1.1 Number Systems

In this course, we will work with many number systems. We will solve problems involving profit and loss, time expressed as fractions, lengths of triangles expressed as square roots, and more.

In this section, we will look at numbers that are part of a larger collection of numbers called the **real numbers**.

Natural Numbers: $\{1, 2, 3...\}$

Whole Numbers: $\{0, 1, 2, 3...\}$

Integers: $\{... -3, -2, -1, 0, 1, 2, 3...\}$

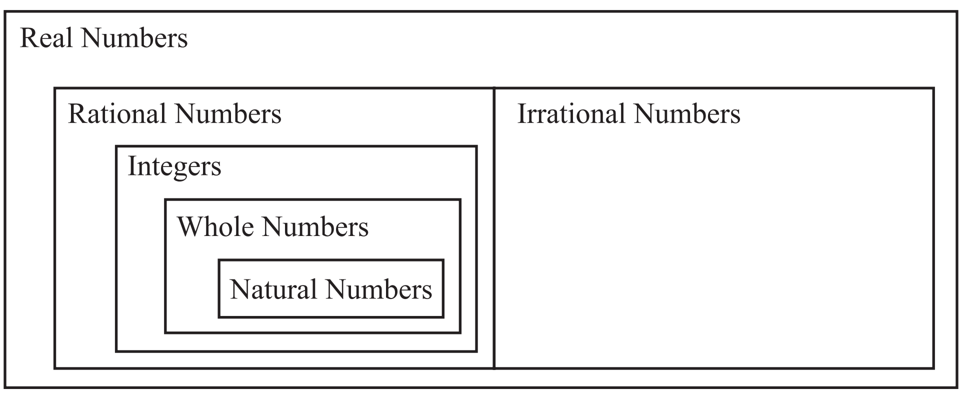
Rational Numbers: All the numbers that can be written as a fraction with the denominator not equal to zero.

Examples: $-3,\ 0,\ 5,\ -\frac{2}{3},\ \frac{10}{7},\ 2.35,\ -2.\overline{35}$

Irrational Numbers: All the numbers that cannot be written as a fraction, a terminating decimal, or a repeating decimal.

Examples: $\sqrt{2},\ \pi,\ 1.62789...$

Real Numbers: All the rational numbers and irrational numbers combined.



Example 1 To which number systems do the following numbers belong?

$$\left\{-2,\ 0,\ 4,\ 1\frac{2}{3},\ \sqrt{2},\ \pi\right\}$$

► *Solution*:

Natural numbers: 4
Whole numbers: $0, 4$
Integers: $-2, 0, 4$
Rational numbers: $-2, 0, 4, 1\frac{2}{3}$
Irrational numbers: $\sqrt{2}, \pi$
Real numbers: $-2, 0, 4, 1\frac{2}{3}, \sqrt{2}, \pi$

1.1 Exercise Set

1. State whether each statement is true or false.

a) Every natural number is positive.

b) Every whole number is positive.

c) Every integer is a whole number.

d) Every whole number is an integer.

e) Every real number is a rational number.

f) Every integer is a rational number.

g) Every decimal number is a rational number.

h) Zero is a rational number.

i) All real numbers are integers.

j) All integers are real numbers.

k) All whole numbers are real numbers.

l) The reciprocal of a non-zero integer is also an integer.

m) The reciprocal of a non-zero rational number is also a rational number.

n) An irrational number times a different irrational number could be rational.

o) Every real number is either rational or irrational.

p) If a number is real then it is irrational.

q) There are an infinite number of real numbers between 0 and 1

r) A rational number times an irrational number is always irrational.

s) Between any two irrational numbers is another irrational number.

t) An irrational number times an irrational number is always an irrational number.

2. Consider the list of numbers: $-12,\ -2.7,\ 0,\ \frac{2}{3},\ \pi,\ \sqrt{13},\ 4.21,\ 50$. List all:

a) Natural numbers

b) Whole numbers

c) Integers

d) Rational numbers

e) Irrational numbers

f) Real numbers

3. Consider the list of numbers: $-4,\ 0.\bar{3},\ 0,\ 0.121121112\ldots,\ 2.3535\ldots,\ 2\frac{1}{3},\ 12,\ \sqrt{10}$. List all:

a) Natural numbers

b) Whole numbers

c) Integers

d) Rational numbers

e) Irrational numbers

f) Real numbers

4. Consider the list of numbers: $-\sqrt{64},\ \sqrt[3]{64},\ -\sqrt{\frac{4}{9}},\ \sqrt[3]{0.008},\ \sqrt{0.04},\ \sqrt{\frac{18}{72}},\ \sqrt{0.4},\ \frac{0}{\sqrt{9}}$. List all:

a) Natural numbers

b) Whole numbers

c) Integers

d) Rational numbers

e) Irrational numbers

f) Real numbers

5. State the number systems each of the following belong to. (Natural, Whole, Integer, Rational, Irrational, Real)

a) $\sqrt{16}$

b) π

c) 0

d) $2.\overline{34}$

e) $4.010010001\ldots$

f) $-3.181818\ldots$

g) $\sqrt{\frac{27}{12}}$

h) $\sqrt{0.0004}$

6. List each of the following:

a) Natural numbers less than 4

b) Natural numbers greater than 5

c) Whole numbers less than 2

d) Integers greater than -3

e) Positive integers greater than -4

f) Whole numbers less than 0

g) Non-negative integers less than 4

h) Non-positive integers greater than -4

i) Negative integers greater than -3

j) Positive and negative integers between -2 and 2

k) Natural numbers less than 1

l) Whole numbers less than 1

m) Negative integers greater than -3.205

n) Positive whole numbers less than 2.758

o) Natural numbers between -3 and 2

p) Whole numbers between -3 and 2

1.2 Greatest Common Factor and Least Common Multiple

A key concept in finding the GCF and LCM is factoring. Factoring is the decomposition of a number into a product of other numbers, or factors, which when multiplied together give the original value.

For example: 4 is a factor of 12 because $4 \times 3 = 12$
4 is not a factor of 18 because there is no whole number k such that $4 \times k = 18$

Next, consider two very special types of whole numbers called prime numbers and composite numbers.

Prime Numbers and Composite Numbers

A prime number is a whole number that has exactly two factors: 1 and itself.

A composite number is a whole number greater than 1 that has a divisor other than on or itself, or in other words, is not prime. Every composite number has more than two factors.

List of Prime Numbers Less Than 100

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Note: The set of prime numbers is infinite. As of August 23, 2008 the largest prime number that has been discovered is $2^{43112609} - 1$. This number has 12 978 189 digits! (Printing this number would take about 2200 pages.)

Zero and One

The whole numbers 0 and 1 are neither prime nor composite.

Zero is not a prime number because it has an infinite number of divisors. One is not a prime number because it does not have two different positive whole number divisors.

Example 1 Which of the following are composite numbers: 3, 10, 18, 23, 29

► *Solution*: $10 = 2 \times 5$ and $18 = 2 \times 3 \times 3$, therefore 10 and 18 are composite. The others are prime.

Divisibility Properties

Divisibility by 2: A whole number is divisible by 2 if its last digit is 0, 2, 4, 6 or 8.

Divisibility by 3: A whole number is divisible by 3 if the sum of its digits are divisible by 3.

Divisibility by 4: A whole number is divisible by 4 if the last two digits are divisible by 4.

Divisibility by 5: A whole number is divisible by 5 if the last digit is 0 or 5.

Divisibility by 6: A whole number is divisible by 6 if it is an even number that is divisible by 3.

Divisibility by 9: A whole number is divisible by 9 if the sum of its digits are divisible by 9.

Divisibility by 10: A whole number is divisible by 10 if it ends in a 0.

Example 2 Determine which of the 4 digit numbers are divisible by the given 1 or 2 digit number.

a) 4735, 2638; 2 **b)** 2783, 5271; 3 **c)** 4384, 6394; 4 **d)** 3753, 8240; 5

e) 7936, 2898; 6 **f)** 3564, 5239; 9 **g)** 9625, 3780; 10

► *Solution*:

a) 2638 ends in an even number, therefore 2638 is divisible by 2.

4735 ends in an odd number, therefore 4735 is not divisible by 2.

b) The digits of 5271 add to 15 $(5 + 2 + 7 + 1 = 15)$, which is divisible by 3, therefore 5721 is divisible by 3.

The digits of 2783 add to 20 $(2 + 7 + 8 + 3 = 20)$, which is not divisible by 3, therefore 2783 is not divisible by 3

c) The last two digits of 4384 are 84, which is divisible by 4, therefore 4384 is divisible by 4.

The last two digits of 6394 are 94, which is not divisible by 4, therefore 6394 is not divisible by 4.

d) The last digit of 8240 is 0, therefore 8240 is divisible by 5.

The last digit of 3753 is 3, therefore 3753 is not divisible by 5.

e) The digits of 2898 add to 27 $(2 + 8 + 9 + 8 = 27)$, which is divisible by 3, and 2898 is an even number, therefore 2898 is divisible by 6.

The digits of 7936 add to 25 $(7 + 9 + 3 + 6 = 25)$, which is not divisible by 3, therefore 7936 is not divisible by 6.

f) The digits of 3564 add to 18 $(3 + 5 + 6 + 4 = 18)$, therefore 3564 is divisible by 9.

The digits of 5239 add to 19 $(5 + 2 + 3 + 9 = 19)$, therefore 5239 is not divisible by 9.

