of as all the points on a graph where y has the same value. From section 5.1, it was shown that the slope of a horizontal line is 0.

Using a slope of 0 in the slope-intercept equation of a line, $y = mx + b \;\rightarrow\; y = 0 \cdot x + b \;\rightarrow\; y = b$

Equation of a Horizontal Line with *y*-Intercept *k*

$y = k$

For example: $y = 3$



Vertical Lines

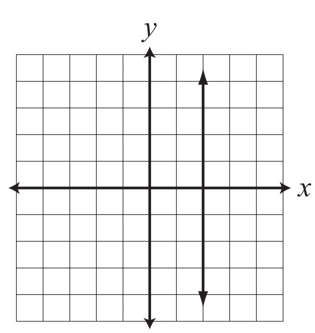
A vertical line can be thought of as all the points on a graph where x has the same value. From section 5.1 it was shown that the slope of a vertical line is undefined.

The equation of a vertical line is $x = k$ by definition, since the slope is undefined.

Equation of a Vertical Line with *x*-Intercept *k*

$x = k$

For example: $x = 3$



Writing the Equation of a Line Through Two Points

With our knowledge from section 6.1 it is possible to write the equation of a line when the coordinates of two points on the line are known.

Example 1 Write the equation of the line passing through A(5, 2) and B(1, − 4) in slope-intercept form.

► *Solution*: First, find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{5-1} = \frac{6}{4} = \frac{3}{2}$$

Pick either point, and substitute it into the point-slope form equation. For this example, A(5, 2) is used.

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 2 = \frac{3}{2}(x-5)$$

$$y - 2 = \frac{3}{2}x - \frac{15}{2}$$

$$y = \frac{3}{2}x - \frac{15}{2} + 2$$

$$y = \frac{3}{2}x - \frac{11}{2}$$

Parallel and Perpendicular Lines

In chapter 5 it was shown that parallel lines have the same slope but different y-intercepts and perpendicular lines have slopes that are negative reciprocals of each other. We can now use these concepts to determine if equations are parallel, perpendicular or neither.

Example 2 In the system of equations $\begin{Bmatrix} x + 2y = 6 \\ -2x + y = 3 \end{Bmatrix}$, determine if the lines are parallel, perpendicular, or neither.

► *Solution*: The slope of the standard form of the equation of a line, $Ax + By = C$, is $-\frac{A}{B}$.

$x + 2y = 6$ has slope $-\frac{1}{2}$

$-2x + y = 3$ has slope 2

The slopes are negative reciprocals of each other, therefore the lines are perpendicular.

Example 3 In the system of equations $\begin{Bmatrix} 3x - y = 5 \\ -6x + 2y = 12 \end{Bmatrix}$, determine if the lines are parallel, perpendicular, or neither.

► *Solution*: This problem can be solved by changing both equations to slope-intercept form.

$$\begin{aligned} 3x - y &= 5 \\ -y &= -3x + 5 \\ y &= 3x - 5,\ m = 3 \end{aligned} \qquad \begin{aligned} -6x + 2y &= 12 \\ 2y &= 6x + 12 \\ y &= 3x + 6,\ m = 3 \end{aligned}$$

The slopes are equal, therefore the lines are parallel.

Example 4 In the system of equations $\begin{Bmatrix} 4x + 3y = 7 \\ 2x - y = 4 \end{Bmatrix}$, determine if the lines are parallel, perpendicular, or neither.

► *Solution*: Leaving the system of equations in standard form:

$4x + 3y = 7$ has slope $m = -\dfrac{A}{B} = -\dfrac{4}{3}$

$2x - y = 4$ has slope $m = -\dfrac{A}{B} = -\dfrac{2}{-1} = 2$

Changing the system of equations to slope-intercept form:

$$\begin{aligned} 4x + 3y &= 7 \\ 3y &= -4x + 7 \\ y &= -\frac{4}{3}x + \frac{7}{3},\ m = -\frac{4}{3} \end{aligned} \qquad \begin{aligned} 2x - y &= 4 \\ -y &= -2x + 4 \\ y &= 2x - 4,\ m = 2 \end{aligned}$$

Both methods produce the same result: the slopes are neither the same, nor negative reciprocals, therefore the lines are neither parallel nor perpendicular.

5.2 Exercise Set

1. Match the graph $y = mx + b$ with its closest description.

a) $m < 0$, $b < 0$ ______

b) $m > 0$, $b < 0$ ______

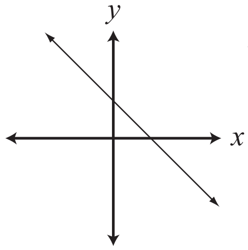
c) $m < 0$, $b > 0$ ______

d) $m > 0$, $b > 0$ ______

e) $m = 0$ ______

f) $b = 0$ ______

i)



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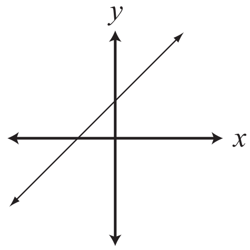

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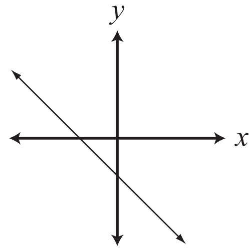

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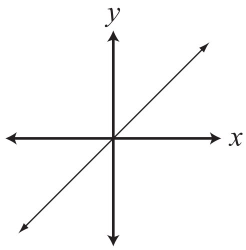

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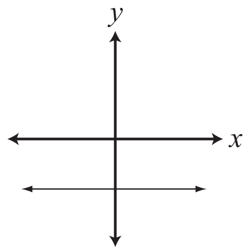

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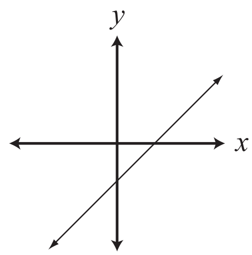


2. Match the graph with the linear relation.

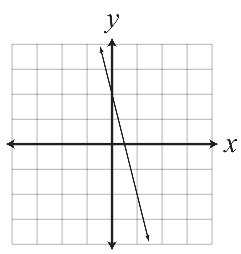
a) $x - 4y = -8$ ______

b) $x + 4y = 8$ ______

c) $4x - y = -2$ ______

d) $4x + y = 2$ ______

i)



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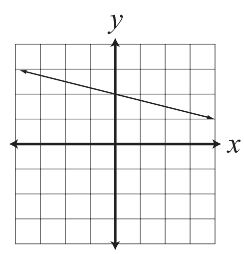

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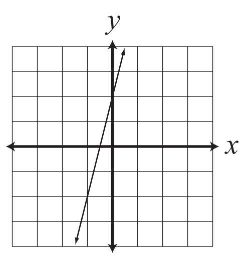

