2.1

Key

**~~3.1~~ – Classifying Polynomials**

**Learning Target:** to classify polynomials, combine like terms, multiply monomials together and multiple a monomial with a polynomial.

**Toolkit:**

- Multiply powers with the same base (add the exponents)
  Ex: $(x^3)(x^4) = x^7$

**Definitions:**

**Term:** a number or a product of a number with one or more variables, which can be raised to a power.

Ex: $7, \; x^2, \; -4xy, \; 10x^2y^2$

**Polynomial:** A term or sum of terms, in which all variables have **whole** number exponents, and which variables only appear in the numerator.

Ex: $3x^2 - 4x, \; 5xy - x,$

Examples of Polynomials

| Polynomials | Non-Polynomials |
|---|---|
| $5$ | $x^{\frac{1}{2}}$ |
| $\sqrt{2}x$ | $2x + \sqrt{y}$ |
| $3a^2 - 2a$ | $\frac{1}{2x} + 4$ |
| $\frac{3}{4}y^2 + 3y - 4$ | $x^{-3} - 2x$ |

Not whole number exponents

Classifying Polynomials

| | | |
|---|---|---|
| Monomial | A polynomial with one term | $3, \; 2x^2y, \; -3a$ |
| Binomial | A polynomial with two terms | $x+2, \; 2x^2y+3, \; x^2-y$ |
| Trinomial | A polynomial with three terms | $3x^2+2x-3, \; \sqrt{2}x+y-z$ |
| Polynomial | General term for expressions with more than three terms (can also be used to describe monomial, binomial, or trinomial) | $x^5 - 2x^4 + 3x^3 - 4x^2$ |

Degree of a Polynomial

The degree of a term ~~polynomial~~ is the **sum** of the exponents of the variables of that term. The degree of a polynomial is the term with the **highest** degree.

Leading Term

The term in a polynomial with the highest degree.

Ex 1)

Fill out the chart for the following polynomial:

$$4x^4y^2 - 10x^2y + 7$$

| | |
|---|---|
| Terms: | $4x^4y^2$ , $-10x^2y$ , $7$ |
| Coefficients: | $4$    $-10$    $7$ |
| Degree of each term: | $6$    $3$    $0$ |
| Leading Term: | $4x^4y^2$ |
| Degree of polynomial: | $6$ |

Combining like Terms:

**Like terms** are any terms in a polynomial that have the same variables to the same power. To combine like terms, add or subtract the coefficients.

**Like terms:** same variables, same power

$3x^3 \qquad -5x^3$

**Not like terms:** different powers on the variables

$4x^2y \qquad 5xy^2$

Ex 2)

Simplify the following expression:

$$-10x^2 - y + 4x^2 + 7y = -6x^2 + 6y$$

Multiplying a monomial by a monomial

**Step 1:** Multiply the constants together.
**Step 2:** Multiply the variables together (add exponents of variables with the same base).

Ex 3)

Multiply the following:

a) $(2x^3)(-3x^4)$

$-6x^7$

b) $(3xy^2)(5x^4y^6)$

$15x^5y^8$

c) $(-2a^3b^2)(5a^5b^2)(-a^2b)$

$10a^{10}b^5$

Multiplying a polynomial by a binomial

Use the Distributive Property to multiply a monomial with a polynomial:

**The Distributive Property:**

$$a(b + c) = a \times b + a \times c$$

Ex 4)

Find each product. Leave answer in descending order of powers.

a) $2x^3(4x^2+6)$

$(2x^3)(4x^2)+(2x^3)(6)$

$8x^5+12x^3$

b) $-3x^2(2x^4y^2-3x^5y+5)$

$-3x^2(2x^4y^2)-3x^2(-3x^5y)-3x^2(5$

$-6x^6y^2+9x^7y-15x^2$

c) $(4x^4)(-2y)(xy-4)$

$(-8x^4y)(xy-4)$

$(-8x^4y)(xy)+(-8x^4y)(-4)$

$-8x^5y^2+32x^4y$

2.2

## 3.2 – Multiplying Polynomials

**Learning Target:** to multiply binomials with polynomials with more than one term.

**Toolkit:**
- Adding, subtracting and multiplying monomials
- Multiplying powers with the same base
- Collecting like terms: same variables with same exponents

