3.1

[redacted] - Relations Key

Learning Target: to discuss the concept of a relation and to represent relations in different ways

Toolkit:	
- anything we already know about graphs and relations	

Definitions

Ordered pair: two numbers written in a certain order, (x, y), that can be used to show a position on a graph. The x value (horizontal) is first and the y value (vertical) is second.

Coordinate System: a system that uses the (x, y) ordered pairs to determine the position of a point.

Ex 1) Plots the points on the grid provided:

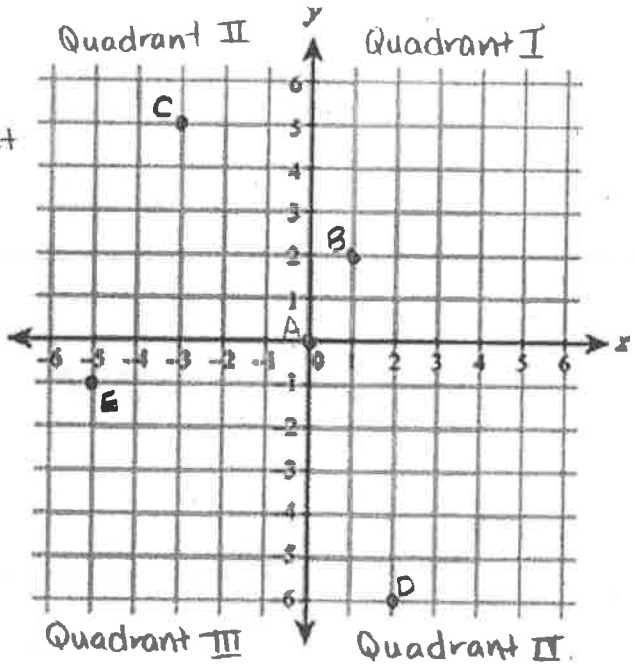
A (0, 0) – the origin. Use the origin as the starting place when plotting any ordered pair.

B (1, 2) – located one unit to the ~~left~~ right and two units up from the origin.

C (-3, 5) - 3 left and 5 up

D (2, -6) → 2 right, 6 down.

E (-5, -1) → 5 left, 1 down.



Definitions 1)

Set – a collection of distinct objects

Element – an element of a set is one object in the set

Relation – a relation associates the elements of one set with the elements of another set

There are many ways to represent a relation between two sets:

Ex 2)

1. Ordered pairs: $(-3,-5),\ (0,1),\ (2,5)$

2. In a table:

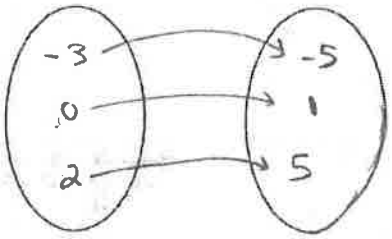
Horizontal:
x value always on the top, y value always on the bottom

x	-3	0	2
y	-5	1	5

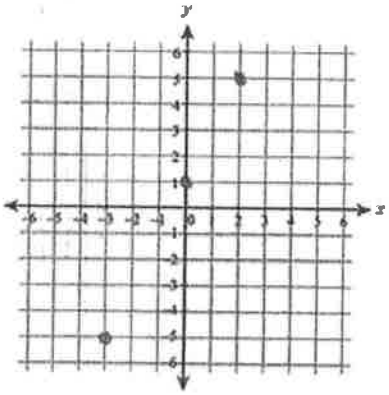
Vertical:
x value always on the left,
y value always on the right

x	y
-3	-5
0	1
2	5

3. Using mapping notation:



4. As a graph:



Definitions 2)

Domain – the set of first elements of a relation

Range – the set of second elements of a relation

Independent Variable – a variable whose value is not determined by another variable. Typically, values are assigned to x arbitrarily which means **x** is the independent variable.

Dependent Variable – a variable whose value is determined by the value of another (the independent) variable. The choice of x produces the output values for **y**, which means the y value is the dependent variable.