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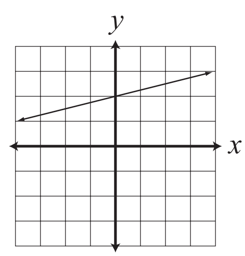
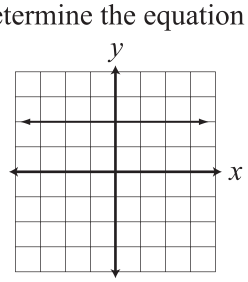
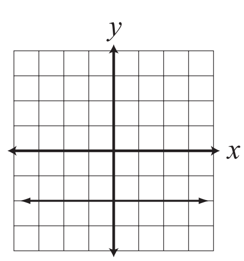


3. Determine the equation of the graph.

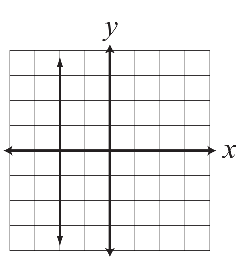
a)



b)



c)



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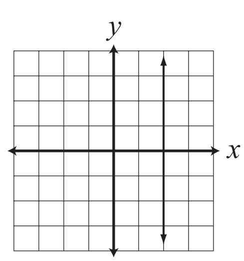


4. Determine the equation of a line through the given pair of points.

a) $(-4, 1)$ and $(6, 1)$

b) $(1, -4)$ and $(1, 6)$

c) $(-2, 0)$ and $(5, 0)$

d) $(0, -2)$ and $(0, 5)$

e) (a, b) and (c, b)

f) (b, a) and (b, c)

5. Write the equation of the line with the given characteristics.

a) vertical, passes through $(3, 6)$

b) vertical, passes through $(-2, -4)$

c) horizontal, passes through $(3, 6)$

d) horizontal, passes through $(-2, -4)$

6. For each pair of equations, determine whether the lines are parallel, perpendicular, or neither parallel nor perpendicular.

a) $2x + 5y = 7$
$4x + 10y = 2$

b) $-4x + 3y = 7$
$-8x + 6y = 0$

c) $4x - 3y = 6$
$4x + 6y = -3$

d) $3x - 5y = 4$
$5x - 3y = 4$

e) $4x - 3y = 5$
$3x + 4y = 2$

f) $2x - 5y = -3$
$10x + 4y = 1$

g) $4x - y = 3$
$x - 4y = -2$

h) $5x - 2y = 7$
$2x + 5y = 7$

7. Write the equation of a line passing through the given set of points in slope-intercept form.

a) $(3, 5)$ and $(2, 4)$

b) $(5, -2)$ and $(-3, 1)$

c) $(-4, 1)$ and $(-2, -3)$

d) $(-1, -2)$ and $(-6, -4)$

e) $(6, -2)$ and $(-3, 2)$

f) $(0, 0)$ and $(-3, 2)$

g) $(0, -6)$ and $(-4, 0)$

h) $(5.2, -6.8)$ and $(-1.6, -3.8)$

i) $(2, 5)$ and $(-2, 5)$

j) $(3, 7)$ and $(3, -1)$

8. Reasoning.

a) If a line is horizontal, what is the slope of any line that is perpendicular to it?

b) If the graph of a linear equation has one point that is both the x-intercept and y-intercept, what point that be?

c) What is the equation of the x-axis?

d) Find the x-intercept of $3x - 2y = 8$.

e) Find the value of c so that the graph of $4x + c = 3y$ has an x-intercept of $(-2, 0)$.

f) Find the y-intercept of $4x = -3y + 2$.

g) Find the value of c so that the graph of $3x - c = 2y$ has a y-intercept of $(0, 5)$.

h) If A is not zero, what will the graph of $Ax + C = D$ look like?

i) If B is not zero, what will the graph of $By + E = F$ look like?

j) What is the equation of a line with x and y coordinates that are equal, and passes through the origin?

k) What is the equation of a line with x and y coordinates that are opposite in value and passes through the origin?

l) What is the equation of a line passing through the point (a, b) with slope 0?

m) What is the equation of a line passing through the point (a, b) with an undefined slope?

n) What is the y-intercept of $ax + by = c$?

o) What is the x-intercept of $ax + by = c$?

p) What is the slope of the line $ax + by = c$?

9. Find the x and y intercepts of the line $ax + by = ab$.

10. Show that the equation of a line with x-intercept $(a, 0)$ and y-intercept $(0, b)$ can be written in the form $\frac{x}{a} + \frac{y}{b} = 1$.

11. Show that the equation of a line with points $(x_1,\ y_1)$ and $(x_2,\ y_2)$ can be written in the form $(y - y_1)(x_2 - x_1) = (y_2 - y_1)(x - x_1)$.

12. Determine the relationship between the graphs of the equations $Ax + By = C$ and $Bx - Ay = C$.

13. If the two points that a line passes through are known, its equation can be found. Explain how this is done.

14. Think of different points on the graph of the horizontal line $y = 2$. What do the points have in common? How do they differ?

15. What is the slope of all ordered pairs of the form $(x, -3x)$?

16. Given that 0°C is the same temperature as 32°F, and 100°C is equivalent to 212°F, determine the equivalent of 68°F in $^\circ$C.

17. In the equation, $ax + by = 2x - 3y + 6$, find a and b if the graph is a horizontal line passing through $(0, 3)$.

18. In the equation, $ax + by = 2x - 3y + 6$, find a and b if the graph is a vertical line passing through $(3, 0)$.

5.3 Equations of Parallel and Perpendicular Lines

To write the equation of a line, a point and slope is needed. However, in some problems this information is not directly given, and further steps must be taken to find a point or slope.

When determining the equation of a line that is parallel to a given slope, the concept to remember is that parallel lines have equal slopes. When determining the equation of a line that is perpendicular to a given slope, the concept to remember is that perpendicular lines have slopes that are negative reciprocals of each other.

Example 1 Write the equation of a line parallel to $3x - 2y = 6$, and which goes through the point A(4, − 2).

► *Solution*: $3x - 2y = 6$ has slope $m = -\frac{A}{B} = -\frac{3}{-2} = \frac{3}{2}$

Therefore, the slope of a line parallel to $3x - 2y = 6$ has slope $m = \frac{3}{2}$

Substituting the given point and slope into the point-slope equation of a line gives:

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - (-2) = \frac{3}{2}(x - 4)$$

$$y + 2 = \frac{3}{2}x - 6$$

$$y = \frac{3}{2}x - 8 \qquad (\textit{slope-intercept form})$$

$$3x - 2y = 16 \qquad (\textit{standard form})$$

Example 2 Write the equation of a line perpendicular to $4x + 2y = 7$ going through the point B(− 2, 5).

► *Solution*: $4x + 2y = 7$ has slope $m = -\frac{A}{B} = -\frac{4}{2} = -2$

Therefore the slope of a line perpendicular to $4x + 2y = 7$ has slope $m = \frac{1}{2}$.