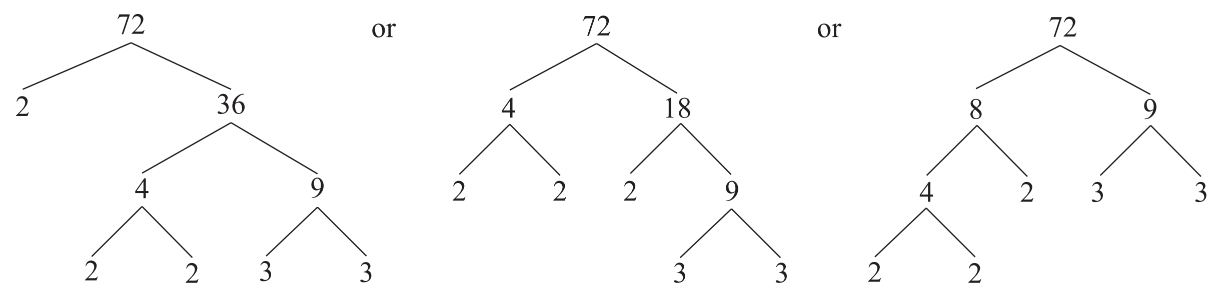
g) The last digit of 3780 is 0, therefore it is divisible by 10.

The last digit of 9625 is 5, therefore it is not divisible by 10.

Finding Prime Factors of Composite Numbers

Three methods for finding the prime factors of the number 72 will be shown.

<u>Method 1: Factor Tree</u>



Therefore 72 factors into $2 \times 2 \times 2 \times 3 \times 3$.

<u>Method 2: Dividing By Primes Until The Quotient Is A Prime Number</u>

Start with the prime number 2 and repeatedly divide into each resulting dividend until it is no longer possible.

$$\begin{array}{r}36\\2\overline{)72}\end{array},\quad \begin{array}{r}18\\2\overline{)36}\end{array},\quad \begin{array}{r}9\\2\overline{)18}\end{array}$$

Once it is no longer possible to divide with 2 try the next prime number 3.

$$\begin{array}{r}3\\3\overline{)9}\end{array}$$

The divisors are prime numbers and the quotient is prime, therefore 72 factors into $2 \times 2 \times 2 \times 3 \times 3$.

<u>Method 3: Continuous Division</u>

$2 \,\underline{|\,72}$ $\leftarrow$ 2 divides into 72. Since 36 is not prime, continue.

$\quad 2 \,\underline{|\,36}$ $\leftarrow$ 2 divides into 36. Since 18 is not prime, continue.

$\quad\quad 2 \,\underline{|\,18}$ $\leftarrow$ 2 divides into 18. Since 9 is not prime, continue.

$\quad\quad\quad 3 \,\underline{|\,9}$ $\leftarrow$ 3 divides into 9. Since the quotient is prime, stop.

$\quad\quad\quad\quad 3$

Therefore 72 factors into $2 \times 2 \times 2 \times 3 \times 3$.

The prime factors of two or more composite numbers can be used to find their greatest common factor.

For example: $60 = 2 \times 2 \times 3 \times 5$
$72 = 2 \times 2 \times 3 \times 3$

Since two 2's and a 3 are common to both, the greatest common factor of 60 and 72 is $2 \times 2 \times 3 = 12$.

Greatest Common Factor (GCF)

The largest number that divides each of the given numbers exactly.

Finding the Greatest Common Factor

1. Find the prime factors of each number.
2. List each common factor the least number of times it appears in any one number.
3. Find the product of the factors.

Example 3 Find the GCF of 36, 48, 60.

► *Solution*:

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

The common factors are 2 and 3.

The least number of 2's is 2^2
The least number of 3's is 3.

Therefore the GCF of 36, 48, 60 is $2^2 \times 3 = 12$.

Example 4 Find the GCF of 32, 40, 60.

► *Solution*:

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$
$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

The only common factor is 2. The least number of 2's is 2^2.

Therefore the GCF of 32, 40, 60 is $2^2 = 4$.

The prime factors of two or more composite numbers can be used to find their least common multiple.

For example: $60 = 2 \times 2 \times 3 \times 5$
$72 = 2 \times 2 \times 3 \times 3$

The only unique prime factor is 5. The highest power of 2 is 2^2. The highest power of 3 is 3^2.

The least common multiple of 60 and 72 is $5 \times 2^2 \times 3^2 = 180$.

Multiple

The product of a number by whole number

For example, the multiples of 7 are: 0, 7, 14, 21, 28 . . .

Least Common Multiple (LCM)

The smallest common non-zero multiple of two or more whole numbers, or the smallest number that is divisible by all the numbers.

Finding the Least Common Multiple (Method 1)

1. Determine if the largest number is a multiple of the smaller number. If so, the larger number is the LCM.
2. If not, write multiples of the larger number until you find one that is a multiple of the smaller numbers.

Example 5 Find the LCM of 15 and 18.

►*Solution*: 15 is not a multiple of 18

$2 \times 18 = 36$ is not a multiple of 15

$3 \times 18 = 54$ is not a multiple of 15

$4 \times 18 = 72$ is not a multiple of 15

$5 \times 18 = 90$ is a multiple of 15, $6 \times 15 = 90$

Therefore the LCM of 15 and 18 is 90.

Finding the Least Common Multiple (Method 2)

1. Write each number as a product of its prime factors.
2. Select the primes that occur the greatest number of times in any one factor.
3. The LCM is the product of these primes.

Example 6 Find the LCM of 60 and 72.

► *Solution*:

$60 = 2 \times 2 \times 3 \times 5 \quad = 2^2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

The unique prime factors are 2, 3 and 5

The highest power of 2 is 2^3.
The highest power of 3 is 3^2.
The highest power of 5 is 5.

Therefore the LCM of 60 and 72 is $2^3 \times 3^2 \times 5 = 360$.

Example 7 Find the LCM of 35, 60 and 75.

► *Solution*:

$35 = 5 \times 7$
$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$
$75 = 3 \times 5 \times 5 \quad = 3 \times 5^2$

The unique prime factors are 2, 3, 5 and 7

The highest power of 2 is 2^2.
The highest power of 3 is 3.
The highest power of 5 is 5^2.
The highest power of 7 is 7.

Therefore the LCM of 35, 60 and 75 is $2^2 \times 3 \times 5^2 \times 7 = 2100$.

Finding the Least Common Multiple (Method 3) (For use with 3 or more numbers)

1. Divide by a prime factor that can divide two or more of your numbers. Bring down any number that does not factor.
2. Repeat, until no remaining numbers have any prime factors in common.

Example 8 Find the LCM of 15, 18 and 24.

► *Solution*:

$$\begin{array}{r|l} 3 & 15,\ 18,\ 24 \\ \hline 2 & 5,\ 6,\ 8 \\ \hline & 5,\ 3,\ 4 \end{array}$$

all three numbers are divisible by 3
6 and 8 are divisible by 2, bring down the 5
no remaining common prime factors

Therefore the LCM of 15, 18 and 24 is $3 \times 2 \times 5 \times 3 \times 4 = 360$

Example 9 Find the LCM of 27, 30 and 36.

► *Solution*:

$$\begin{array}{r|l} 2 & 27,\ 30,\ 36 \\ \hline 3 & 27,\ 15,\ 18 \\ \hline 3 & 9,\ 5,\ 6 \\ \hline & 3,\ 5,\ 2 \end{array}$$

30 and 36 are divisible by 2, bring down the 27
all three numbers are divisible by 3
6 and 9 are divisible by 3, bring down the 5
no remaining common prime factors

Therefore the LCM of 27, 30 and 36 is $2 \times 3 \times 3 \times 3 \times 5 \times 2 = 540$

Example 10 Simplify $\frac{140}{728}$ using primes.

► *Solution*:

$140 = 2 \times 2 \times 5 \times 7$

$728 = 2 \times 2 \times 2 \times 7 \times 13$

Therefore $\frac{140}{728} = \frac{\cancel{2} \times \cancel{2} \times 5 \times \cancel{7}}{\cancel{2} \times \cancel{2} \times 2 \times \cancel{7} \times 13} = \frac{5}{26}$

1.2 Exercise Set

1. Without the use of a calculator, determine which of the following numbers: 63, 126, 280, 396, 575, 2610, 7800 are divisible by:

a) 2

b) 3

c) 4

d) 5

e) 6

f) 9

g) 10

2. The number 530__ has a missing last digit. Determine the smallest digit so that the 4 digit number is divisible by:

a) 2 ________

b) 3 ________

c) 4 ________

d) 5 ________

e) 6 ________

f) 9 ________

g) 10 ________

3. The number 3689__ has a missing last digit. Determine the smallest digit so that the 5 digit number is divisible by

a) 2 _______ **b)** 3 _______

c) 4 _______ **d)** 5 _______

e) 6 _______ **f)** 9 _______

g) 10 _______

4. By testing numbers, determine which of the following statements are true.

a) A number is divisible by 8 if the last three digits are divisible by 8. T / F

b) A number is divisible by 11 if, starting from the first digit, you find the sum of every alternate digit, and then subtract the sum of the remaining digits to get a new value which is divisible by 11. T / F

c) A number is divisible by 12 if it is divisible by both 3 and 4. T / F

d) A number is divisible by 15 if it is divisible by both 3 and 5. T / F

e) A number is divisible by 18 if it is divisible by both 2 and 9. T / F

5. List the first eight multiples.

a) 3 ______________________ **b)** 6 ______________________

c) 8 ______________________ **d)** 12 ______________________

6. Decide whether the number is prime or composite. If the number is composite, factor it into primes.

a) 19 **b)** 51

c) 87 **d)** 101

e) 117 **f)** 199

g) 611 **h)** 997

i) 629 **j)** 551

7. Simplify using prime factorization.

a) $\frac{385}{455}$ **b)** $\frac{1155}{1188}$

c) $\frac{1848}{2310}$ **d)** $\frac{4950}{5775}$

e) $\frac{2600}{3575}$ **f)** $\frac{2210}{2618}$

8. Completely factor each of the numbers.

a) 36

b) 78

c) 84

d) 169

e) 178

f) 425

g) 1000

h) 6250

i) 2431

j) 5681

9. Find the greatest common factor.

a) 12, 28

b) 54, 66

c) 48, 136

d) 65, 169

e) 81, 108

f) 30, 45, 60

g) 12, 27, 42

h) 28, 42, 84

i) 52, 130, 182

j) 66, 165, 231

10. Find the least common multiple.

a) 5, 10

b) 12, 16

c) 21, 48

d) 24, 56

e) 30, 55

f) 6, 12, 15

g) 12, 16, 24

h) 20, 36, 48

i) 7, 11, 13

j) 28, 35, 42

k) 22, 33, 66

l) 4, 36, 225

m) 8, 27, 125

n) 14, 84, 98

o) 2, 8, 12, 18

p) 9, 15, 25, 45

11. Fill in the missing numbers.

Product	6	48	18	72	108			32	75		
Factor	2					7				7	11
Factor	3						9			6	9
Sum	5	16	11	22	31	12	13	12	20		

12. Mercury, Venus and Earth revolve around the Sun every 3, 7 and 12 months respectively. If the three planets are currently lined up, how many months will pass before this happens again?