**Warm up**

Multiply the following:

a) $2ab(-3a^4 + 5b^6)$

$-6a^5b + 10ab^7$

b) $(-4xy)(5x)(x^2y^2 - 1)$

$-20x^2y(x^2y^2 - 1)$

$-20x^4y^3 + 20x^2y$

**Binomial Multiplication**

In order to multiply polynomials with more than one term, we need to use the distributive property more than once. We use the acronym **FOIL** to help us.

**First** $(a + b)(c + d) = ac$

**Outside** $(a + b)(c + d) = ad$

**Inside** $(a + b)(c + d) = bc$

**Last** $(a + b)(c + d) = bd$

**FOIL** $(a + b)(c + d) = ac + ad + bc + bd$

Ex 1) Multiply $(x + 2)(x + 3)$

$(x)(x) + (x)(3) + (2)(x) + (2)(3)$

$x^2 + \underbrace{3x + 2x} + 6$

$x^2 + 5x + 6$

**Ex 2)** Multiply $(2y + z)(3y - 2z)$

$(2y)(3y) + (2y)(-2z) + (z)(3y) + (z)(2z)$

$6y \underbrace{- 4yz + 3yz} + 2z^2$

$6y - yz + 2z^2$

**Ex 3)** Multiply $(2x + 1)^2 \Rightarrow (2x+1)(2x+1)$

$4x^2 + 2x + 2x + 1$

$4x^2 + 4x + 1$

When there are more than two terms in each polynomial, we cannot use the term **FOIL** but we can use the same concept and multiple every term in one polynomial with every term in the other.
**note: keep your work organized here! It's easy to get mixed up.**

Ex 4)

Multiply the following:

a) $(x+y)(x+y-3)$

$x^2+xy-3x+xy+y^2-3y$

$x^2+2xy-3x-3y+y^2$

b) $(r^2+3r-1)(2r^2-r+2)$

$2r^4-r^3+2r^2+6r^3-3r^2+6r-2r^2+r$

$2r^4+5r^3-3r^2+7r-2$

Special Case (like #3 in WB)

**Product of Sum and Difference**

$$(a+b)(a-b)) = a^2-b^2$$

Ex 5)

Multiply: $(3x+2y)(3x-2y)$

$9x^2\underbrace{-6xy+6xy}-4y^2$

$9x^2-4y^2$

2.3

# ~~1.3~~ – Removing Common Factors

**Learning Target:** To determine the factors of a polynomial by identifying the GCF

**Toolkit:**

- Finding the GCF
- The Distributive Property

| | |
|---|---|
| Factor the polynomial | Factoring is the **opposite** of multiplying. We can factor to rewrite a polynomial as a product.<br><br>To factor a polynomial, we must look for the largest factor that is the same in each term of the polynomial. We call this the **common factor**. |
| Ex 1) | Factor the following:<br><br>a) $3g + 6$    $3(g+2)$<br>b) $-8y + 16y^2$    $-8y(1-2y)$<br>c) $3x^2 + 12x - 6$    $3(x^2+4x-2)$ |
| Polynomials with more than one variable | $-20c^4d - 30c^3d^2 - 25cd$<br>$-5cd(4c^3+6c^2d+5)$ |
| Factoring out a common binomial | a) $x(x+1) + 3(x+1)$<br>$(x+1)(x+3)$<br><br>b) $2x(3x+2) - 7y(3x+2)$<br>$(3x+2)(2x-7y)$ |
| Factor by grouping | **Step 1:** Group the polynomials in pairs of two.<br>**Step 2:** Factor each pair of two.<br>**Step 3:** Remove the common factor. |
| Ex 2) | Factoring by grouping:<br>a) $(3x^3 - 6x^2) + (2x - 4)$<br>$3x^2(x-2) + 2(x-2)$<br>$(x-2)(3x^2+2)$<br><br>b) $(3x^2 - 12x) - (x + 4)$<br>$3x(x-4) - 1(x-4)$<br>$(x-4)(3x-1)$ |