Substituting the given point and slope into the point-slope equation of a line gives:

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 5 = \frac{1}{2}(x - (-2))$$

$$y - 5 = \frac{1}{2}(x + 2)$$

$$y - 5 = \frac{1}{2}x + 1$$

$$y = \frac{1}{2}x + 6 \qquad (\textit{slope-intercept form})$$

$$x - 2y = 12 \qquad (\textit{standard form})$$

5.3 Exercise Set

1. Find the slopes of lines parallel and perpendicular to the equation.

a) $y = 3x$

$m_{\parallel}$ _______

$m_{\perp}$ _______

b) $y = -2x$

$m_{\parallel}$ _______

$m_{\perp}$ _______

c) $y = -\frac{2}{3}x + 2$

$m_{\parallel}$ _______

$m_{\perp}$ _______

d) $y = \frac{3}{5}x - 1$

$m_{\parallel}$ _______

$m_{\perp}$ _______

e) $2x - 3y = 4$

$m_{\parallel}$ _______

$m_{\perp}$ _______

f) $3x + y = 2$

$m_{\parallel}$ _______

$m_{\perp}$ _______

g) $5x - y = 0$

$m_{\parallel}$ _______

$m_{\perp}$ _______

h) $x = 2$

$m_{\parallel}$ _______

$m_{\perp}$ _______

i) $y = -2$

$m_{\parallel}$ _______

$m_{\perp}$ _______

j) $x = 2y - 1$

$m_{\parallel}$ _______

$m_{\perp}$ _______

k) $\frac{3}{4}x = \frac{1}{3}y + \frac{1}{2}$

$m_{\parallel}$ _______

$m_{\perp}$ _______

l) $0.2x + 2.3 = 1.4y$

$m_{\parallel}$ _______

$m_{\perp}$ _______

2. Find the equation of the line, in standard form, that passes through the given point and is parallel to the given line.

a) $P(0, 0)$; $y = 2x - 5$

b) $P(0, 0)$; $x = 2y + 5$

c) $P(1, 3)$; $3x - y = 6$

d) $P(-2, 0)$; $2x + 5y = 3$

e) $P(-6, 3)$; $y + 4x = -8$

f) $P(5, -2)$; $3y + 1 = -4x$

g) $P(-4, -3)$; $x = \frac{3}{4}y - 2$

h) $P(0, -5)$; $x = -\frac{2}{3}y + 1$

i) $P(-5, 2)$; $x = 3$

j) $P(-5, 2)$; $y = -4$

k) $P(-4, 1)$; $\frac{2}{3}x + \frac{3}{4}y = 12$

l) $P\left(\frac{1}{2}, -\frac{2}{3}\right)$; $\frac{1}{3}x - 0.4y = 2$

3. Find the equation of the line, in standard form, that passes through the given point and is perpendicular to the given line.

a) $P(0, 0)$; $y = 2x - 5$

b) $P(0, 0)$; $x = 2y + 5$

c) $P(1, 3)$; $3x - y = 6$

d) $P(-2, 0)$; $2x + 5y = 3$

e) $P(-6, 3)$; $y + 4x = -8$

f) $P(5, -2)$; $3y + 1 = -4x$

g) $P(-4, -3)$; $x = \frac{3}{4}y - 2$

h) $P(0, -5)$; $x = -\frac{2}{3}y + 1$

i) $P(-5, 2)$; $x = 3$

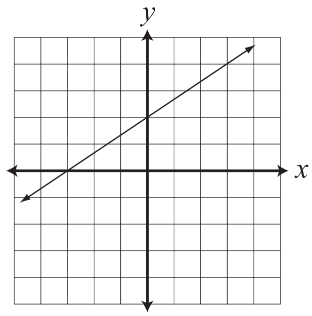
j) $P(-5, 2)$; $y = -4$

k) $P(-4, 1)$; $\frac{2}{3}x + \frac{3}{4}y = 12$

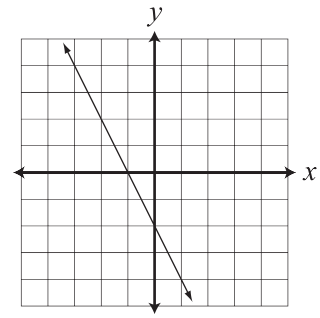
l) $P\left(\frac{1}{2}, -\frac{2}{3}\right)$; $\frac{1}{3}x - 0.4y = 2$

4. Determine the equation of a line parallel to the graph going through the given point, in standard form.

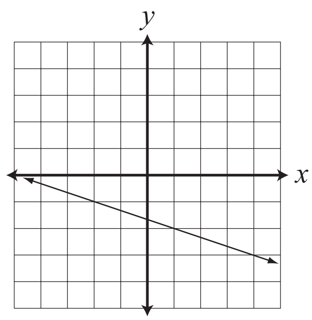
a) (4, − 2)



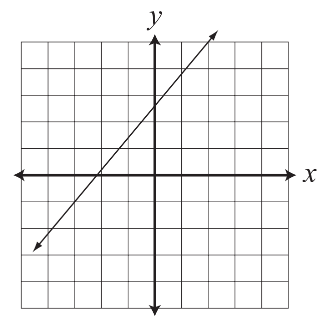
b) (− 5, − 4)



c) (2, 1)

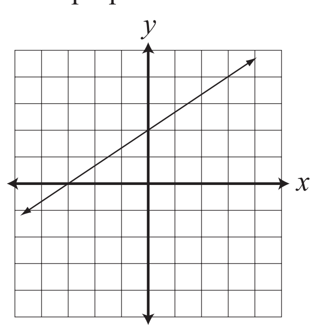


d) (5, − 3)

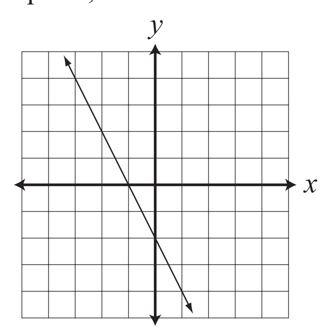


5. Determine the equation of a line perpendicular to the graph going through the given point, in standard form.

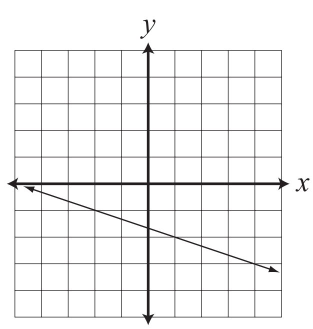
a) (4, − 2)



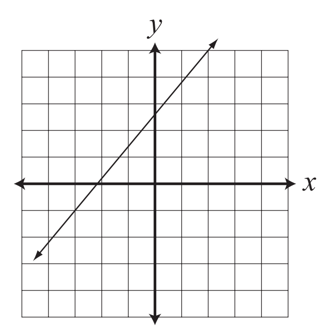
b) (− 5, − 4)



c) (2, 1)



d) (5, − 3)



6. Find the equation of a line parallel to $3x + 4y = 8$ with the same y-intercept as $5x - 3y = 10$.

7. Find the equation of a line parallel to $x - 3y = 8$ with the same y-intercept as $3x + 2y = 6$.

8. Find the equation of a line parallel to $2x + 7y = 10$ with the same x-intercept as $3x - 4y = 5$.

9. Find the equation of a line perpendicular to $2x - 3y = 7$ with the same y-intercept as $5x - 2y = 10$.

10. Find the equation of a line perpendicular to $3x + 2y = 9$ with the same x-intercept as $2x - 5y = 0$.

11. Find the equation of a line perpendicular to $\frac{3}{2}x = \frac{1}{2}y + 1$ with the same x-intercept as $2x + 3y = 9$.