13. Earth, Jupiter, Saturn, and Uranus revolve around the Sun every 1, 12, 30, and 84 years respectively. If the four planets currently line up, how many years will pass before they would line up again?

14. Tom, Dick and Harry get half days off with pay. Tom gets a half day off with pay every eight days, Dick every 10 days and Harry every 12 days. If all three are off together on April 1, what is the next date when all three will be off together again?

15. There are 6 players on a volleyball team, 9 players on a baseball team, and 11 players on a soccer team. What is the smallest number of students in a school that can be split evenly into any of the three teams?

16. There is a unique way of finding the LCM of two numbers. Take the product of both numbers, and divide that product by the GCF. Find the LCM of 18, 24 by this method.

17. Find the LCM of 72, 108 by taking the product of both numbers and dividing that product by the GCF.

18. The Fields Medal, the highest scientific award for Mathematics, is awarded every 4 years. It was awarded in 2010. The Birkhoff Prize for Applied Mathematics is awarded every 3 years. It was awarded in 2009. What are the first two times both awards were given in the same year in the 21st century?

19. What is the smallest whole number divisible by every whole number from 1 to 10?

1.3 Squares and Square Roots

To square a number means to raise the number to the second power.

For example: $4^2 = 4 \times 4 = 16$
$9^2 = 9 \times 9 = 81$

Some numbers can be written as the product of two identical factors.

For example: $25 = 5 \times 5$
$64 = 8 \times 8$

These identical factors are called the **square root** of the number. The symbol $\sqrt{}$ (called a radical sign) is used to indicate square roots. $\sqrt{25} = 5$ and $\sqrt{64} = 8$.

All numbers with square roots that are rational are called **perfect squares**.

A List of Perfect Square Whole Numbers

0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625

Example 1 Determine which are perfect squares.

a) 4 **b)** $\frac{4}{9}$ **c)** 7 **d)** $\frac{4}{15}$

►*Solution*: **a)** Yes, because $7 \times 7 = 49$.

b) Yes, because $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

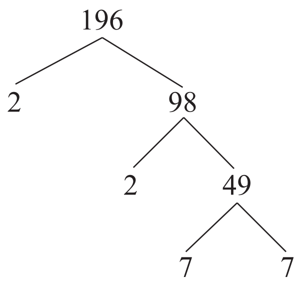
c) No, because 7 cannot be written as the product of two identical rational numbers.

d) No, because $\frac{4}{15}$ cannot be written as the ratio of two identical rational numbers.

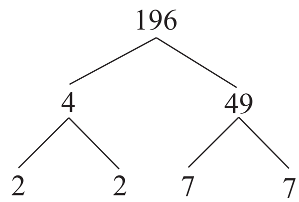
Finding Square Roots Without a Calculator or Table

Method 1: Factor Tree

Example: Determining the square root of 196.



or



Therefore $\sqrt{196} = \sqrt{2 \times 2 \times 7 \times 7} = \sqrt{2 \times 2} \times \sqrt{7 \times 7} = 2 \times 7 = 14$

Note: For whole numbers $\sqrt{x^2} = \sqrt{x \times x} = x$

Method 2: Continuous Division

Example: Determining the square root of 441

441 is not divisible by 2, but is divisible by 3 so:

$3 \,\underline{|\ 441}$ ← 3 divides into 441. Since 147 is not prime, continue.
$3 \,\underline{|\ 147}$ ← 3 divides into 147. Since 49 is not prime, continue.
$7 \,\underline{|\ 49}$ ← 7 divides into 49. Since 7 is prime, stop.
7

Therefore $\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7} = \sqrt{3 \times 3} \times \sqrt{7 \times 7} = 3 \times 7 = 21$

Cubes and Cube Roots

To cube a number means to raise the number to the third power, or to multiply the number by itself three times.

For example: $4^3 = 4 \times 4 \times 4 = 64$
$7^3 = 7 \times 7 \times 7 = 343$

Some numbers can be written as the product of three identical factors.

For example: $27 = 3 \times 3 \times 3$
$125 = 5 \times 5 \times 5$

These identical factors are called the **cube root** of the number. The symbol $\sqrt[3]{}$ (called a radical sign) is used to indicate cube roots. $\sqrt[3]{27} = 3$, and $\sqrt[3]{125} = 5$

A List of Perfect Cube Whole Numbers

0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

Example 2 Determine which are perfect cubes.

a) 8 **b)** $\frac{27}{64}$ **c)** 25 **d)** $\frac{8}{9}$

► *Solution*: **a)** Yes, because $2 \times 2 \times 2 = 8$.

b) Yes, because $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$.

c) No, because 25 cannot be written as the product of three identical integers.

d) No, because it cannot be written as the product of three identical rational numbers.

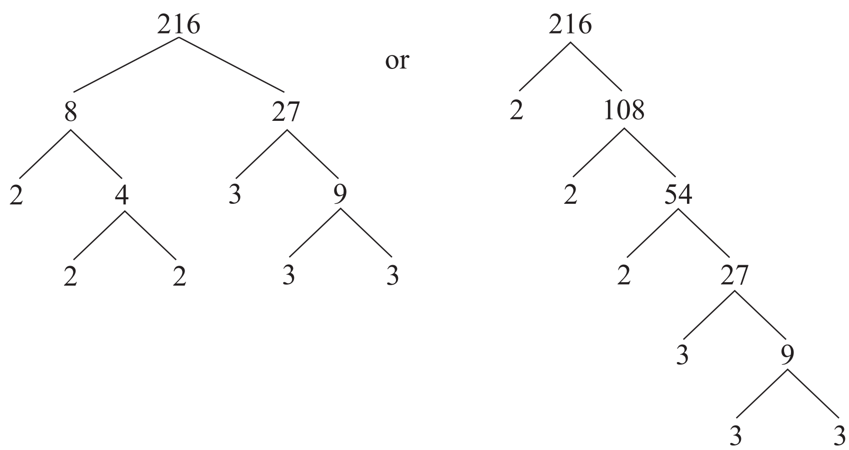
Note: In the expression $\sqrt[k]{a}$, we call k the index, and assume $k \geq 2$. If the index is not written, the expression is assumed to be a square root, i.e. $k = 2$.

eg. $\sqrt[5]{32} = 2$ because $2 \times 2 \times 2 \times 2 \times 2 = 32$

Finding Cube Roots Without a Calculator or Table

Method 1: Factor Tree

Example: Find the cube root of 216.



Therefore $\sqrt[3]{216} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} = \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{3 \times 3 \times 3} = 2 \times 3 = 6$

Note: For whole numbers, $\sqrt[3]{x^3} = \sqrt[3]{x \times x \times x} = x$

Method 2: Continuous Division

Example: Find the cube root of 729.

729 is not divisible by 2, but is divisible by 3 so:

3 | 729 ← 3 divides into 729. Since 243 is not prime, continue.
3 | 243 ← 3 divides into 243. Since 81 is not prime, continue.
3 | 81 ← 3 divides into 81. Since 27 is a cube root number, stop.
27

Therefore $\sqrt[3]{729} = \sqrt[3]{3 \times 3 \times 3 \times 27} = \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{27} = 3 \times 3 = 9$

1.3 Exercise Set

1. Find the square root of the perfect squares without a calculator.

a) $\sqrt{100}$ __________

b) $\sqrt{441}$ __________

c) $\sqrt{225}$ __________

d) $\sqrt{361}$ __________

e) $\sqrt{529}$ __________

f) $\sqrt{2\,890\,000}$ __________

g) $\sqrt{25\,000\,000}$ __________

h) $-\sqrt{72\,900}$ __________

i) $-\sqrt{9\,610\,000}$ __________

j) $\sqrt{0.000004}$ __________

2. Find the cube root of the perfect cubes without a calculator.

a) $\sqrt[3]{27}$ __________

b) $\sqrt[3]{1000}$ __________

c) $\sqrt[3]{343}$ __________

d) $\sqrt[3]{1728}$ __________

e) $\sqrt[3]{3375}$ __________

f) $\sqrt[3]{8000}$ __________

g) $\sqrt[3]{125\,000\,000}$ __________

h) $-\sqrt[3]{216\,000}$ __________

i) $-\sqrt[3]{64\,000\,000\,000}$ __________

j) $\sqrt[3]{0.000008}$ __________

3. Find the perfect square root, if it exists, without a calculator.

a) 25 ________ **b)** 29 ________

c) 80 ________ **d)** 81 ________

e) 169 ________ **f)** 99 ________

g) 1600 ________ **h)** 900 ________

i) $\frac{81}{400}$ ________ **j)** $\frac{8}{18}$ ________

4. Find the perfect cube root, if it exists, without a calculator.

a) 8 ________ **b)** 9 ________

c) 64 ________ **d)** 81 ________

e) 100 ________ **f)** 216 ________

g) 1000 ________ **h)** 144 ________

i) 625 ________ **j)** 729 ________

5. The area of a baseball diamond is 8100 ft^2.

a) How far is it from home plate to first base?

b) How far is it from 1st base to 3rd base?

6. Police use the formula $S = 2\sqrt{5x}$ to determine the speed, S in miles per hour of a skid of length x ft. Determine the speed of a car whose skid length is

a) 45 ft.

b) 180 ft.