2.4

## ~~3.4~~ – Factoring $x^2 + bx + c$

**Learning Target:** to factor trinomials in the form $x^2 + bx + c$, where the leading coefficient is 1 (the $a$ value = 1)

**Toolkit:**

- Factoring → write as a multiplication
  Ex. $2x - 10 = 2(x - 5)$

**Definitions:**

**Descending order**: the terms are written in order from the term with the greatest exponent to the term with the least exponent

Consider the example: $(x+a)(x+b) = x^2 + bx + ax + ab$

$$= x^2 + (b+a)x + ab$$

From this example we can see:

1. The product of $(x+a)(x+b)$ is a trinomial.
2. The first term, $x^2$, is the product of $x$ and $x$.
3. The coefficient of the middle term is the sum of $a$ and $b$.
4. The last term is the product of $a$ and $b$.

(**notice the $x^2$ has a coefficient of 1 here…this is what we call the "$a$" value… $a = 1$ in these trinomials)

From this, we can come up with a general rule for factoring polynomials of the type $x^2 + bx + c$ :

Steps for factoring $x^2 + bx + c$

**Steps for factoring a Trinomial in the form $x^2 + bx + c$, where $a = 1$**

**Step 1:** As with any factoring question, check to see if you can factor out a GCF

**Step 2:** If needed, re-order the terms in **descending** powers of the variable

**Step 3:** Find two numbers that multiply to equal the c term and add to equal the b term *(what multiplies to the end and adds to the middle!)*

**Step 4:** Factor into two binomials using the numbers from step 3 (one in each binomial), with the variable from the trinomial placed first in each bracket

you can check!
$(x-4)(x+2)$ FOIL!
$= x^2 + 2x - 4x - 8$
$= x^2 - 2x - 8$
which is what you started with! ✓

Ex 1) Factor:

a) $x^2 - 2x - 8$

GCF? no
reorder? no

$-4 \times 2 = -8$ end
$-4 + 2 = -2$ middle

$= \boxed{(x-4)(x+2)}$

or $= (x+2)(x-4)$

b) $z^2 - 8z + 7$

GCF? no
reorder? no

$-1 \times -7 = 7$ (end)
$-1 + -7 = -8$ (middle)

$= \boxed{(z-1)(z-7)}$

Re-order

Ex 2) Factor:

GCF? no
reorder? yes

a) $x^2 + 8 - 6x$

$= x^2 - 6x + 8$

$\underline{-2} \times \underline{-4} = 8$

$\underline{-2} + \underline{-4} = -6$

$\boxed{= (x-2)(x-4)}$

GCF? no
reorder? yes

b) $30 + x^2 + 13x$

$= x^2 + 13x + 30$

$\underline{10} \times \underline{3} = 30$

$\underline{10} + \underline{3} = 13$

$= (x+10)(x+3)$

Remove a GCF first

Ex 3) Factor:

GCF? yes!
reorder? no

now...
a = 1...

a) $\frac{3x^2}{3} + \frac{21x}{3} + \frac{36}{3}$

$= 3(x^2 + 7x + 12)$

$\underline{3} \times \underline{4} = 12$

$\underline{3} + \underline{4} = 7$

$\boxed{= 3(x+3)(x+4)}$

GCF? yes!
reorder? no

now, a = 1!

b) $\frac{-5h^3}{-5h} - \frac{20h^2}{-5h} + \frac{60h}{-5h}$

$= -5h(h^2 + 4h - 12)$

$\underline{6} \times \underline{-2} = -12$

$\underline{6} + \underline{-2} = +4$

$= -5h(h+6)(h-2)$

Trinomials with two variables

attach the second variable (the y here), to the second term in each bracket

Ex 4) Factor:

GCF? no
reorder? no

a) $x^2 + 4xy - 21y^2$

$\underline{7} \times \underline{-3} = -21$

$\underline{7} + \underline{-3} = 4$

$= (x+7y)(x-3y)$

GCF? no
reorder? no

b) $c^2 + cd - 42d^2$

$\underline{7} \times \underline{-6} = -42$

$\underline{7} + \underline{-6} = 1$

$= (c+7d)(c-6d)$

2.5

## ~~3.5~~ – Factoring $ax^2 + bx + c$

**Learning Target:** to factor trinomials of the form $ax^2 + bx + c$, where the $a$ value $\neq 1$

**Toolkit:**

- Multiplying binomials
- Factoring by grouping

When $a \neq 1$in a trinomial of the form $ax^2 + bx + c$, and it can't be factored out, then another process is needed.

This process is called DECOMPOSITION, and uses factoring by grouping.

**Steps for factoring $ax^2 + bx + c$, $a \neq 1$ by DECOMPOSITION**

**Step 1:** As with any factoring question, check to see if you can factor out a GCF

**Step 2:** If needed, re-order the terms in **descending** powers of the variable

**Step 3:** Find two numbers that multiply to equal $ac$ and add to equal $b$
*(multiply to the product (×) of the first and last, and add to the middle!)*

**Step 4:** Rewrite the expression but split or *decompose* the middle ($b$) term, using the two numbers from step 3

**Step 5:** Now the expression has FOUR terms, so we can **factor by grouping** the first two terms and the last two terms

**Step 6:** When factoring by grouping, the two resulting binomials need to be identical! These matching binomials are now the COMMON FACTOR, and can be factored out…and what is left become the components of the second bracket.

Ex. 1) Factor: a=4, decomp!

GCF? no
reorder? no

$$4x^2 + 11x + 6$$

$$8 \times 3 = \underset{(4)(6)}{24} \quad (a \times c)$$
$$8 + 3 = 11 \quad (b)$$

so… split $11x$ into $8x+3x$ (or $3x+8x$)

$$= 4x^2 + 8x + 3x + 6$$

now, factor by grouping
*remove GCF from both pairs

$$= 4x(x+2) + 3(x+2)$$

so, binomials match; factor out $(x+2)$ from both terms.

$$= \boxed{(x+2)(4x+3)}$$

*note: if you factor by grouping and the binomials that result DO NOT MATCH, you have made a mistake.

Ex 2) $-7m - 10 + 6m^2$

GCF? no
reorder? yes!

$= 6m^2 - 7m - 10$

$= \underbrace{6m^2 - 12m} + \underbrace{5m - 10}$

$= 6m(m-2) + 5(m-2)$

$= \boxed{(m-2)(6m+5)}$

a = 6, so decomp!

$\underline{-12} \times \underline{5} = \underset{(6)(-10)}{\underline{-60}}$ (a×c)

$\underline{-12} + \underline{5} = \underline{-7}$ (b)

so... split $-7m$ into $-12m + 5m$!

Ex 3) $8p^2 - 18p - 5$

GCF? no
reorder? no

$= \underbrace{8p^2 + 2p} \underbrace{- 20p - 5}$

$= 2p(4p+1) - 5(4p+1)$

$= \boxed{(4p+1)(2p-5)}$

a = 8, so decomp!

$\underline{2} \times \underline{-20} = \underset{(8)(-5)}{\underline{-40}}$ (a×c)

$\underline{2} + \underline{-20} = \underline{-18}$ (b)

so... split $-18p$ into $2p$ and $-20p$!

Ex 4) $\frac{24}{2}x^2 - \frac{10}{2}x - \frac{4}{2}$

GCF? yes!
reorder? no

$= 2(12x^2 - 5x - 2)$

$= 2(\underbrace{12x^2 + 3x} \underbrace{- 8x - 2})$

$= 2[3x(4x+1) - 2(4x+1)]$

$= \boxed{2(4x+1)(3x-2)}$

a = 12, so decomp!

$\underline{3} \times \underline{-8} = \underset{(12)(-2)}{\underline{-24}}$

$\underline{3} + \underline{-8} = \underline{-5}$

so... split $-5x$ into $3x - 8x$!