12. A circle centred at the origin passes through the point $(-3, 4)$. What is the equation of a line perpendicular to the radius at this point?

13. A rhombus has coordinates $(0, 0)$, $(3, 4)$, $(8, 4)$, and $(5, 0)$. What are the equations of the diagonals of the rhombus? What relationship is there between the diagonals?

5.4 Linear Applications and Modelling

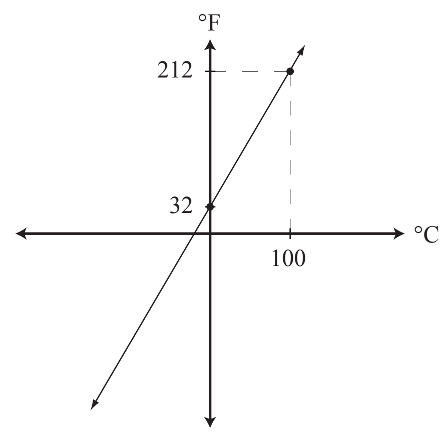
Graphs are effective visual tools because they present information quickly and easily. Sometimes, data can be better understood when presented by a graph than by a table because the graph can reveal a trend or comparison.

Example 1 Water freezes at 32° F, or 0° C. Water boils at 212° F, or 100° C. Graph the linear relation between °C and °F, and find a formula that converts Celsius to Fahrenheit.

►*Solution*: The freezing point on the graph is (0, 32)
The boiling point on the graph is (100, 212)

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

By slope-intercept, $F = \frac{9}{5}C + 32$



Example 2 It costs a popcorn vendor \$490 to make 150 bags of popcorn and \$610 to make 350 bags.

a) Graph the linear relation between cost and number of bags.
b) Find the cost equation.
c) Find the fixed cost.
d) Find the cost of 250 bags of popcorn.
e) How many bags of popcorn can be bought for \$724?

►*Solution*: **b)** $m = \frac{610 - 490}{350 - 150} = \frac{120}{200} = 0.60$

$C - 490 = 0.60(B - 150)$

$C - 490 = 0.60B - 90$

$C = 0.60B + 400$

c) The fixed cost is \$400

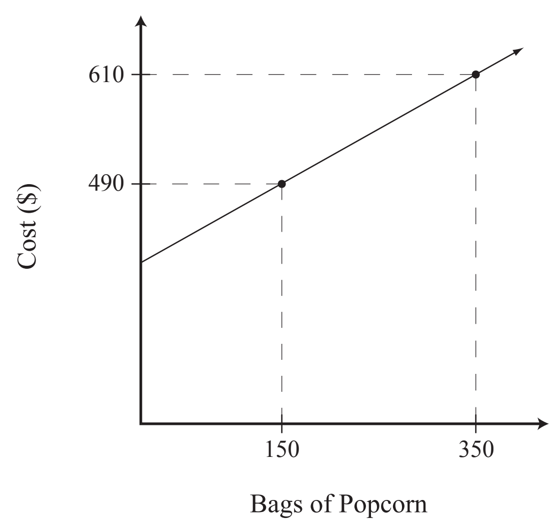
d) $C = 0.60(250) + 400 = \$550$

e) $724 = 0.60B + 400$

$724 - 400 = 0.60B$

$0.60B = 324$

$B = 540$



Example 3 A family has a medical plan that pays 70% of all prescription costs, less a $200 deductible each year.

a) Write a function that models the family's responsibility for prescription costs.
b) Determine the amount the medical plan will pay on $1250 in prescription costs.
c) Determine the amount spent on prescription purchases if the amount the plan paid was $1250.
d) Graph this function and label the answers from b) and c).

► *Solution*: **a)** Let R be the refund, and C be the prescription cost.

$m = 0.70$, y-intercept $= -200$, $R = 0.70C - 200$

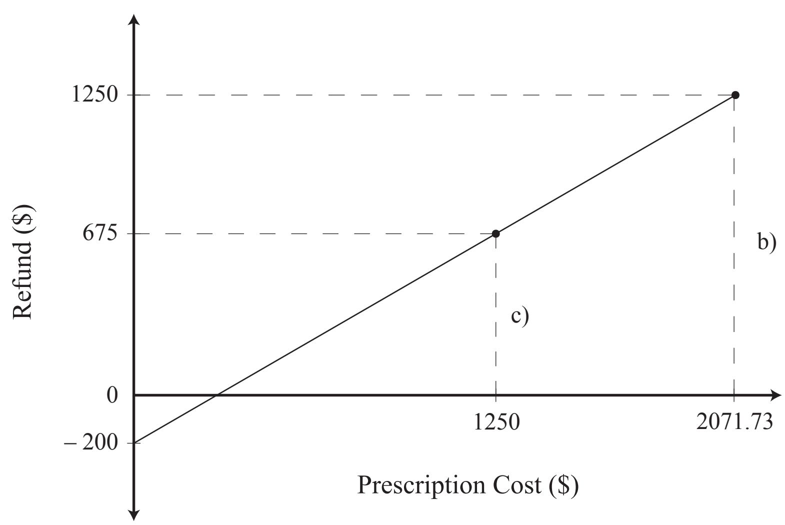
b)
$$\begin{aligned} R &= 0.70C - 200 \\ &= 0.70(1250) - 200 \\ &= 675 \end{aligned}$$

The plan will pay $675 on $1250 in prescription costs.

c)
$$\begin{aligned} R &= 0.70C - 200 \\ 1250 &= 0.70C - 200 \\ 1450 &= 0.70C \\ C &= 2071.43 \end{aligned}$$

$2071.43 is spent on prescription purchases, to get a $1250 refund.

d)



5.4 Exercise Set

Assume linear appreciation or linear depreciation for all problems.

1. An insurance company purchased computers for its office. The value of the computers after two years was \$80 000, and \$56 000 after four years. Determine the purchase price of the computers.

2. In her first year of practice, a psychologist has 160 patients. By the third year, the number of patients grew to 246. If this trend continues, how many patients would she have in the fourth year?

3. The percent of 18-25 year olds who smoke worldwide has changed from 46.8% in 1987 to 37.2% in 2000. Predict the percentage of 18-25 year olds that will smoke in 2012.