7. The area of a rectangle with length twice as long as the width is 1250 m^2. Determine the length and width of the rectangle.

8. A cube has a volume of 216 cm^3. Determine the length of each side of the cube.

9. A rectangular solid has a length three times the width and a height twice its width. If the volume of the rectangular solid is 384 in^3, determine the dimensions of the rectangular solid.

10. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, with r the radius of the sphere. Determine the radius of a sphere with volume $972\,\pi$ mm^3.

1.4 Rational and Irrational Numbers

If a real number is not rational, it must be irrational. All numbers with square roots that are rational must be perfect squares. This is also true for cube roots.

For example: $\sqrt{25}$ This is a rational number since 25 is a perfect square; $\sqrt{25} = 5$

$\sqrt[3]{25}$ This is an irrational number since 25 is not a perfect cube.

Remember, a rational number is a number that can be represented as a fraction. An irrational number is a non-repeating, or non-terminating decimal value. Therefore, when writing a value for an irrational number, it is just an approximation.

Irrational numbers do not have to be the n th root of an integer. Here are some other examples of irrational numbers:

π, e, 1.010010001..., $\sqrt{2}$

Approximating Irrational Numbers

With the use of a calculator, an approximation of the following values were obtained.

$\sqrt{7} \doteq 2.65 \qquad \sqrt{70} \doteq 8.37 \qquad \sqrt{700} \doteq 26.5 \qquad \sqrt{7000} \doteq 83.7$

Notice that $\sqrt{7}$ and $\sqrt{700}$ have the same numerals, but different decimal point answers. This is because $\sqrt{700} = \sqrt{7} \times \sqrt{100} = \sqrt{7} \times 10$. The same is also true for $\sqrt{70}$ and $\sqrt{7000} = \sqrt{70} \times \sqrt{100}$.

Without the use of a calculator, an approximation of an n th root can be found by determining where the value lies on a number line.

Example 1 Between what two consecutive integers is $\sqrt{7}$?

► *Solution*: Let a and b be consecutive integers so that $a < \sqrt{7} < b$.

Therefore $a^2 < 7 < b^2$.

Since $2^2 = 4$ and $3^2 = 9$: $2^2 < 7 < 3^2 \rightarrow 2 < \sqrt{7} < 3$

So $\sqrt{7}$ lies between 2 and 3.

Example 2 Between what two consecutive integers is $\sqrt[3]{100}$?

► *Solution*: Let a and b be consecutive integers so that $a < \sqrt[3]{100} < b$.

Therefore $a^3 < 100 < b^3$.

Since $4^3 = 64$ and $5^3 = 125$: $4^3 < 100 < 5^3 \rightarrow 4 < \sqrt[3]{100} < 5$

So $\sqrt[3]{100}$ lies between 4 and 5.

Further Approximation of Irrational Numbers

The previous examples show rough approximations of irrational numbers. Adding some additional steps to the process will allow our estimates to be closer to the actual value.

Example 3 Approximate $\sqrt{11}$ to one decimal place.

► *Solution*: Let a and b be consecutive integers so that $a < \sqrt{11} < b$. Therefore $a^2 < 11 < b^2$.

Since $3^2 = 9$ and $4^2 = 16$: $3^2 < 11 < 4^2 \rightarrow 3 < \sqrt{11} < 4$

Then $9 < 11 < 16$ shows us 11 is 2 units from 9, and 5 units from 16.

By ratios: $\dfrac{2}{2+5} \doteq 0.3$ units

Therefore $\sqrt{11} \doteq 3.3$ (Check: $3.3^2 \simeq 10.89$)

Example 4 Approximate $\sqrt[3]{140}$ to one decimal place.

► *Solution*: Let a and b be consecutive integers so that $a < \sqrt[3]{140} < b$. Therefore $a^3 < 140 < b^3$.

Since $5^3 = 125$ and $6^3 = 216$: $5^3 < 140 < 6^3 \rightarrow 5 < \sqrt[3]{140} < 6$

Then $125 < 140 < 216$ shows us 140 is 15 units from 125 and 76 units from 216.

By ratios: $\dfrac{15}{15+76} \doteq 0.2$ units

Therefore $\sqrt[3]{140} \doteq 5.2$ (Check: $5.2^3 \simeq 140.6$)

1.4 Exercise Set

1. Without a calculator, decide if the numbers are rational or irrational.

a) $\sqrt{81}$

b) $\sqrt{810}$

c) $\sqrt{40}$

d) $\sqrt{400}$

e) $\sqrt{6.4}$

f) $\sqrt{0.64}$

g) $\sqrt{0.004}$

h) $\sqrt{0.0004}$

i) $\sqrt{22\,500}$

j) $\sqrt{22.5}$

k) $\sqrt{32^2}$

l) $\sqrt{0.32^2}$

2. Without a calculator, decide if the numbers are rational or irrational.

a) $\sqrt[3]{1}$

b) $\sqrt[3]{10}$

c) $\sqrt[3]{100}$

d) $\sqrt[3]{1000}$

e) $\sqrt[3]{8}$

f) $\sqrt[3]{80}$

g) $\sqrt[3]{800}$

h) $\sqrt[3]{8000}$

i) $\sqrt[3]{0.8}$

j) $\sqrt[3]{0.08}$

k) $\sqrt[3]{0.008}$

l) $\sqrt[3]{0.0008}$

3. Using a calculator, approximate the irrational numbers to 2 decimal places.

a) $\sqrt{2.3}$

b) $\sqrt{23}$

c) $\sqrt{230}$

d) $\sqrt{0.23}$

e) $\sqrt{2300}$

f) $\sqrt{0.023}$

g) $\sqrt[3]{7.9}$

h) $\sqrt[3]{79}$

i) $\sqrt[3]{790}$

j) $\sqrt[3]{7900}$

k) $\sqrt[3]{0.79}$

l) $\sqrt[3]{0.0079}$

4. Given $\sqrt{21} \doteq 4.58$ and $\sqrt{210} \doteq 14.49$, determine the value of the square root.

a) $\sqrt{2.1}$

b) $\sqrt{0.21}$

c) $\sqrt{2100}$

d) $\sqrt{0.021}$

e) $\sqrt{21\,000}$

f) $\sqrt{0.0021}$

g) $\sqrt{210\,000}$

h) $\sqrt{0.00021}$

i) $\sqrt{2\,100\,000}$

j) $\sqrt{0.000021}$

5. Given $\sqrt[3]{21} \doteq 2.76$, $\sqrt[3]{210} \doteq 5.94$ and $\sqrt[3]{2100} \doteq 12.81$, determine the value of the cube root.

a) $\sqrt[3]{21\,000}$

b) $\sqrt[3]{2.1}$

c) $\sqrt[3]{210\,000}$

d) $\sqrt[3]{0.21}$

e) $\sqrt[3]{0.021}$

f) $\sqrt[3]{0.0021}$

g) $\sqrt[3]{2\,100\,000}$

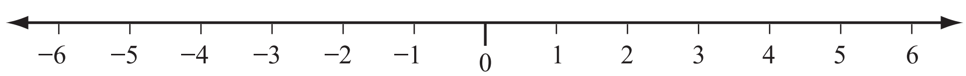
h) $\sqrt[3]{0.00021}$

i) $\sqrt[3]{21\,000\,000}$

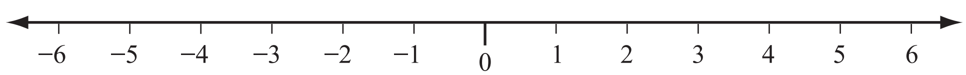
j) $\sqrt[3]{0.000021}$

6. Estimate the position of each irrational number on the number line provided.

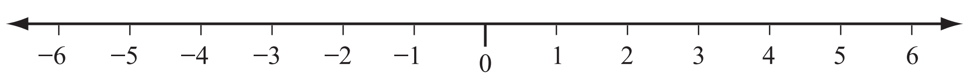
a) $\sqrt{11}$



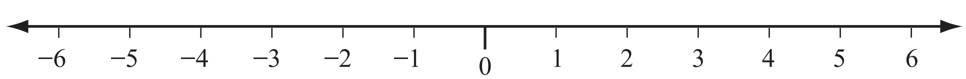
b) $-\sqrt{32}$



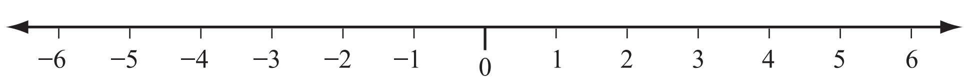
c) $\sqrt{29}$



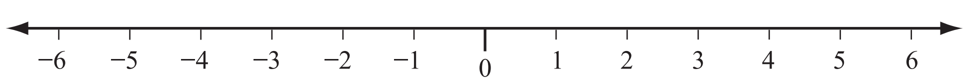
d) $-\sqrt{1.9}$



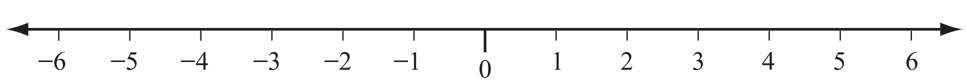
e) $\sqrt[3]{100}$



f) $-\sqrt[3]{87}$



g) $\sqrt[3]{153}$



7. Without using the root functions of a calculator, approximate the irrational number to one decimal point.

a) $\sqrt{30}$

b) $-\sqrt{58}$

c) $\sqrt{88}$

d) $-\sqrt{76}$

e) $\sqrt{130}$

f) $\sqrt[3]{98}$

g) $-\sqrt[3]{74}$

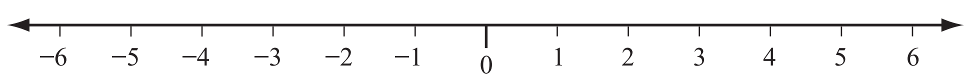
h) $\sqrt[3]{4}$

i) $-\sqrt[3]{11}$

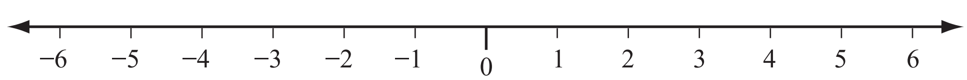
j) $\sqrt[3]{19}$

8. Order the irrational numbers on the number line provided.

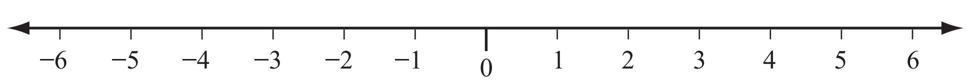
a) $\sqrt{18}$, π, $-\sqrt{27}$, $-\sqrt{30}$, $\sqrt{22}$



b) $\sqrt[3]{73}$, e, $\sqrt[3]{130}$, $-\sqrt[3]{30}$, $\sqrt[3]{110}$



c) $\sqrt{10}$, $\sqrt[3]{10}$, $-\sqrt{17}$, $-\sqrt[3]{17}$, $\sqrt{28}$, $\sqrt[3]{28}$



1.5 Exponential Notation

An exponent tells how many times the base is used as a factor. In the statement $a \times a \times a \times a \times a = a^5$, the exponent is 5 and the base is *a*. This is read "*a* to the fifth power".