Ex 5) $\frac{-16x^3}{-4x} + \frac{20x^2}{-4x} + \frac{24x}{-4x}$

GCF? yes!
reorder? no

$= -4x(4x^2 - 5x - 6)$

$= -4x(\underbrace{4x^2 - 8x} + \underbrace{3x - 6})$

square brackets here... just to help keep work organized

$= -4x[4x(x-2) + 3(x-2)]$

$= \boxed{-4x(x-2)(4x+3)}$

a = 4, so decomp!

$\underline{-8} \times \underline{3} = \underset{(4)(-6)}{\underline{-24}}$

$\underline{-8} + \underline{3} = -5$

so... split $-5x$ into $-8x + 3x$!

2.6

~~3.6~~ – **Special Factors**

**Learning Target:** To factor a difference of squares and perfect square trinomials

**Toolkit:**

- Finding a square root
- Decomposition

### Factoring the Difference of Squares

perfect square, perfect square

$$a^2 - b^2 = (a+b)(a-b)$$

a subtraction (difference!)

Ex 1) Factor: a) $x^2 - 16$

$\sqrt{x^2} = x$
$\sqrt{16} = 4$

$= \boxed{(x+4)(x-4)}$

or $= (x-4)(x+4)$

b) $81m^2 - 49$

$\sqrt{81m^2} = 9m$
$\sqrt{49} = 7$

$= \boxed{(9m+7)(9m-7)}$

c) $162v^2 - 2w^6$

162 and 2 not perfect squares... GCF of 2!

$= 2(81v^2 - w^6)$

now...
$\sqrt{81v^2} = 9v$
$\sqrt{w^6} = w^3$

$= \boxed{2(9v+w^3)(9v-w^3)}$

d) $45r^2 + 5s^2$

$= \boxed{5(9r^2+s^2)}$

that's it!
since we have no way to factor the SUM (+) of squares!

e) $16x^4 - 1$

$\sqrt{16x^4} = 4x^2$
$\sqrt{1} = 1$

$= (4x^2+1)(4x^2-1)$ → $4x^2-1$ is another diff of squares!

$\sqrt{4x^2} = 2x$
$\sqrt{1} = 1$

$= \boxed{(4x^2+1)(2x+1)(2x-1)}$

sum of squares here

## Perfect Square Trinomials

A trinomial that is a square of a binomial is called a perfect square trinomial.

**Factoring Perfect Square Trinomials**

$$a^2 + 2ab + b^2 = (a+b)^2$$

$$a^2 - 2ab + b^2 = (a-b)^2$$

Ex 2) Factor: $36x^2 + 12x + 1$

$= 36x^2 + 6x + 6x + 1$

$= 6x(6x+1) + 1(6x+1)$

$= (6x+1)(6x+1)$

$= \boxed{(6x+1)^2}$

GCF? no
reorder? no
a = 36... decomp!

$\underline{6} \times \underline{6} = \underset{(36)(1)}{36}$

$\underline{6} + \underline{6} = \underline{12}$

so split $12x$ into $6x + 6x$!

identical binomials, so $(6x+1)(6x+1) = (6x+1)^2$!

Ex 3) Factor: $4c^2 - 12c + 9$

$= 4c^2 - 6c - 6c + 9$

$= 2c(2c-3) - 3(2c-3)$

$= (2c-3)(2c-3)$

$= \boxed{(2c-3)^2}$

GCF? no
reorder? no
a = 4, ... decomp!

$\underline{-6} \times \underline{-6} = \underset{(4)(9)}{36}$

$\underline{-6} + \underline{-6} = \underline{-12}$

so split $-12c$ into $-6c - 6c$!

Ex 4) Factor: $\frac{3x^2}{3} + \frac{24x}{3} + \frac{48}{3}$

$= 3(x^2 + 8x + 16)$

$\underline{4} \times \underline{4} = 16$

$\underline{4} + \underline{4} = 8$

$= 3(x+4)(x+4)$

$= \boxed{3(x+4)^2}$

GCF? yes!
reorder? no

a = 1, so decomp not necessary.
(but would work ☺)