4. A taxi cab is purchased for \$36 000. At the end of 10 years it is sold for scrap for \$1800. Find the depreciation equation.

5. A home was purchased for \$410 000. The owner expects the home to double in value in the next 10 years. Find the appreciation equation.

6. A printer costs \$960 new and is expected to be worth \$140 after six years. What will it be worth after four years?

7. A painting is expected to appreciate \$75 each year. If the painting will be worth \$620 in two years, what will it be worth in 14 years?

8. A grandfather clock is expected to be worth \$2700 in three years and \$3200 in five years. What will it be worth in eight years?

9. A time share cottage purchased four years ago is now worth \$36 200. If the cottage has appreciated \$2150 per year, find its original purchase price.

10. A printing company charges a fixed rate to set up the printing press, plus a cost of \$3.50 for every 100 copies. If 800 copies cost \$64.00, how much will it cost to print 1500 copies?

11. A city with a population of 62 000 had 480 police investigations in a year. When the population of the city rose to 74 000, the number of investigations was 640 in a year. If this trend continues, how many investigations will the city have when its population reaches 100 000?

12. An electrical substation is worth \$246 000 when it is installed new, but is worth nothing after its 15 year life cycle. Find the depreciation equation.

13. The total cost of a computer is the sum of the selling price, plus a sales tax of 12%, plus a $20 disposal fee.

a) Express the total cost of the computer as a linear function of the selling price.

b) What is the total cost of a computer that sells for $1540?

c) What is the selling price of a computer whose total cost was $1061.60?

14. It costs $1200 to start up a business selling hot dogs on the beach. Each hot dog costs 40¢ to produce.

a) Find the cost equation.

b) How many hot dogs are produced if the total cost is $1560?

c) How many hot dogs must be sold at two for $1.00 to break even?

15. It costs a company $2140 to produce 500 widgets and $3660 to produce 900 widgets.

a) What is the fixed cost for producing widgets?

b) Find an equation relating the cost of producing widgets.

c) What is the total cost of producing 200 widgets?

d) How many widgets can be produced for $7308?

16. To ship a package from Vancouver to Winnipeg overnight costs $27.30 for a one pound package, and $38.80 for a three pound package.

a) Find the cost equation.

b) Find the cost of shipping a 6.5 pound package.

c) If a package cost $45.70 to ship, how much does it weigh?

5.5 Function Notation

The notation $f(x)$ is another way of writing y as a function. For example, the function $y = 2x - 4$ may be written as $f(x) = 2x - 4$, where $f(x)$ is read "f of x".

Without function notation, a problem could be stated: Given $y = 2x - 4$, find y when $x = 5$. Using function notation, the same problem would be stated: Given $f(x) = 2x - 4$, find $f(5)$. **The notation $f(5)$ implies the value of *y* when *x* is 5**. The statement $f(5) = 6$ says the value of y is 6 when x is 5. This is the point (5, 6).

Example 1 Given $f(x) = 3x + 5$, determine the coordinates of one point on the line for $f(2)$.

► *Solution*: $f(2) = 3(2) + 5 = 11$

Therefore the point is (2, 11).

Example 2 Given $f(x) = 3x + 5$, determine the coordinates of the point where $f(x) = -7$.

► *Solution*:

$$\begin{aligned} f(x) &= 3x + 5 \\ -7 &= 3x + 5 \\ -7 - 5 &= 3x \\ -12 &= 3x \\ -4 &= x \end{aligned}$$

Therefore the point is (− 4, − 7).

Example 3 Complete the table for $f(x) = 3x + 5$.

x	$3x + 5$	$f(x)$	(x, y)
3			

► *Solution*:

x	$3x + 5$	$f(x)$	(x, y)
3	14	$f(3)$	(3, 14)

Example 4 Determine the slope-intercept function $f(x) = mx + b$ if $f(1) = 4$ and $f(3) = -2$.

► *Solution*: $f(1) = 4$ means the point (1, 4)

$f(3) = -2$ means the point (3, − 2)

$$m = \frac{4-(-2)}{1-3} = \frac{6}{-2} = -3$$

$$\begin{aligned} f(x) &= mx + b \\ f(1) &= -3(1) + b \\ 4 &= -3 + b \\ b &= 7 \end{aligned}$$

Therefore $f(x) = -3x + 7$

Example 5 If $f(x) = 2x + 1$,

a) What is $f(3x)$? **b)** What is $f(x + 3)$?

► *Solution*: **a)** $$\begin{aligned} f(3x) &= 2(3x) + 1 \\ &= 6x + 1 \end{aligned}$$

b) $$\begin{aligned} f(x+3) &= 2(x+3) + 1 \\ &= 2x + 7 \end{aligned}$$

Example 6 If $f(x) = 2x + 1$, determine $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

► *Solution*: $f(x) = 2x + 1$, $f(x+h) = 2(x+h) + 1$

Therefore $$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{[2(x+h)+1]-[2x+1]}{h} \\ &= \frac{2x+2h+1-2x-1}{h} \\ &= \frac{2h}{h} \\ &= 2 \end{aligned}$$