For example: 2^4 means $2 \times 2 \times 2 \times 2$

$(3a)^2$ means $3a \times 3a$

$(-y)^3$ means $(-y) \times (-y) \times (-y)$

$10p^3$ means $10 \times p \times p \times p$

One and Zero as Exponents

Consider the following example:

$$\begin{aligned} 3 \times 3 \times 3 \times 3 &= 3^4 \\ 3 \times 3 \times 3 &= 3^3 \\ 3 \times 3 &= 3^2 \end{aligned}$$

On the left side of the equation each step is being divided by 3. On the right side of the equation, the exponent decreases by 1 on each step.

To continue the pattern:

$$\begin{aligned} 3 &= 3^1 \\ 1 &= 3^0 \end{aligned}$$

Exponents of 0 and 1

$a^1 = a$, for any number *a*.
$a^0 = 1$, for any non-zero number *a*.

Note: 0^0 is not defined.

Examples: $5^0 = 1$

$17^1 = 17$

$\left(\frac{2}{3}\right)^0 = 1$

$-5^0 = -1$

$(-5)^0 = 1$

The Product Rule

Consider the following example: $a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$

The exponent in the expression a^5 is the sum of the exponents in the expression $a^2 \times a^3$.
Therefore: $a^2 \times a^3 = a^{2+3} = a^5$.

Product Rule

For any numbers *a* and *b* with exponents *m* and *n*:

$a^m \times a^n = a^{m+n}, \quad a \neq 0$

Examples: $(-3)^4 \times (-3)^5 = (-3)^{4+5} = (-3)^9$

$$\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^{1+5} = \left(\frac{2}{3}\right)^6$$

$$2^{3x} \times 8^{2x} = 2^{3x} \times (2^3)^{2x} = 2^{3x} \times 2^{6x} = 2^{3x+6x} = 2^{9x}$$

The Quotient Rule

Consider the following example: $\dfrac{a^7}{a^3} = \dfrac{a \times a \times a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a \times a \times a \times a = a^4$

The exponent in the expression a^4 is the difference of the exponents in the expression $\dfrac{a^7}{a^3}$.
Therefore: $\dfrac{a^7}{a^3} = a^{7-3} = a^4$.

Quotient Rule

For any number *a* with exponents *m* and *n*:

$\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$

Examples: $\dfrac{(-4)^8}{(-4)^3} = (-4)^{8-3} = (-4)^5$

$$\frac{\left(\frac{3}{5}\right)^6}{\left(\frac{3}{5}\right)} = \left(\frac{3}{5}\right)^{6-1} = \left(\frac{3}{5}\right)^5$$

The Power Rule

An expression such as $(5^2)^3$ means $5^2 \times 5^2 \times 5^2$. By the product rule $5^2 \times 5^2 \times 5^2 = 5^{2+2+2} = 5^6$. But we can also get to 5^6 by multiplying the exponents: $(5^2)^3 = 5^{2\times3} = 5^6$.

> **Power Rule**
>
> For any numbers *a* and *b* with exponents *m* and *n*:
>
> $(a^m)^n = a^{m\times n}$

Examples: $(3^5)^4 = 3^{5\times4} = 3^{20}$

$[(-2)^3]^3 = (-2)^{3\times3} = (-2)^9$

$\left[\left(\frac{2}{5}\right)^3\right]^4 = \left(\frac{2}{5}\right)^{3\times4} = \left(\frac{2}{5}\right)^{12}$

Raising a Product to a Power

An expression such as $(2x)^3$ means $2x \times 2x \times 2x$.
This expression can be written: $(2\times2\times2)\times(x\times x\times x) = 2^3\times x^3 = 8x^3$.

> **A Product to a Power**
>
> For any numbers *a* and *b* with exponent *n*:
>
> $(ab)^n = a^n \times b^n$

Examples: $(3x)^3 = 3^3 \times x^3 = 27x^3$

$(2y^2)^4 = 2^4 \times y^{2\times4} = 16y^8$

$(-3a^3b^4)^2 = (-3)^2 \times a^{3\times2} \times b^{4\times2} = 9a^6b^8$

Raising a Fraction to a Power

An expression such as $\left(\frac{2}{3}\right)^4$ means $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$. Multiplying the fractions gives: $\frac{2\times2\times2\times2}{3\times3\times3\times3} = \frac{2^4}{3^4} = \frac{16}{81}$.

A Fraction to a Power

For any numbers a and b, $b \neq 0$, with exponent n:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Examples: $\left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3} = \frac{8}{x^3}$

$$\left(\frac{1}{a^4}\right)^2 = \frac{1^2}{a^{4\times2}} = \frac{1}{a^8}$$

Negative Exponents

Consider the following example: $\frac{5^2}{5^6} = \frac{\cancel{5}\times\cancel{5}}{5\times5\times5\times5\times\cancel{5}\times\cancel{5}} = \frac{1}{5\times5\times5\times5} = \frac{1}{5^4}$.

Solving the previous example with the quotient rule gives: $\frac{5^2}{5^6} = 5^{2-6} = 5^{-4}$

Therefore $\frac{1}{5^4} = 5^{-4}$

Negative Exponents

For any number a, $a \neq 0$, with exponent n:

$$a^{-n} = \frac{1}{a^n}$$

Examples: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$$(2x)^{-4} = \frac{1}{(2x)^4} = \frac{1}{16x^4}$$

Changing from Negative to Positive Exponents

Consider the expression $\frac{1}{2^{-3}}$. By the negative exponent rule, this is equivalent to $2^{-(-3)} = 2^3$.

Changing from Negative to Positive Exponents

For any non-zero numbers a and b, with exponents m and n:

$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$ and $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Examples: $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$

$\frac{3^{-2}}{4^{-3}} = \frac{4^3}{3^2} = \frac{64}{9}$

Rational Exponents: $a^{\frac{1}{n}}$

Consider the square root example: $\sqrt{2} \times \sqrt{2} = 2$.

Now consider the exponent rule example: $2^{\frac{1}{2}} \times 2^{\frac{1}{2}} = 2^{\frac{1}{2}+\frac{1}{2}} = 2^1 = 2$.

Since $\sqrt{2} \times \sqrt{2}$ and $2^{\frac{1}{2}} \times 2^{\frac{1}{2}}$ equal 2, $\sqrt{2}$ should equal $2^{\frac{1}{2}}$.

Rational Exponents: $a^{\frac{1}{n}}$

For any non-negative real number a and any positive integer n.

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Examples: $x^{\frac{1}{2}} = \sqrt{x}$

$\sqrt[4]{2} = 2^{\frac{1}{4}}$

$\sqrt{2} \times \sqrt[4]{2} = 2^{\frac{1}{2}} \times 2^{\frac{1}{4}} = 2^{\frac{1}{2}+\frac{1}{4}} = 2^{\frac{3}{4}} = \sqrt[4]{2^3} = \sqrt[4]{8}$

Rational Exponents: $a^{\frac{m}{n}}$

This is a more general version of rational exponents.

By the power rule, $8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = \left(\sqrt[3]{8}\right)^4 = 2^4 = 16$

However, $8^{\frac{4}{3}}$ can also be written as $(8^4)^{\frac{1}{3}} = \sqrt[3]{8^4} = \sqrt[3]{4096} = 16$

> **Rational Exponents: $a^{\frac{m}{n}}$**
>
> For any non-negative real number a and any positive integer n.
>
> $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Examples: $4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^{\left(2\times\frac{3}{2}\right)} = 2^3 = 8$

$$\left(\sqrt[5]{32}\right)^4 = 32^{\frac{4}{5}} = (2^5)^{\frac{4}{5}} = 2^4 = 16$$

$$16^{-\frac{3}{4}} = \frac{1}{(2^4)^{\frac{3}{4}}} = \frac{1}{2^3} = \frac{1}{8}$$

$$\frac{\sqrt[3]{4}}{\sqrt[4]{2}} = \frac{4^{\frac{1}{3}}}{2^{\frac{1}{4}}} = \frac{(2^2)^{\frac{1}{3}}}{2^{\frac{1}{4}}} = \frac{2^{\frac{2}{3}}}{2^{\frac{1}{4}}} = 2^{\frac{2}{3}-\frac{1}{4}} = 2^{\frac{8}{12}-\frac{3}{12}} = \sqrt[12]{2^5} = \sqrt[12]{32}$$

$$\sqrt[3]{9}\cdot\sqrt[4]{27} = 9^{\frac{1}{3}}\cdot 27^{\frac{1}{4}} = (3^2)^{\frac{1}{3}}\cdot(3^3)^{\frac{1}{4}} = 3^{\frac{2}{3}}\cdot 3^{\frac{3}{4}} = 3^{\frac{2}{3}+\frac{3}{4}} = 3^{\frac{8}{12}+\frac{9}{12}} = 3^{\frac{17}{12}} = \sqrt[12]{2^{17}} = \sqrt[12]{2^{12}\cdot 2^5} = 2\sqrt[12]{32}$$