Ex 3) Determine the domain and range of the ordered pairs:

$A = \{(-3,-5),\ (0,1),\ (2,5)\}$

Domain of A = {-3, 0, 2}
Range of A = {-5, 1, 5}

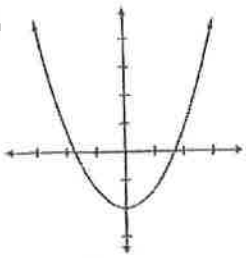
$B = \{(2,0), (-3,0), (-4,2), (9,1)\}$

Domain of B = {-4, -3, 2, 9}
Range of B = {0, 1, 2}

Do not repeat elements. Write in order from smallest to largest

Ex 4) Determine the domain and the range of the following graphs:

a)



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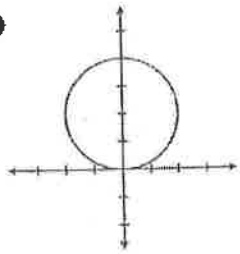
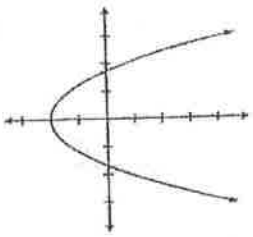
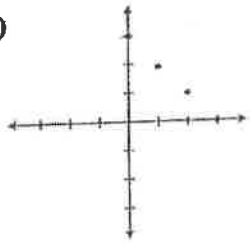


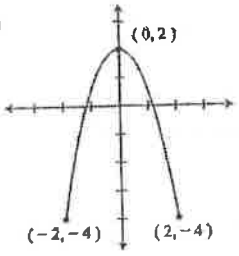
c)



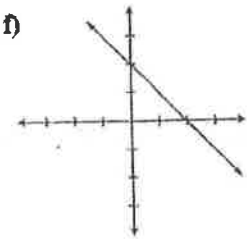
d)



e)



f)



a) Domain: $x \in \mathbb{R}$ all real numbers

Range: $y \geq -2$,

b) Domain: $-2 \leq x \leq 2$

Range: $0 \leq y \leq 4$

c) Domain: $x \geq -2$

Range: all real numbers $y \in \mathbb{R}$

d) Domain: $\{0, 1, 2, 3\}$

Range: $\{0, 1, 2, 3\}$

e) Domain: $-2 \leq x \leq 2$

Range: $-2 \leq y \leq 2$

f) Domain: $x \in \mathbb{R}$ all real numbers

Range: $y \in \mathbb{R}$ all real numbers

3.2

- Functions

Learning Target: to develop the concept of a function and to be able to recognize functions and one-to-one functions

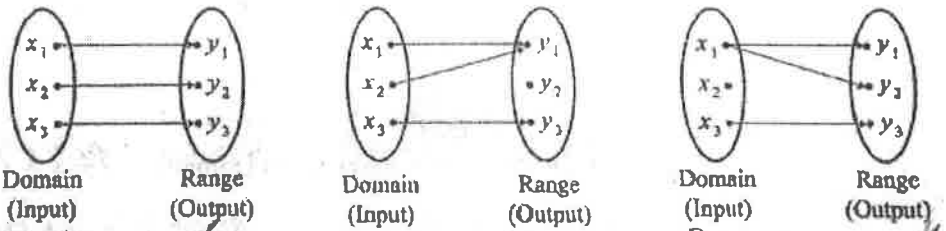
Toolkit:	
• Relations • Ordered pairs, independent/dependent variables	

Definitions

Function: a special type of relation where each element in the domain is associated with exactly one element in the range (OR a set of ordered pairs in which no two ordered pairs have the same first coordinate).

One-to-one Function: a function in which every one value of the domain is associated with one value in the range and vice-versa.