5.5 Exercise Set

1. Complete the table for the linear function defined by $g(x) = -2x + 3$.

x	$-2x+3$	$g(x)$	(x, y)
2			
-4			
$2c$			
$c-2$			
		$-c+1$	

2. For $f(x) = 3x - 2$, find:

a) $f(3)$

b) $f(-4)$

c) $f(k)$

d) $f(2x-1)$

e) $f(x+h)$

f) $f(x)+f(h)$

3. For $f(x) = 4x + 5$, find:

a) $f(3)$

b) $f(-4)$

c) $f(k)$

d) $f(2x-1)$

e) $f(x+h)$

f) $f(x)+f(h)$

4. For $f(x) = -5x + 2$, find:

a) $f(x) = -3$

b) $f(x) = 7$

c) $f(x) = -12$

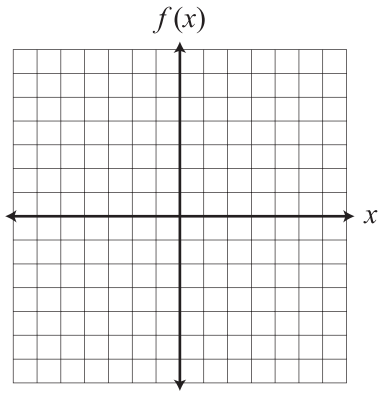
d) $f(x) = -5$

e) $f(x) = a$

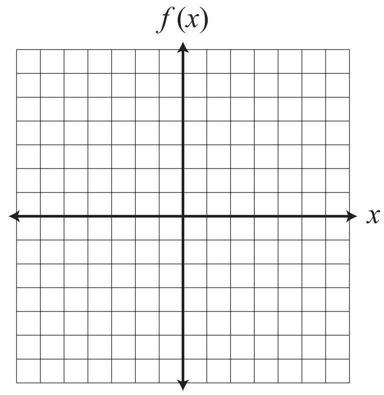
f) $f(x) = -5a + 7$

5. Graph each function over the real numbers.

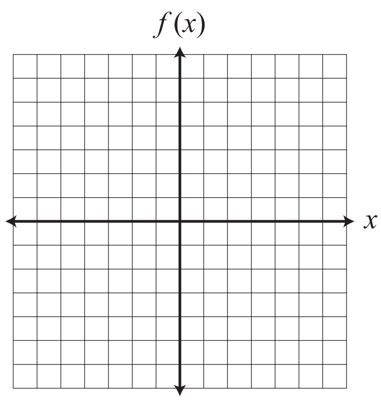
a) $f(x) = 2x + 1$



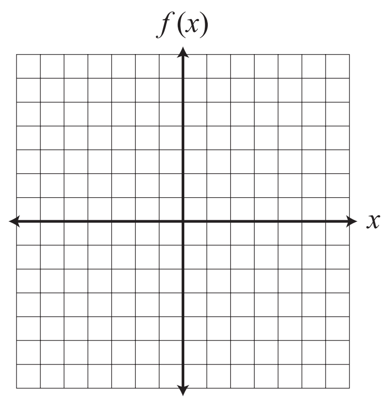
b) $f(x) = -\frac{1}{2}x + 3$



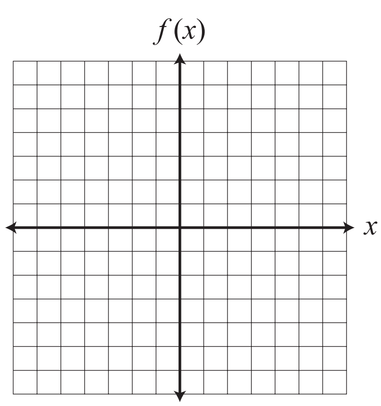
c) $f(x) = \frac{3}{4}x - 2$



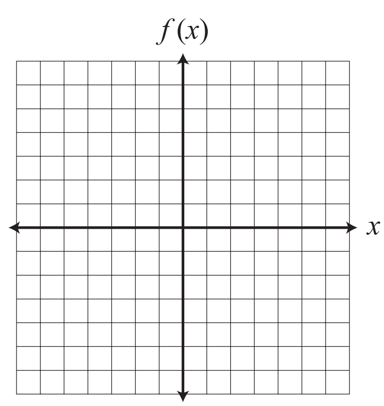
d) $f(x) = -\frac{2}{3}x - 4$



e) $f(x) = 3$

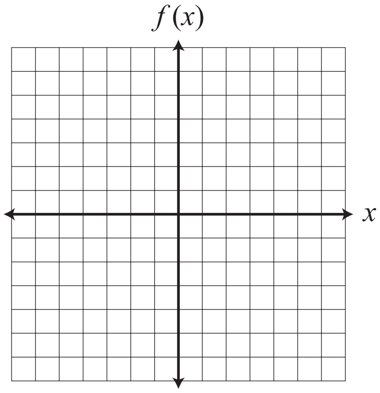


f) $f(x) = -\frac{1}{4}x^2 + 4$

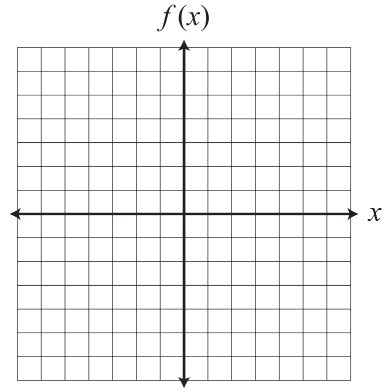


6. Graph each function if the domain is $\{-3, -2, -1, 0, 1, 2\}$

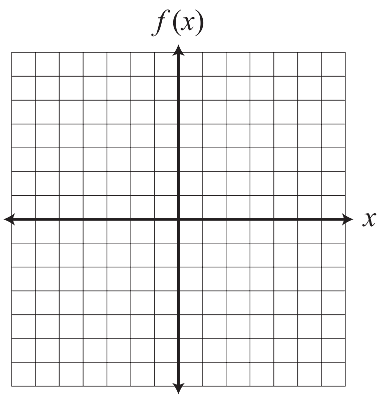
a) $f(x) = 2x + 1$



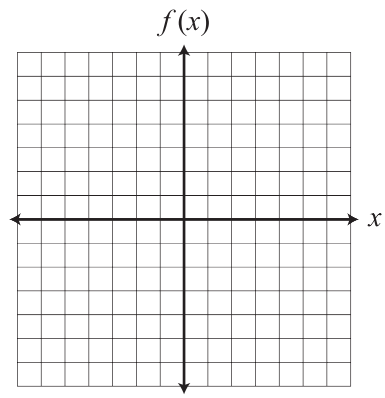
b) $f(x) = -\frac{1}{2}x + 3$



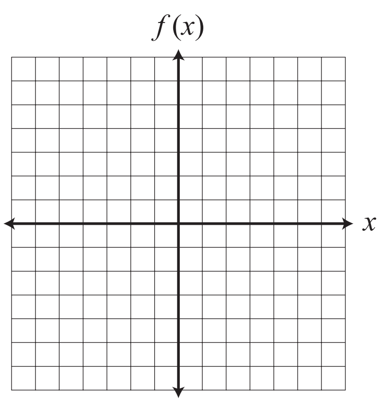
c) $f(x) = \frac{3}{4}x - 2$



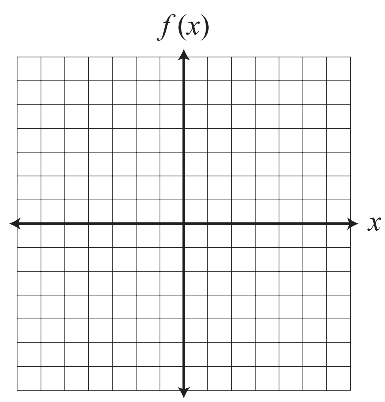
d) $f(x) = -\frac{2}{3}x - 4$



e) $f(x) = -3$



f) $f(x) = \frac{1}{4}x^3$



7. For $g(r) = 2\pi r$, find:

a) $g(0.5)$

b) $g\left(\frac{8}{3}\right)$

c) $g(h)$

d) $g(h+2)$

8. For $g(r) = 2\pi rh$, find:

a) $g(0.5)$

b) $g\left(\frac{8}{3}\right)$

c) $g(h)$

d) $g(h+2)$

9. For $g(r) = \pi r^2$, find:

a) $g\left(\frac{1}{2}\right)$

b) $g\left(\frac{8}{3}\right)$

c) $g(h)$

d) $g(h+2)$

10. For $g(r) = \pi r^2 h$, find:

a) $g\left(\frac{1}{2}\right)$

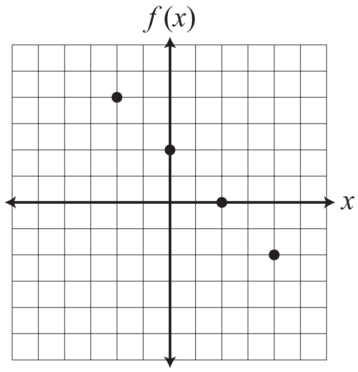
b) $g\left(\frac{8}{3}\right)$

c) $g(h)$

d) $g(h+2)$

11. Use the graph of each function to state the domain, state the range, determine $f(2)$, and solve $f(x) = 2$ for x.

a)