Summary of Exponent Rules

For any integers m and n:		
Exponent of 1	$a^1 = a$	$3^1 = 3$
Exponent of 0	$a^0 = 1, \quad a \neq 0$	$(-5)^0 = 1$
Product Rule	$a^m \times a^n = a^{m+n}, \quad a \neq 0$	$2^3 \times 2^4 = 2^{3+4} = 2^7$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$	$\frac{3^5}{3^3} = 3^{5-3} = 3^2$
Power Rules	$(a^m)^n = a^{m \times n}$ $(ab)^n = a^n \times b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(2^3)^4 = 2^{3\times 4} = 2^{12}$ $(2x)^3 = 2^3 \times x^3$ $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
Negative Exponents	$a^{-n} = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$	$2^{-3} = \frac{1}{2^3}$ $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$ $\frac{2^{-3}}{3^{-4}} = \frac{3^4}{2^3}$
Rational Exponents	$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[3]{5} = 5^{\frac{1}{3}}$ $\sqrt[4]{5^3} = 5^{\frac{3}{4}}$

1.5 Exercise Set

1. Multiply. Leave answer in exponential form.

a) $2^3 \times 2^4$ ________ **b)** $3^5 \times 3^7$ ________

c) $4^{-3} \times 4^2$ ________ **d)** $5^0 \times 5^3$ ________

e) $a \times a^2 \times a^3$ ________ **f)** $y^{-3} \times y^2 \times y$ ________

g) $8^0 \times 8^1 \times 8^2$ ________ **h)** $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4$ ________

i) $(-3)^4 \times (-3)^2 \times (-3)^1$ ________ **j)** $\left(-\frac{1}{2}\right)^5 \times \left(-\frac{1}{2}\right)^{-3} \times \left(-\frac{1}{2}\right)$ ________

2. Divide. Leave answer in exponential form

a) $\frac{5^6}{5^3}$ ________ **b)** $\frac{4^8}{4^4}$ ________

c) $\frac{2^8}{2^2}$ ________ **d)** $\frac{3^9}{3^3}$ ________

e) $\frac{t^6}{t^2}$ ________ **f)** $\frac{x^7}{x^7}$ ________

g) $\frac{(-6)^4}{(-6)^{-3}}$ ________ **h)** $\frac{(-9)^{-3}}{(-9)^{-6}}$ ________

i) $\frac{(-2x)^3}{(-2x)^{-4}}$ ________ **j)** $\frac{z^{-2}}{z^{-6}}$ ________

3. Simplify. Express without brackets or negative exponents.

a) $(2^4)^2$ ________ **b)** $(5^3)^{-2}$ ________

c) $(3^{-4})^{-2}$ ________ **d)** $(-3x^{-2})^0$ ________

e) $(2x)^3$ ________ **f)** $(3x^{-4})^2$ ________

g) $(2a^{-4})^3$ ________ **h)** $(3x^4y^{-2})^4$ ________

i) $(-4a^{-3}b^{-2})^2$ ________ **j)** $(-2^{-3}x^{-2}y)^3$ ________

4. Simplify. Express without brackets or negative exponents.

a) $\dfrac{3^4 \times 3^7}{3^5}$ ________ **b)** $\dfrac{2^5}{2^4 \times 2^3}$ ________

c) $\dfrac{4^{-3} \times 4}{4^{-1}}$ ________ **d)** $\dfrac{5^4 \times 5^{-2}}{5^3 \times 5^{-1}}$ ________

e) $\dfrac{7^0 \times 7^{-3}}{7 \times 7^{-2}}$ ________ **f)** $\dfrac{11^2 \times 11^3}{11^{-1}}$ ________

g) $\dfrac{3(x^3)^2}{x^{-2}}$ ________ **h)** $\dfrac{(3x^2)^{-3}}{x^3}$ ________

i) $(2a^2b^{-4}c^{-5})^3$ ________ **j)** $\left(\dfrac{2a^2}{3b^4}\right)^{-3}$ ________

5. Evaluate.

a) 3^2 ________ **b)** 3^{-2} ________

c) $\left(\frac{1}{3}\right)^2$ ________ **d)** $\left(\frac{1}{3}\right)^{-2}$ ________

e) -3^2 ________ **f)** $(-3)^2$ ________

g) $-\left(-\frac{1}{3}\right)^2$ ________ **h)** $\left(-\frac{1}{3}\right)^2$ ________

i) $\left(-\frac{1}{3}\right)^{-2}$ ________ **j)** $-\left(-\frac{1}{3}\right)^{-2}$ ________

k) 2^3 ________ **l)** 2^{-3} ________

m) $\left(\frac{1}{2}\right)^3$ ________ **n)** $\left(\frac{1}{2}\right)^{-3}$ ________

o) -2^3 ________ **p)** $(-2)^3$ ________

q) $-\left(-\frac{1}{2}\right)^3$ ________ **r)** $\left(-\frac{1}{2}\right)^3$ ________

s) $\left(-\frac{1}{2}\right)^{-3}$ ________ **t)** $-\left(-\frac{1}{2}\right)^{-3}$ ________

6. Simplify. Express without brackets or negative exponents.

a) $\dfrac{(2a^2b^3)^{-2}\times(4ab^{-1})^3}{(a^3b)^{-4}}$

b) $\dfrac{(x^5y^2)^{-2}\times(x^2y^{-2})^3}{x^{-1}y^{-2}}$

c) $\dfrac{(5m^{-1}n^2)^2\times(2m^{-2}n^{-3})^3}{(2m^3n^2)^{-1}}$

d) $\dfrac{(3a^{-2}b^3)^2\times(3a^{-1}b^{-4})^{-1}}{(3a^2b^{-2})^{-3}}$

e) $\dfrac{(3^{-1}x^{-2}y)^{-1}\times(5x^2y^4)^{-2}}{(4x^{-2}y^{-3})^2}$

f) $\dfrac{(3^{-1}a^{-1}b^{-2})^{-2}\times(4a^{-3}b^4)^{-2}}{(3a^{-3}b^{-4})^2}$

g) $\left(\dfrac{4^{-2}x^2y^{-3}}{x^{-2}y}\right)^3\left(\dfrac{8^{-1}x^{-3}y}{x^3y^{-1}}\right)^{-2}$

h) $\left(\dfrac{9ab^{-1}}{8a^{-2}b^2}\right)^{-2}\left(\dfrac{3a^{-2}b^2}{2a^2b^{-1}}\right)^3$

i) $\dfrac{(2x^{-1}y^2)(4x^2y^{-3})^{-2}}{(12x^2y^2)}$

j) $\left[\dfrac{(5x^{-3}y^4)^{-2}(6x^2y^{-5})}{15x^2y^{-4}}\right]^{-2}$

7. Evaluate.

a) $16^{\frac{3}{4}}$ ________ **b)** $16^{-\frac{3}{4}}$ ________

c) $8^{\frac{2}{3}}$ ________ **d)** $8^{-\frac{2}{3}}$ ________

e) $27^{\frac{4}{3}}$ ________ **f)** $27^{-\frac{4}{3}}$ ________

g) $-16^{\frac{5}{4}}$ ________ **h)** $-16^{-\frac{5}{4}}$ ________

i) $-32^{\frac{4}{5}}$ ________ **j)** $-32^{-\frac{4}{5}}$ ________

k) $216^{\frac{2}{3}}$ ________ **l)** $216^{-\frac{2}{3}}$ ________

m) $-125^{\frac{4}{3}}$ ________ **n)** $-125^{-\frac{4}{3}}$ ________

o) $64^{\frac{7}{6}}$ ________ **p)** $64^{-\frac{7}{6}}$ ________

q) $-49^{\frac{3}{2}}$ ________ **r)** $-49^{-\frac{3}{2}}$ ________

s) $128^{\frac{5}{7}}$ ________ **t)** $128^{-\frac{5}{7}}$ ________

u) $-243^{\frac{6}{5}}$ ________ **v)** $-243^{-\frac{6}{5}}$ ________

w) $81^{\frac{5}{4}}$ ________ **x)** $81^{-\frac{5}{4}}$ ________

8. Simplify. Leave answer with positive exponents.

a) $2^{\frac{1}{4}} \times 2^{\frac{5}{4}}$ ________

b) $3^{\frac{2}{3}} \times 3^{\frac{7}{3}}$ ________

c) $4^{\frac{1}{4}} \times 4^{-\frac{3}{4}}$ ________

d) $5^{-\frac{2}{3}} \times 5^{-\frac{1}{3}}$ ________

e) $\dfrac{6^{\frac{3}{4}}}{6^{\frac{5}{4}}}$ ________

f) $\dfrac{7^{\frac{2}{5}}}{7^{-\frac{1}{5}}}$ ________

g) $\dfrac{8^{-\frac{2}{7}} \times 8^{\frac{4}{7}}}{8^{-\frac{3}{7}}}$ ________

h) $\dfrac{9^{\frac{3}{5}}}{9^{\frac{2}{5}} \times 9^{-\frac{4}{5}}}$ ________

i) $a^{\frac{3}{4}} \times a^{\frac{5}{4}}$ ________

j) $b^{\frac{5}{6}} \times b^{-\frac{1}{3}}$ ________

k) $\dfrac{c^{\frac{2}{3}}}{c^{\frac{5}{6}}}$ ________

l) $\dfrac{d^{\frac{1}{3}}}{d^{-\frac{1}{2}}}$ ________

m) $\left(\dfrac{9}{4}\right)^{\frac{3}{2}}$ ________

n) $\left(\dfrac{9}{4}\right)^{-\frac{3}{2}}$ ________

o) $\left(\dfrac{81}{16}\right)^{\frac{3}{4}}$ ________

p) $\left(\dfrac{81}{16}\right)^{-\frac{3}{4}}$ ________

q) $\left(a^{3}b^{\frac{1}{4}}\right)^{\frac{2}{3}}$ ________

r) $\left(x^{4}y^{\frac{1}{2}}\right)^{\frac{4}{3}}$ ________

s) $\left(a^{\frac{2}{3}}b^{\frac{5}{6}}c^{\frac{1}{2}}\right)^{\frac{6}{7}}$ ________

t) $\left(x^{\frac{4}{3}}y^{\frac{3}{4}}z^{\frac{5}{2}}\right)^{-\frac{12}{5}}$ ________