Ex 1)



Relation ✓ Function ✓ One-to-one ✓

Relation ✓ Function ✓ One-to-one X

Relation ✓ Function X One-to-one X

Ex 2) Are the following sets of ordered pairs relations, functions and/or one-to-one functions?

a) (Baker, PE), (Johnston, Math), (Fogelklou, Math)

Relation ✓, Function ✓ One-to-one X

↳ "Math" not a unique range

b) $(-3,-5)$, $(0,1)$, $(2,5)$

Relation ✓ Function ✓ One-to-one ✓

c) $(1,2)$, $(2,3)$, $(1,4)$

Relation ✓ Function X → 1 associated with 2 and 4 in range

When given a graph, we can use two tests to determine whether a graph is a function and/or a one-to-one function.

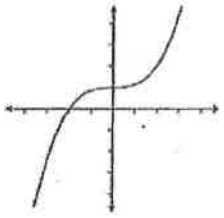
Vertical Line Test: a graph represents a function when no two points on the graph lie on the same vertical line.

-- run a ruler vertically along a graph: should only ever touch once --

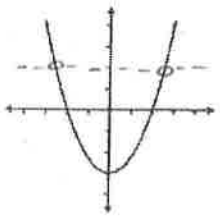
Horizontal Line Test: a graph represents a one-to-one function if it passes the vertical line test and no two points on the graph lie on the same horizontal line.

-- run a ruler horizontally along the graph: should only every touch once --

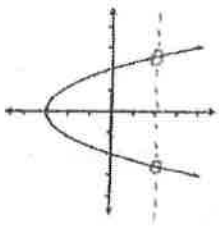
Ex 3) State whether each of the following relations is a function, a one-to-one function, or neither.

a) 

vertical line test ✓
⇒ Function
horizontal line test ✓
⇒ one to one

b) 

V.L.T. ✓
⇒ Function
H.L.T. X
⇒ not one to one

c) 

V.L.T. X
⇒ not a function
∴ cannot be one to one if it is not a function

3.3

– Linear Equations

Learning Target: to recognize functions from algebraic equations, and to graph linear equations using ordered pairs

Toolkit:	
- graphing ordered pairs - solving an equation for a variable	

Definitions

Expression: a collection of numbers, variables, and operation signs.

Ex. 5, $3x-7$, $2x^2+5x-4$, $\sqrt{x-1}$

Equation: a mathematical statement that two expressions are equal

Ex. $y=2$, $y=6x+1$, $x+4y=7$, $x=y^2$

Linear Equation: and equation of the form $x + By = C$, where $A, B, and\ C$ are constants, and $x\ and\ y$ are variables. All linear equations are functions, except a vertical line

Ex. $x=3$ → a vertical line @ $x=3$, so NOT a function

$y=-1$ → a horizontal line @ $y=-1$, so a function

$2x+3y=5$ → A function

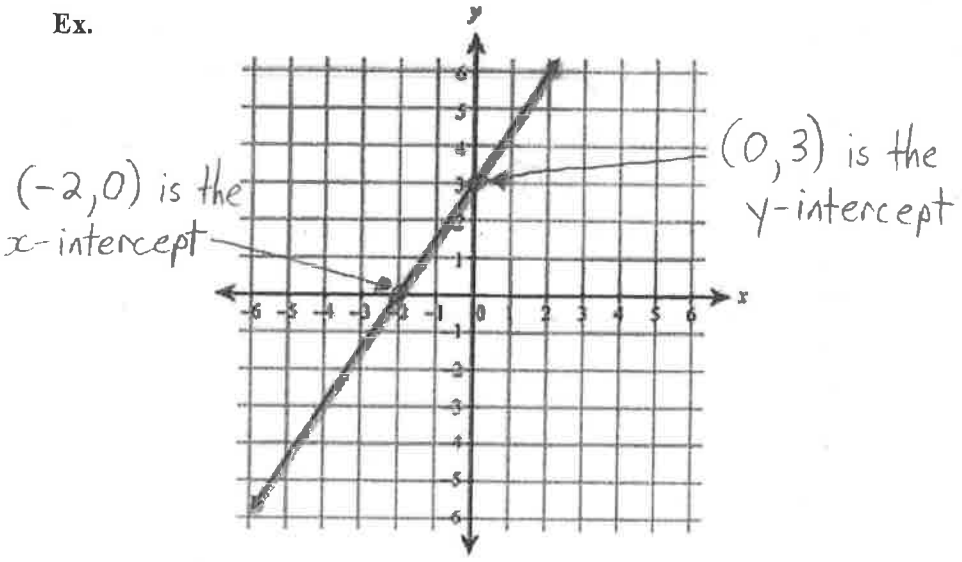
***y*-intercept:** where a graph of a relation crosses or touches the y-axis (the vertical axis). The coordinates of any y-intercepts are always $(0, b)$;