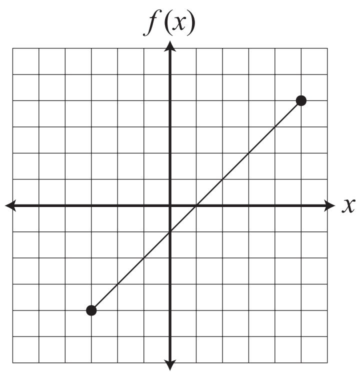
domain __________

range __________

$f(2)$ __________

$f(x) = 2$ __________

b)



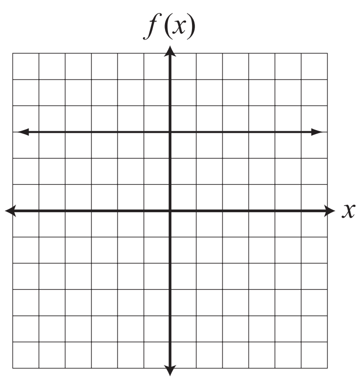
domain __________

range __________

$f(2)$ __________

$f(x) = 2$ __________

c)



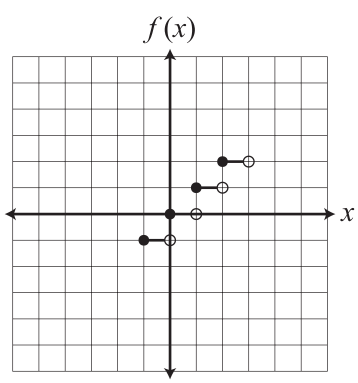
domain __________

range __________

$f(2)$ __________

$f(x) = 2$ __________

d)



domain __________

range __________

$f(2)$ __________

$f(x) = 2$ __________

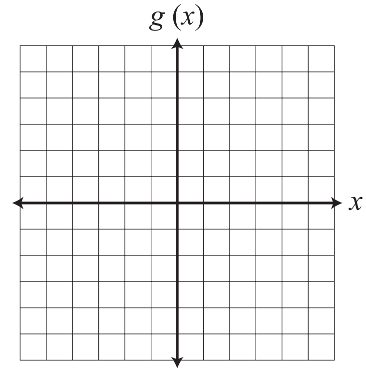
12. Complete the table, and graph the function. Also, give the domain and range of the function.

a) $g(x) = -2x - 1$

domain ____________

range ____________

x	$g(x)$
0	
1	
-1	
-2	

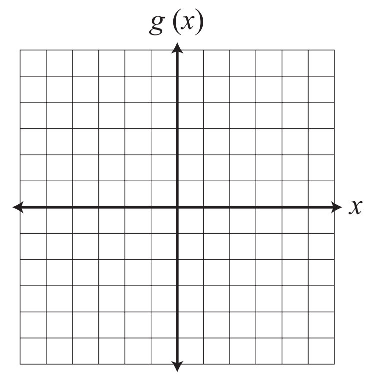


b) $g(x) = 3 - \frac{x}{2}$

domain ____________

range ____________

x	$g(x)$
-4	
-2	
0	
4	

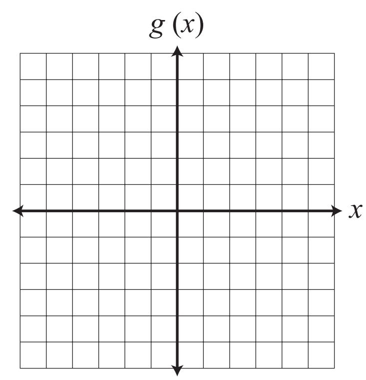


c) $g(x) = x - 4$

domain ____________

range ____________

x	$g(x)$
	1
	-1
	-3
	-5

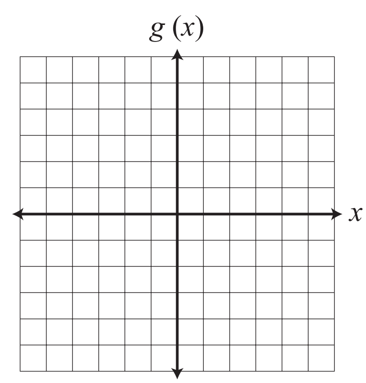


d) $g(x) = \frac{x}{2} - 1$

domain ____________

range ____________

x	$g(x)$
	1
	0
	-1
	-2



13. Determine $f(x) = mx + b$.

a) $f(0) = -3$
$f(-2) = 5$

b) $f(2) = 4$
$f(-1) = -4$

c) $f(2) = 5$
$f(-3) = 3$

d) $f(-3) = 6$
$f(1) = -2$

e) $f(3) = 2$
$f(-3) = 2$

f) $f\left(\frac{1}{2}\right) = -\frac{2}{3}$
$f\left(-\frac{5}{2}\right) = \frac{8}{3}$

14. Determine $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

a) $f(x) = 3x$

b) $f(x) = 3x - 4$

c) $f(x) = 5 - 2x$

d) $f(x) = x^2$

15. The function $f(c) = \frac{9}{5}c + 32$ determines the Fahrenheit equivalent of degrees Celsius. Find the Fahrenheit equivalent of:

a) 30°C

b) 0°C

c) − 40°C

16. A ball is dropped from a high rise building. The height of the ball in metres, *t* seconds after it is dropped, is given by the function $h(t) = -9.8t^2 + 100$.

a) Find $h(0)$.

b) Find the height of the ball after 2 seconds.

c) Find the time it takes for the ball to hit the ground.

17. The function $P(d) = \frac{d}{32} + 1$ gives the pressure in atmospheres at a depth of *d* feet in the ocean.

a) Find the pressure at 160 feet.

b) At what depth is the pressure 9.6 atmospheres?

18. The temperature below the surface of the Earth is given by $T(d) = 10d + 20$, where *T* is in celsius and *d* is in kilometres.

a) Find the temperature 12 km below the surface of the earth.

b) What depth has a temperature of 166° C?