9. Simplify each radical. Assume variables are positive.

a) $\sqrt[4]{4}$ ________ **b)** $\sqrt[3]{8^6}$ ________

c) $\sqrt[8]{16^3}$ ________ **d)** $\sqrt[3]{27^2}$ ________

e) $\sqrt[12]{9^3}$ ________ **f)** $\sqrt[8]{4^2}$ ________

g) $\sqrt[4]{a^2}$ ________ **h)** $\sqrt[9]{b^3}$ ________

i) $\sqrt[6]{c^4}$ ________ **j)** $\sqrt[8]{d^2}$ ________

10. Simplify.

a) $\sqrt{2}\times\sqrt[3]{2}$ ________ **b)** $\sqrt{3}\times\sqrt[4]{3}$ ________

c) $\sqrt[3]{2}\times\sqrt[4]{2}$ ________ **d)** $\dfrac{\sqrt[3]{4}}{\sqrt[4]{4}}$ ________

e) $\dfrac{\sqrt{27}}{\sqrt[3]{9}}$ ________ **f)** $\dfrac{\sqrt[3]{16}}{\sqrt[4]{8}}$ ________

g) $\dfrac{\left(\frac{1}{2}\right)^x\cdot 8^x}{4^x}$ ________ **h)** $\dfrac{3^x\cdot 27^x}{9^x}$ ________

i) $\dfrac{\left(\frac{1}{3}\right)^x\cdot 81^x}{27^x}$ ________ **j)** $\dfrac{5^{-x}\cdot 125^{2x}}{25^{3x}}$ ________

11. Identify the errors, then correct the result.

a) $2^5 + 2^3 = 2^8$

b) $3^4 \times 3^2 = 9^6$

c) $4^3 \times 4^5 = 4^{15}$

d) $\dfrac{2^{12}}{2^4} = 2^3$

e) $\dfrac{2^{12}}{2^4} = 1^8$

f) $(x^2)^3 = x^5$

g) $x^{-3} = -x^3$

h) $\left(\dfrac{16^3}{2^2}\right) = 8$

i) $-3^4 = 81$

j) $(2x^2)^4 = 8x^4$

k) $(-2)^0 = -1$

l) $0^0 = 1$

m) $(2+2)^{-1} = 1$

n) $\dfrac{16}{64} = \dfrac{1\cancel{6}}{\cancel{6}4} = \dfrac{1}{4}$

1.6 Irrational Numbers

To factor a square root, the product rule can be used.

The Product Rule for Square Roots

For any real numbers $\sqrt{A}$ and $\sqrt{B}$: $\sqrt{A \times B} = \sqrt{A} \times \sqrt{B}$

The product rule is used when there is a perfect square as a factor.

Consider $\sqrt{72}$. To simplify this expression, there are many ways to factor 72.

Method 1:
$$\begin{aligned}\sqrt{72} &= \sqrt{4 \times 18} \\ &= \sqrt{4} \times \sqrt{18} \\ &= 2 \times \sqrt{18} \\ &= 2 \times \sqrt{9 \times 2} \\ &= 2 \times \sqrt{9} \times \sqrt{2} \\ &= 2 \times 3 \times \sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

An alternative way to approach this problem is to look for the largest perfect square factor of 72, which is 36.

Method 2:
$$\begin{aligned}\sqrt{72} &= \sqrt{36 \times 2} \\ &= \sqrt{36} \times \sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

Simplifying is much easier if the largest perfect square factor is identified in the first step.

Example 1 Simplify $\sqrt{48}$.

► *Solution*:
$$\begin{aligned}\sqrt{48} &= \sqrt{16 \times 3} \\ &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3}\end{aligned}$$

Square root expressions such as $\sqrt{14}$ or $\sqrt{2}$ cannot be simplified, since neither have perfect square factors. Sometimes, after multiplication, it is possible to simplify the product.

For example:
$$\begin{aligned}\sqrt{2}\times\sqrt{14} &= \sqrt{28}\\ &= \sqrt{4\times 7}\\ &= \sqrt{4}\times\sqrt{7}\\ &= 2\sqrt{7}\end{aligned}$$

To factor a cube root, the product rule is used again.

The Product Rule for Cube Roots

For any real numbers $\sqrt[3]{A}$ and $\sqrt[3]{B}$: $\sqrt[3]{A\times B} = \sqrt[3]{A}\times\sqrt[3]{B}$

The product rule is used when there is a perfect cube as a factor.

Example 2 Simplify $\sqrt[3]{40}$.

► *Solution*:
$$\begin{aligned}\sqrt[3]{40} &= \sqrt[3]{8\times 5}\\ &= \sqrt[3]{8}\times\sqrt[3]{5}\\ &= 2\sqrt[3]{5}\end{aligned}$$

Simplifying products of cube roots follows the method used for simplifying products of square roots.

For example:
$$\begin{aligned}\sqrt[3]{9}\times\sqrt[3]{6} &= \sqrt[3]{54}\\ &= \sqrt[3]{27\times 2}\\ &= \sqrt[3]{27}\times\sqrt[3]{2}\\ &= 3\sqrt[3]{2}\end{aligned}$$

Entire Root

An expression such as $2\sqrt{7}$ is called a **mixed root**, and the expression $\sqrt{28}$ is called an **entire root**. Both expressions have the same value. Any mixed root can be changed to an entire root.

For example:
$$\begin{aligned}9\sqrt{3} &= \sqrt{9\times 9\times 3}\\ &= \sqrt{243}\end{aligned} \qquad \begin{aligned}2\sqrt[3]{5} &= \sqrt[3]{2\times 2\times 2\times 5}\\ &= \sqrt[3]{40}\end{aligned}$$

1.6 Exercise Set

1. Find each product.

a) $\sqrt{3} \times \sqrt{5}$

b) $\sqrt{7} \times \sqrt{2}$

c) $\sqrt{13} \times \sqrt{13}$

d) $\sqrt{5} \times \sqrt{6}$

e) $\sqrt{2} \times \sqrt{11}$

f) $\sqrt{2} \times \sqrt{3} \times \sqrt{5}$

g) $\sqrt[3]{4} \times \sqrt[3]{5}$

h) $\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2}$

i) $\sqrt[3]{2} \times \sqrt[3]{3} \times \sqrt[3]{5}$

j) $\sqrt[3]{6} \times \sqrt[3]{7} \times \sqrt[3]{5}$

2. Which one of the entire roots cannot be simplified to a mixed root?

a) $\sqrt{44}$, $\sqrt{46}$, $\sqrt{48}$, $\sqrt{50}$

b) $\sqrt{18}$, $\sqrt{20}$, $\sqrt{21}$, $\sqrt{24}$

c) $\sqrt[3]{40}$, $\sqrt[3]{81}$, $\sqrt[3]{100}$, $\sqrt[3]{125}$

d) $\sqrt[3]{16}$, $\sqrt[3]{36}$, $\sqrt[3]{54}$, $\sqrt[3]{128}$

e) $\sqrt{32}$, $\sqrt[3]{32}$, $\sqrt{100}$, $\sqrt[3]{100}$

f) $\sqrt{64}$, $\sqrt[3]{64}$, $\sqrt{75}$, $\sqrt[3]{75}$

g) $\sqrt{27}$, $\sqrt[3]{27}$, $\sqrt{50}$, $\sqrt[3]{50}$

h) $\sqrt{8}$, $\sqrt{10}$, $\sqrt{12}$, $\sqrt{20}$

i) $\sqrt[3]{24}$, $\sqrt[3]{36}$, $\sqrt[3]{54}$, $\sqrt[3]{72}$

j) $\sqrt{25}$, $\sqrt[3]{25}$, $\sqrt{125}$, $\sqrt[3]{125}$

3. Simplify to a mixed root.

a) $\sqrt{20}$

b) $\sqrt{72}$

c) $\sqrt{45}$

d) $\sqrt{24}$

e) $\sqrt{75}$

f) $\sqrt{125}$

g) $\sqrt{140}$

h) $\sqrt{128}$

i) $-\sqrt{80}$

j) $-\sqrt{160}$

4. Simplify.

a) $2\sqrt{9}$

b) $4\sqrt{25}$

c) $6\sqrt{40}$

d) $3\sqrt{8}$

e) $4\sqrt{27}$

f) $6\sqrt{50}$

g) $-\frac{5}{2}\sqrt{32}$

h) $-2\frac{1}{3}\sqrt{72}$

i) $-0.8\sqrt{125}$

j) $-1.25\sqrt{128}$

5. Simplify each root.

a) $\sqrt[3]{40}$

b) $\sqrt[3]{48}$

c) $\sqrt[3]{54}$

d) $\sqrt[3]{135}$

e) $\sqrt[3]{128}$

f) $\sqrt[3]{192}$

g) $2\sqrt[3]{27}$

h) $-3\sqrt[3]{16}$

i) $\frac{1}{2}\sqrt[3]{64}$

j) $-\frac{3}{5}\sqrt[3]{250}$

6. Multiply, and simplify if possible.

a) $\sqrt{3}\times\sqrt{6}$

b) $\sqrt{7}\times\sqrt{14}$

c) $\sqrt{3}\times\sqrt{27}$

d) $\sqrt{2}\times\sqrt{8}$

e) $\sqrt{3}\times\sqrt{24}$

f) $\sqrt{10}\times\sqrt{20}$

g) $5\sqrt{6}\times2\sqrt{18}$

h) $-4\sqrt{10}\times\sqrt{21}$

i) $2\sqrt{10}\times3\sqrt{50}$

j) $(-3\sqrt{12})\times(-2\sqrt{18})$

7. Multiply, and simplify if possible.

a) $\sqrt[3]{4} \times \sqrt[3]{6}$

b) $\sqrt[3]{9} \times \sqrt[3]{24}$

c) $\sqrt[3]{5} \times \sqrt[3]{5}$

d) $\sqrt[3]{4} \times \sqrt[3]{54}$

e) $2\sqrt[3]{12} \times \sqrt[3]{30}$

f) $-3\sqrt[3]{25} \times 4\sqrt[3]{75}$

g) $2\sqrt[3]{10} \times 3\sqrt[3]{50}$

h) $(-3\sqrt[3]{12})(-2\sqrt[3]{18})$

i) $(-3\sqrt[3]{4})(-2\sqrt[3]{32})$

j) $(-5\sqrt[3]{49})(2\sqrt[3]{56})$

8. Without a calculator, compare each expression using $>$, $<$ or $=$.
(Hint: Convert each expression to an entire root first)