** The x value at any y-intercept is always 0.

***x*-intercept:** where a graph of a relation crosses or touches the x-axis (the horizontal axis). The coordinates of any x-intercepts are always $(a, 0)$;

**The y value at any x-intercept is always 0.

Ex.



Graphing Linear Equations of the Type $Ax + By = C$

1. To find the y-intercept, set $x = 0$ and solve for y.
   To find the x-intercept, set $y = 0$ and solve for x.
2. To get a third point, pick another value for x, and solve for y.
3. Plot three points from steps 1 and 2. Draw a straight line through the points.

Note: If a straight line does not go through all three points, a mistake has been made, and you must check your work to find the error.

Ex 1) **Graph $3x + 2y = 6$**

Find the **y-intercept**: (set x=0)

$3x + 2y = 6$
$3(0) + 2y = 6$
$2y = 6$
$y = 3$

so... $(0, 3)$

Find the **x-intercept**: (set y=0)

$3x + 2y = 6$
$3x + 2(0) = 6$
$3x = 6$
$x = 2$

so... $(2, 0)$

Find a third point. (easy value for x:)

let's choose $x = -2$

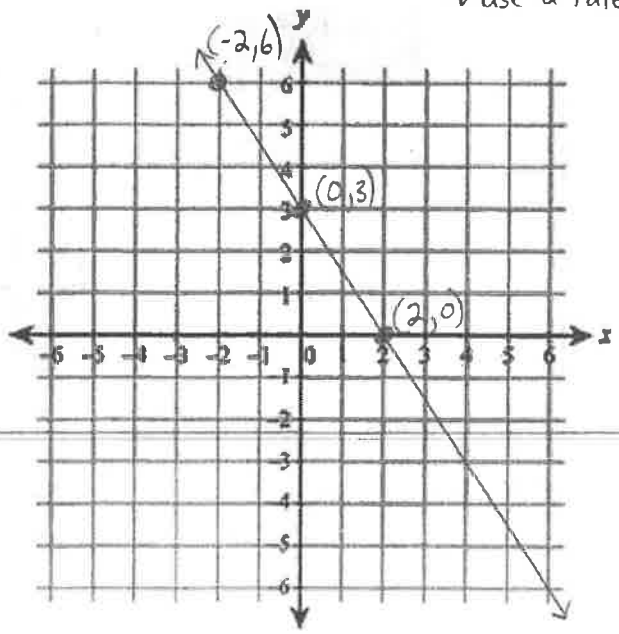
$3x + 2y = 6$
$3(-2) + 2y = 6$
$-6 + 2y = 6$
$+6 \quad +6$
$2y = 12$
$y = 6$

so... $(-2, 6)$

Now, we have three ordered pairs. **Plot these points** and draw a straight line through these points, and extend the line in both directions.

↳ use a ruler!

x	y
0	3
2	0
-2	6



Summary of the Ordered Pair (x, y)

x	y
Domain	Range
Input	Output
Independent Variable	Dependent Variable

Graphing Linear Equations of the Type $y = mx + b$

1. Select three values of x that are divisible by the denominator of the slope.
2. Solve for y in each case.
3. Plot three points from steps 1 and 2. Draw a straight line through the points.