5.6 Chapter Review

Section 5.1

1. Find the slope and y-intercept.

a) $2x - 5y = 7$ slope ________ y-intercept ________

b) $5x + y = -2$ slope ________ y-intercept ________

2. Write the standard form equation in slope-intercept form.

a) $6x - y = 3$

b) $2x + 5y = 7$

3. Write the slope-intercept equation in standard form.

a) $y = -\frac{2}{3}x + 4$

b) $y = -3x + \frac{2}{5}$

4. Write the point-slope equation in slope-intercept form.

a) $y + 1 = -\frac{2}{3}(x - 4)$

b) $y - \frac{2}{3} = -4\left(x + \frac{1}{2}\right)$

5. Write the point-slope equation in standard form.

a) $y + 1 = -\frac{2}{3}(x - 4)$

b) $y - \frac{2}{3} = -4\left(x + \frac{1}{2}\right)$

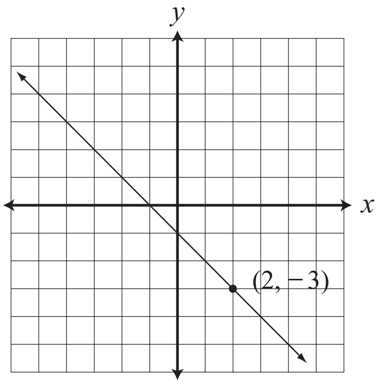
6. Write the equation of each line in standard form.

a) $(0, -3)$, $m = -4$

b) $(2, 0)$, $m = -\frac{1}{3}$

7. Determine the equation in: standard form, slope-intercept form and point-slope form.

a)

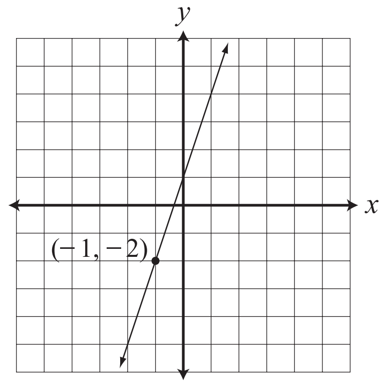


standard form ______________

slope-intercept form ______________

point-slope form ______________

b)



standard form ______________

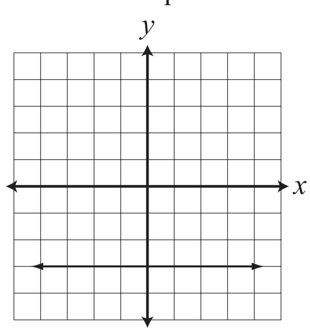
slope-intercept form ______________

point-slope form ______________

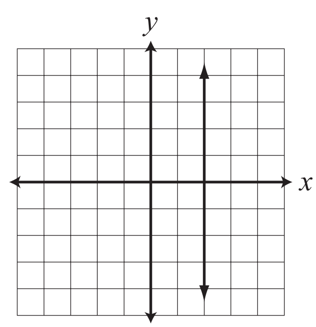
Section 5.2

8. Determine the equation of the graph.

a)



b)



9. Write the equation of the line with the given characteristics.

a) vertical, passes through $(-2, 5)$

b) horizontal, passes through $(-2, 5)$

c) vertical, passes through (a, b)

d) horizontal, passes through (a, b)

10. For each pair of equations, determine whether the lines are parallel, perpendicular or neither parallel nor perpendicular.

a) $3x + 2y = 7$
$4x + 6y = 2$

b) $5x - 2y = 4$
$4x + 10y = 3$

c) $y = 2x - 3$
$2x + y = -3$

d) $3x - y = 2$
$6x - 2y = 2$

11. Write the equation of the line passing through the given set of points in standard form.

a) $(-3, 1)$ and $(-4, -6)$

b) $(-2, -3)$ and $(-5, -1)$

Section 5.3

12. Find the slopes of lines parallel and perpendicular to the following equations.

a) $3x - 4y = -6$

$m_{||}$ ________

$m_{\perp}$ ________

b) $x = 3y + 2$

$m_{||}$ ________

$m_{\perp}$ ________

13. Find the equation of the line that passes through the given point and is parallel to the given line.

a) $P(-2, 4)$; $2x - 3y = 5$

b) $P(4, -1)$; $4x + 7y = -2$

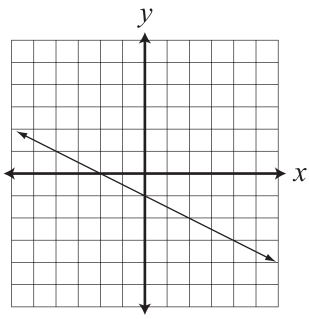
14. Find the equation of the line that passes through the given point and is perpendicular to the given line.

a) $P(-2, 4)$; $2x - 3y = 5$

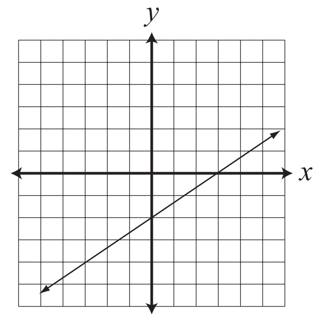
b) $P(4, -1)$; $4x + 7y = -2$

15. Determine the equation of a line, in standard form which is parallel to the line and which goes through the given point.

a) (5, 2)

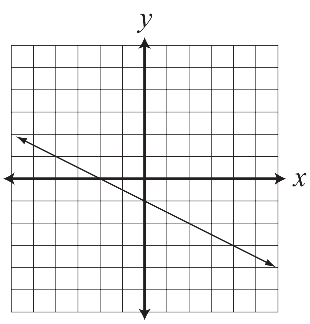


b) (− 3, 4)

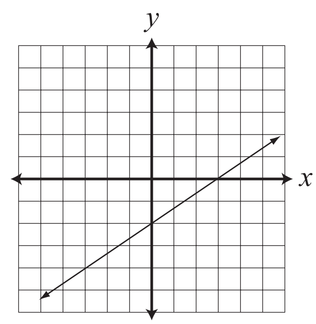


16. Determine the equation of a line, in standard form which is perpendicular to the line and which goes through the given point.

a) (5, 2)



b) (− 3, 4)



Section 5.4

17. The cost to print 1200 books is \$11 140, and the cost to print 2000 books is \$17 940. Assuming there is a linear relation between the costs and the number of books printed.

a) Find the cost equation.

b) Find the “set up” cost of printing the books.

c) Find the cost of 3000 books.

d) How many books can be purchased for \$24 740.

Section 5.5

18. For $f(x) = -3x - 2$, find:

a) $f(3)$

b) $f(-4)$

c) $f(x) = 3$

d) $f(x) = -4$

e) $f(a)$

f) $f(x) = a$

g) $f(x + h)$

h) $f(x) + f(h)$

19. Determine $f(x) = mx + b$.

a) $f(3) = 4$
$f(-2) = 6$

b) $f(-1) = -4$
$f(3) = 7$

c) $f(-4) = -2$
$f(1) = 5$

d) $f(4) = 2$
$f(2) = 4$

e) $f(a) = 2a$
$f(b) = 2b$

f) $f(a) = b$
$f(b) = a$