3.1 – Solving Quadratic Equations by Factoring

Key

A function of *degree 2* (meaning the highest exponent on the variable is 2), is called a **Quadratic Function.**

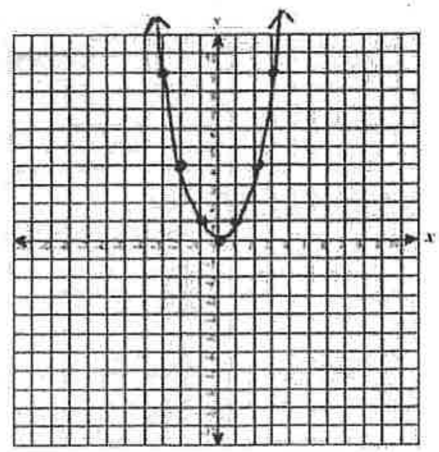
Quadratic functions are written as, for example, $f(x) = x^2 - x - 6$ OR $y = x^2 - x - 6$. *f(x)* is '*f* of *x*' and means that the *y* value is dependent upon the value of *x*. Once you have an *x* value and you substitute it into the function, the value of *f(x)* will result. *f(x)* is really just a Type equation here.another way of writing *y*, and is used when the graph is a function (passes the vertical line test).

When a quadratic function is graphed, a **parabola** results.

Example 1: $y = x^2$

$y = (-3)^2 = 9$
$y = (-2)^2 = 4$
$y = (-1)^2 = 1$
$y = 0^2 = 0$
etc

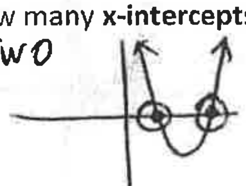
x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



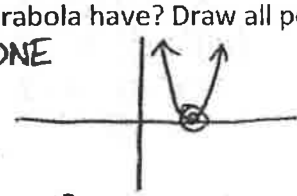
A very important feature of the parabola that results from a quadratic function is where it touches or crosses the *x*-axis. These are the *x*-intercepts of the parabola.

How many **x-intercepts** can a parabola have? Draw all possibilities:

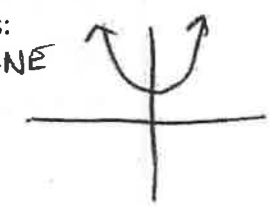
TWO



ONE



NONE



What is the *y* value at an *x*-intercept? $y=0$ at an x-int

Therefore, to find the *x*-intercepts of a parabola, we can set ***y* = 0 (or *f(x)* = 0)** and solve the resulting **quadratic equation.**

When ***y*** is set to 0, we call the question a **quadratic equation** instead of a quadratic function.

Quadratic function: $f(x) = x^2 - x - 6$ OR $y = x^2 - x - 6$

Quadratic equation: $x^2 - x - 6 = 0$

The ***x-intercepts*** of the parabola are the **zeros** of the quadratic *function.* They are also called the **solutions** or **roots** of the quadratic *equation.*

Therefore, how many **zeros** can there be for a quadratic function? 0, 1, or 2

How many **roots** or **solutions** can there be for a quadratic equation? 0, 1, or 2

One method to find the **zeros** of a **quadratic function** is to graph it and visually determine the ***x-intercepts.***

You can often find the **roots** of a **quadratic equation** by factoring when in general form $ax^2 + bx + c = 0$. Remember, the **roots** or **solutions** of the quadratic **equation** correspond to the **zeros** of the quadratic **function**, and the **x-intercepts** of the **parabola**.

if $a = 1$

Example 2 - Solve and check $x^2 + 3x = -2$

To '**solve**' a quadratic equation means to find the **roots or solutions**. Steps are as follows:

1) Get everything to one side so that only zero is on the other.
2) Identify a, b, and c values, and factor accordingly into binomials.
3) The roots are the x-values that will make the product of the binomials zero. If either of the binomials equal zero, then the product of the binomials will equal zero (this is called the **Zero Factor Property**). Therefore, identify the x values that make each binomial equal to zero.

Solve and check $x^2 + 3x = -2$

① $x^2+3x+2=0$

② $a=1,\ b=3,\ c=2$

Find two numbers that multiply to c and add to b

$\times 2,\ +3$

2, 1

$(x+2)(x+1)=0$

If either bracket equals 0, the eqn equals 0

③ $(x+2)(x+1)=0$

-2 makes this bracket zero

-1 makes this bracket zero

$x = -2, -1$

Check: $\underline{-2}$

$x^2+3x=-2$

$(-2)^2+3(-2)=-2$

$4+-6=-2$

✓

$\underline{-1}$

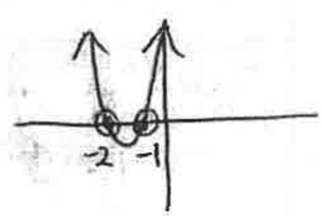
$x^2+3x=-2$

$(-1)^2+3(-1)=-2$

$1+-3=-2$

✓

Sketch the Graph:



Example 3: a) Solve $x^2 - 8x - 40 = 8$

$-8 \quad -8$

$x^2-8x-48=0$

$(x-12)(x+4)=0$

$x = 12, -4$

b) Solve $2x^2 + 6x - 108 = 0$

$2(x^2+3x-54)=0$

$2(x+9)(x-6)=0$

$x=-9, 6$

c) Solve $\frac{1}{2}x^2 - x - 4 = 0$

$\frac{1}{2}(x^2-2x-8)=0$

$\frac{1}{2}(x-4)(x+2)=0$

$x=4, -2$

d) Solve $\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$

$\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{(x-5)(x+1)}$

$\boxed{x \neq 5, -1}$

$x(x+1)-3(x-5)=30$

$x^2+x-3x+15=30$

$x^2-2x-15=0$

$(x-5)(x+3)=0$

$x=\cancel{5}, -3$

$x=-3$

if $a \neq 1$

When $a \neq 1$, factor by "decomposition."

Example 4 – Solve $3x^2 - 5x + 2 = 0$

$a=3,\ b=-5,\ c=2$ $\quad \times ac \quad +b$

$\times 6 \quad + -5$

$-3, -2$

$3x^2-3x-2x+2=0$

$3x(x-1)-2(x-1)=0$

$(x-1)(3x-2)=0$

$x-1=0 \qquad 3x-2=0$

$x=1 \qquad 3x=2$

$x=\frac{2}{3}$

$x=1, \frac{2}{3}$

Trick for finding roots:

$(3x-2)=0$

'Right over left, switch sign'

$x=\frac{2}{3}$

Example 5 – a) Solve $3(x^2-2)=-7x$

$3x^2-6=-7x$

$3x^2+7x-6=0$

$3x^2+9x-2x-6=0$

$3x(x+3)-2(x+3)=0$

$(x+3)(3x-2)=0$

$x=-3, \frac{2}{3}$

b) Solve $2x^2(3x+2)-5x(3x+2)=-2(3x+2)$

$2x^2(3x+2)-5x(3x+2)+2(3x+2)=0$

$(3x+2)(2x^2-5x+2)=0$

$(3x+2)(2x^2-4x-x+2)=0$

$(3x+2)[2x(x-2)-1(x-2)]=0$

$(3x+2)(x-2)(2x-1)=0$

$x=-\frac{2}{3}, 2, \frac{1}{2}$

* This is a cubic equation so can have 3 roots!

If $c = 0$

Example 6 - Solve $3x^2 = -5x$

$3x^2+5x=0$

$x(3x+5)=0$

$x=0, -\frac{5}{3}$

difference of squares

Example 7 – a) Solve $x^2 - 25 = 0$

$(x+5)(x-5)=0$

$x=\pm 5$

b) Solve $8p^2 - 18 = 0$

$2(4p^2-9)=0$

$2(2p+3)(2p-3)=0$

$p=\pm\frac{3}{2}$

c) Solve $49 - 4x^2 = 0$

$(7+2x)(7-2x)=0$

can't use 'trick' as x term in second position.

$7+2x=0 \qquad 7-2x=0$

$2x=-7 \qquad 7=2x$

$x=-\frac{7}{2} \qquad x=\frac{7}{2}$

Example 8 - a) Write a quadratic equation with roots 6 and -1. b) $\frac{2}{3}$ and $-\frac{1}{2}$

a $\quad (x-6)(x+1)=0$

$x^2-5x-6=0$

b $\quad (3x-2)(2x+1)=0$

$6x^2-x-2=0$

3.2A – Solving Quadratic Equations by Completing the Square

square root property

When there is no *bx* term in a quadratic equation, first look to see if it is a *difference of squares* (is each term a perfect square, and is there a subtraction sign in between?). If it is not a difference of squares, it can be solved by the *square root property.*

The general form of a quadratic equation that you could solve using the square root method is $ax^2 \pm c = 0$.

Example 1 – Solve $9y^2 - 21 = 0$

1) Get everything that <u>isn't</u> squared to one side.
2) Square root both sides.
3) Consider both the positive and negative square roots.
4) Simplify any radical solutions as much as possible in exact form.