Ex 2) Graph $y = -\frac{2}{3}x + 4$

Denominator of slope ($m = -\frac{2}{3}$) is 3...so pick 3 x-values divisible by 3 and solve for y:

$x = 0$

$y = -\frac{2}{3}x + 4$

$y = -\frac{2}{3}(0) + 4$

$y = 4$

so... $(0, 4)$

$x = 3$

$y = -\frac{2}{3}x + 4$

$y = -\frac{2}{3}(3) + 4$

$y = -2 + 4$

$y = 2$

so... $(3, 2)$

$x = -3$

$y = -\frac{2}{3}x + 4$

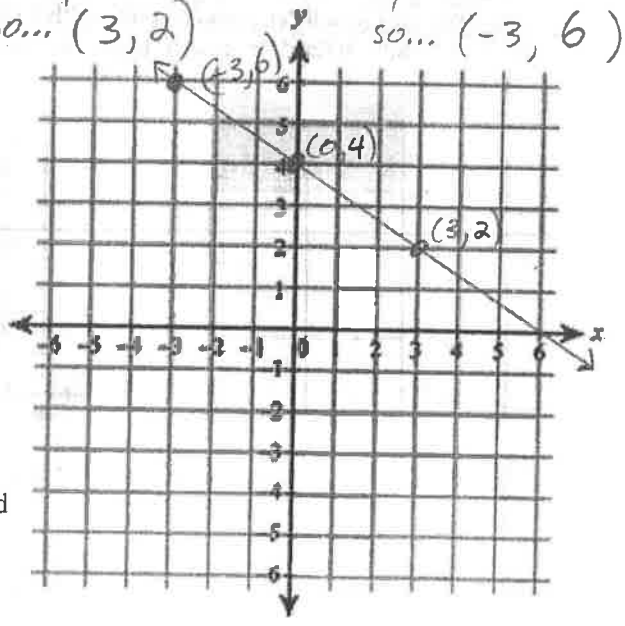
$y = -\frac{2}{3}(-3) + 4$

$y = 2 + 4$

$y = 6$

so... $(-3, 6)$

x	y
0	4
3	2
-3	-2



Now, plot these three points and draw a straight line through them, and extend the line in both directions

Ex. 3) **Determine whether the given ordered pair is a solution to the equation.**

* substitute in the values of x and y into equation, and see if the left side = right side.

a) $(0, 4)$; $y = -\frac{1}{3}x + 4$

$4 = -\frac{1}{3}(0) + 4$

$4 = 0 + 4$

$4 = 4$ ✓ left side = right side, so $(0,4)$ is a solution!

b) $(6, 8)$; $\frac{1}{3}x - \frac{1}{4}y = 4$

$\frac{1}{3}(6) - \frac{1}{4}(8) = 4$

$\frac{6}{3} - \frac{8}{4} = 4$

$2 - 2 = 4$

$0 \neq 4$ X Left side ≠ right side, so $(6,8)$ not a solution!

c) $(-1, 3)$; $y = -1$

$3 \neq -1$ x left side ≠ right side, so $(-1,3)$ not a solution!

3.4

– Non-Linear Equations

Learning Target: to investigate graphs produced by non-linear equations

Toolkit:	
• Creating a table of values to graph a relation • Plotting coordinates on a plane	

Rules for Graphing Non-Linear Equations:

1. Use positive numbers, negative numbers, and zero whenever possible.
2. If any value is to an even power, both positive and negative values must be used.
3. Use values between 0 and 1 when the variable is in the denominator, or is in the exponent.

Ex 1) **Is $y = x^2 - 1$ a function?**

- *Since x is an even power (has an even exponent), both positive and negative values of x are used to create a table of values*