a) $4\sqrt{14}$ ____ 15

b) $\sqrt{162}$ ____ $9\sqrt{2}$

c) $3\sqrt{11}$ ____ $7\sqrt{2}$

d) 12 ____ $\sqrt{11} \times \sqrt{13}$

e) $4\sqrt[3]{2}$ ____ 5

f) $2\sqrt[3]{7}$ ____ $\sqrt[3]{56}$

g) $2\sqrt[3]{15}$ ____ $\sqrt[3]{125}$

h) $5\sqrt[3]{3}$ ____ $4\sqrt[3]{4}$

i) $-3\sqrt[3]{25}$ ____ $-\sqrt[3]{676}$

j) $-2\sqrt[3]{7}$ ____ $-\sqrt[3]{55}$

9. Express as an entire root.

a) $4\sqrt{3}$

b) $2\sqrt{5}$

c) $7\sqrt{6}$

d) $12\sqrt{2}$

e) $5\sqrt{11}$

f) $4\sqrt{14}$

g) $6\sqrt{3}$

h) $2\sqrt{3}\times 3\sqrt{2}$

i) $2\sqrt{5}\times\sqrt{3}$

j) $4\sqrt{5}\times 3\sqrt{3}$

10. Express as an entire root.

a) $3\sqrt[3]{2}$

b) $4\sqrt[3]{3}$

c) $5\sqrt[3]{4}$

d) $3\sqrt[3]{5}$

e) $6\sqrt[3]{6}$

f) $4\sqrt[3]{7}$

g) $7\sqrt[3]{8}$

h) $3\sqrt[3]{2}\times 4\sqrt[3]{3}$

i) $2\sqrt[3]{4}\times 5\sqrt[3]{5}$

j) $3\sqrt[3]{6}\times\sqrt[3]{7}$

11. A square has an area of 150 mm^2. What are the lengths of the sides of the square?

12. A cube has a volume of 192 cm^3. What are the lengths of each edge of the cube?

13. The dimensions of a rectangle are $9\sqrt{30}$ cm by $4\sqrt{105}$ cm. Calculate the area of the rectangle.

14. The dimensions of a rectangle are $5\sqrt{6}$ cm by $4\sqrt{3}$ cm. Calculate the area of the rectangle.

15. The base of a triangle is $6\sqrt{14}$ ft and its height is $3\sqrt{21}$ ft. Calculate the area of the triangle.

16. The base of a triangle is $5\sqrt{33}$ m and its height is $6\sqrt{55}$ m. Calculate the area of the triangle.

17. The dimensions of a rectangular prism are: length $2\sqrt{10}$ cm, width $3\sqrt{14}$ cm, and height $\sqrt{35}$ cm. Determine the volume of the rectangular prism.

18. The dimensions of a rectangular prism are: length $3\sqrt{110}$ m, width $2\sqrt{22}$ m and height $4\sqrt{15}$ m. Determine the volume of the rectangular prism.

19. The dimensions of a rectangular base pyramid are: length $2\sqrt{42}$ cm, width $3\sqrt{30}$ cm, and height $4\sqrt{70}$ cm. Determine the volume of the pyramid.

20. The dimensions of a rectangular base pyramid are: length $3\sqrt{26}$ m, width $2\sqrt{39}$ m, and height $4\sqrt{42}$ m. Determine the volume of the pyramid.

1.7 Chapter Review

Section 1.1

1. Consider the list of numbers: $-2,\ 0.\overline{4},\ 0,\ 0.343343334\ldots,\ 4.222\ldots,\ \frac{5}{3},\ 7,\ \sqrt{2}$. List all:

a) Natural numbers

b) Whole numbers

c) Integers

d) Rational numbers

e) Irrational numbers

f) Real numbers

Section 1.2

2. Simplify the composite numbers to a product of prime numbers.

a) 4950

b) 1848

c) 2618

d) 264 264

3. Find the greatest common factor.

a) 126, 588

b) 1755, 2475

c) 7007, 13 013

d) 544, 600, 2250

4. Find the least common multiple.

a) 56, 196

b) 90, 300

c) 15, 20, 30

d) 30, 45, 84

Section 1.3

5. Determine the roots without a calculator.

a) $\sqrt{64}$

b) $\sqrt[3]{64}$

c) $\sqrt{729}$

d) $\sqrt[3]{729}$

e) $\sqrt{1296}$

f) $\sqrt[3]{2744}$

g) $\sqrt{1764}$

h) $\sqrt[3]{5832}$

Section 1.4

6. Without a calculator, determine if the number is a rational or irrational number.

a) $\sqrt{0.4}$

b) $\sqrt{0.04}$

c) $\sqrt{90}$

d) $\sqrt{900}$

e) $\sqrt[3]{0.27}$

f) $\sqrt[3]{0.027}$

g) $\sqrt[3]{800}$

h) $\sqrt[3]{8000}$

7. Using $\sqrt{18} \doteq 4.24$, $\sqrt{180} \doteq 13.42$, $\sqrt[3]{18} \doteq 2.62$, $\sqrt[3]{180} \doteq 5.64$, $\sqrt[3]{1800} \doteq 12.16$, determine the value of the radical.

a) $\sqrt{1.8}$

b) $\sqrt{0.18}$

c) $\sqrt{0.018}$

d) $\sqrt{1800}$

e) $\sqrt{18\,000}$

f) $\sqrt[3]{1.8}$

g) $\sqrt[3]{0.18}$

h) $\sqrt[3]{0.018}$

i) $\sqrt[3]{18\,000}$

j) $\sqrt[3]{180\,000}$

Section 1.5

8. Simplify. Express without brackets or negative exponents.

a) $\dfrac{2^4 \times 2^3}{2^5}$

b) $\dfrac{(3^2)^3 \times (3^4)^2}{(3^5)^2}$

c) $\dfrac{(2x^3)^{-2}}{(x^{-2})^4}$

d) $\left(\dfrac{3x^2}{2y^3}\right)^{-2}$

e) $(3a^{-2}b^{-3}c^4)^{-2}$

f) $\left(\dfrac{3a^{-2}}{2b^3}\right)^{-4}$

g) $\dfrac{(2x^2y^{-3})^2 \times (2x^{-1}y^4)^{-1}}{(2x^{-2}y^2)^{-3}}$

h) $\left(\dfrac{3x^{-1}y}{4x^2y^{-2}}\right)^{-2}\left(\dfrac{3x^2y^{-2}}{2x^{-2}y}\right)^3$

9. Simplify. Evaluate if possible.

a) $32^{\frac{4}{5}}$

b) $32^{-\frac{4}{5}}$

c) $-625^{\frac{3}{4}}$

d) $-625^{-\frac{3}{4}}$

e) $(27 \times 64)^{\frac{4}{3}}$

f) $(27 \times 64)^{-\frac{4}{3}}$

g) $\left(\frac{16}{81}\right)^{\frac{3}{4}}$

h) $\left(\frac{16}{81}\right)^{-\frac{3}{4}}$

i) $\sqrt[3]{x^6}$

j) $\sqrt[12]{x^4},\ x \geq 0$

k) $\dfrac{\sqrt{2}}{\sqrt[3]{2}}$

l) $\dfrac{\sqrt[3]{4}}{\sqrt{2}}$

m) $\dfrac{\sqrt[3]{3}}{\sqrt[4]{3}}$

n) $\dfrac{\sqrt[3]{4}}{\sqrt[4]{2}}$

o) $\dfrac{\sqrt{2} \times \sqrt[3]{2}}{\sqrt[4]{2}}$

p) $\dfrac{\sqrt{3} \times \sqrt[3]{9}}{\sqrt[4]{27}}$

Section 1.6

10. Simplify each radical.

a) $\sqrt{108}$

b) $\sqrt[3]{108}$

c) $\sqrt{288}$

d) $\sqrt[3]{288}$

e) $3\sqrt{54}$

f) $3\sqrt[3]{54}$

g) $2\sqrt{14} \times \sqrt{28}$

h) $2\sqrt[3]{14} \times \sqrt[3]{28}$

i) $-5\sqrt{12} \times \sqrt{54}$

j) $-5\sqrt[3]{12} \times \sqrt[3]{54}$

11. Express as an entire radical.

a) $3\sqrt{2}$

b) $3\sqrt[3]{2}$

c) $-2\sqrt{5}$

d) $-2\sqrt[3]{5}$

e) $5\sqrt{2} \times 2\sqrt{3}$

f) $5\sqrt[3]{2} \times 2\sqrt[3]{3}$

g) $3\sqrt{3} \times 2\sqrt{5} \times 4\sqrt{2}$

h) $4\sqrt[3]{3} \times 3\sqrt[3]{4} \times 2\sqrt[3]{2}$