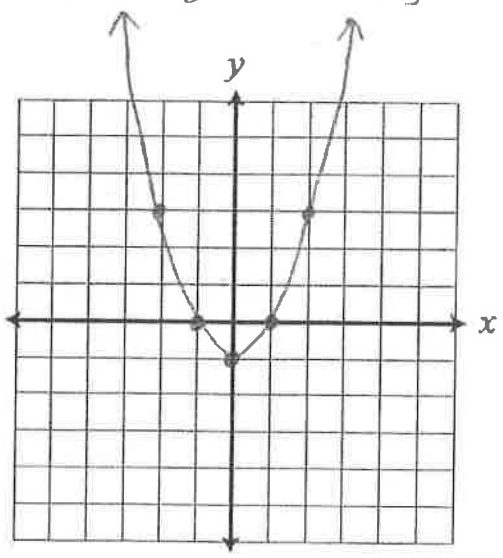
x	0	1	-1	2	-2	3	-3
y	-1	0	0	3	3	8	8

x^2-1 x^2-1 x^2-1 x^2-1 x^2-1 x^2-1 x^2-1

$=0^2-1$ $=(1)^2-1$ $=(-1)^2-1$ $=(2)^2-1$ $=(-2)^2-1$ $=(3)^2-1$ $=(-3)^2-1$

$=-1$ $=0$ $=1-1$ $=4-1$ $=4-1$ $=9-1$ $=9-1$

$=0$ $=3$ $=3$ $=8$ $=8$



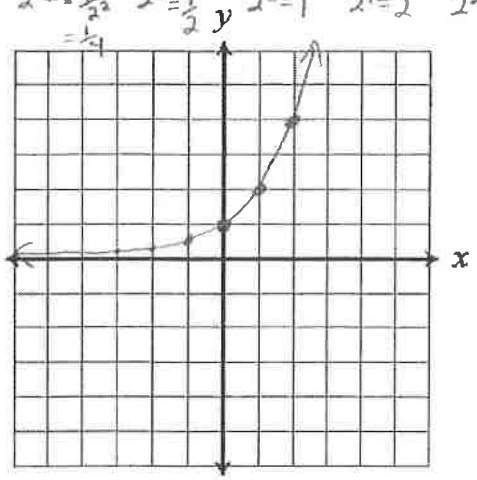
Is it a function? Why or why not? yes, since any vertical line intersects the graph only once.

Ex 2) **Is $y = 2^x$ a function?**

- Since x is the exponent, both positive and negative values of x are used

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $\quad 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ $\quad 2^{-1} = \frac{1}{2}$ $\quad 2^0 = 1$ $\quad 2^1 = 2$ $\quad 2^2 = 4$ $\quad 2^3 = 8$



Is it a function? Why or why not? yes, since any vertical line intersects the graph only once.

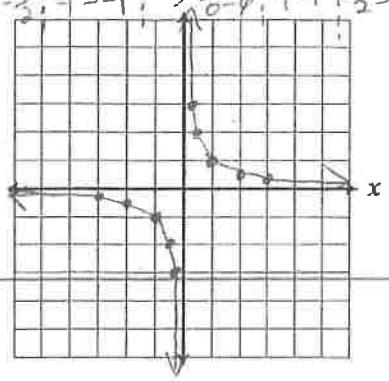
Ex 3) **Is $y = \frac{1}{x}$ a function?**

- *Since x is in the denominator, values between 0 and 1 must be used.*

x	-3	-2	-1	0	1	2	3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
y	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	$\emptyset$	1	$\frac{1}{2}$	$\frac{1}{3}$	2	-2	3	-3

$\frac{1}{-3} = -\frac{1}{3}$ $\quad \frac{1}{-2} = -\frac{1}{2}$ $\quad \frac{1}{-1} = -1$ $\quad \frac{1}{0} = \emptyset$ $\quad \frac{1}{1} = 1$ $\quad \frac{1}{2} = \frac{1}{2}$ $\quad \frac{1}{3} = \frac{1}{3}$ $\quad \frac{1}{\frac{1}{2}} = 1 \times \frac{2}{1} = 2$ $\quad \frac{1}{\frac{1}{3}} = 1 \times \frac{3}{1} = 3$

$\frac{1}{-\frac{1}{2}} = 1 \times \frac{-2}{1} = -2$ $\quad \frac{1}{-\frac{1}{3}} = 1 \times \frac{-3}{1} = -3$



Is it a function? Why or why not? yes, since any vertical line intersects the graph only once