① $9y^2 - 21 = 0$

$9y^2 = 21$

$y^2 = \frac{21}{9}$

② $\sqrt{y^2} = \pm\sqrt{\frac{21}{9}}$

③ $y = \pm\frac{\sqrt{21}}{\sqrt{9}}$

$y = \pm\frac{\sqrt{21}}{3}$

Example 2 - Solve: a) $2x^2 - 11 = 87$ b) $50y^2 = 72$ c) $(x+3)^2 = 16$

ⓐ $\frac{2x^2}{2} = \frac{98}{2}$

$x^2 = 49$

$\sqrt{x^2} = \pm\sqrt{49}$

$x = \pm 7$

ⓑ $\frac{50y^2}{50} = \frac{72}{50}$

$y^2 = \frac{36}{25}$

$\sqrt{y^2} = \pm\sqrt{\frac{36}{25}}$

$y = \pm\frac{\sqrt{36}}{\sqrt{25}}$

$y = \pm\frac{6}{5}$

ⓒ $\sqrt{(x+3)^2} = \pm\sqrt{16}$

$x+3 = \pm 4$

$x+3 = 4$ $x+3 = -4$

$x = 1$ $x = -7$

d) $3x^2 - 8 = 0$ e) $(x-1)^2 = 12$

$\frac{3x^2}{3} = \frac{8}{3}$

$x^2 = \frac{8}{3}$

$\sqrt{x^2} = \pm\sqrt{\frac{8}{3}}$

$x = \pm\frac{\sqrt{8}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{6}}{3}$

ⓔ $\sqrt{(x-1)^2} = \pm\sqrt{12}$

$x-1 = \pm 2\sqrt{3}$

$x = \pm 2\sqrt{3} + 1$

or

$x = 1 \pm 2\sqrt{3}$

completing the square when a = 1

Sometimes factoring quadratic equations (6.1) is not possible, as you cannot find the two numbers that multiply to c (or ac) and add to b. When this is the case, you can still solve the quadratic equation by a method called *completing the square.*

Example 3 – Solve $x^2 - 24 = -10x$ by completing the square. This example can easily be solved by factoring, but we will use it to introduce how to complete the square.

1) Get c to one side of the equation.
2) If the a value is 1, find the b value, halve it, and square it.
3) Add the number from step 2 to BOTH sides of the equation.
4) Factor the trinomial on the left (we created a perfect square trinomial, so it will lead to 2 brackets that are exactly the same).
5) Solve using the square root property.

$x^2 - 24 = -10x$

1. $x^2 + 10x = 24$
2. $b = 10$; 5 ; 25
3. $x^2 + 10x + 25 = 24 + 25$
4. $(x+5)^2 = 49$
5. $\sqrt{(x+5)^2} = \pm\sqrt{49}$

$x + 5 = \pm 7$

$x + 5 = 7$ $\quad$ $x + 5 = -7$

$x = 2$ $\quad$ $x = -12$

Example 4 – Solve by completing the square. Express the solutions in exact form.

a) $w^2 - 4w - 11 = 0$

$w^2 - 4w = 11$

$b = -4$; -2 ; 4

$w^2 - 4w + 4 = 11 + 4$

$(w-2)^2 = 15$

$w - 2 = \pm\sqrt{15}$

$w = 2 \pm \sqrt{15}$

b) $x^2 + 5x + 7 = 0$

$x^2 + 5x = -7$

$b = 5,\ \frac{5}{2},\ \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = \frac{-7^{\times 4}}{1_{\times 4}} + \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = \frac{-3}{4}$

$(x + \frac{5}{2})^2 = \frac{-3}{4}$

cannot sq. root right side, therefore no solutions (the parabola has no x-ints)

c) $m^2 - 5m + 3 = 0$

$m^2 - 5m = -3$

$b = -5,\ -\frac{5}{2},\ \frac{25}{4}$

$m^2 - 5m + \frac{25}{4} = \frac{-3^{\times 4}}{1_{\times 4}} + \frac{25}{4}$

$(m - \frac{5}{2})^2 = \frac{13}{4}$

$m - \frac{5}{2} = \pm\sqrt{\frac{13}{4}}$

$m - \frac{5}{2} = \pm\frac{\sqrt{13}}{\sqrt{4}}$

$m = \pm\frac{\sqrt{13}}{2} + \frac{5}{2}$

$m = \frac{5 \pm \sqrt{13}}{2}$

3.2B – Solving Quadratic Equations by Completing the Square Part 2

completing the square when $a \neq 1$

If $a \neq 1$, there are a few more considerations when *completing the square.*

Example 1 – Solve $2x^2 - 5x - 1 = 0$ by completing the square

1) Get c to one side of the equation.
2) Factor the a value out of the left side.
3) Divide both sides by the a value to leave $x^2 + bx$ on the left side of the equation. THEN find the b value, halve it, and square it.
4) Add the number from step 3 to BOTH sides of the equation.
5) Factor the resulting trinomial on the left.
6) Solve using the square root principle and answer in exact form.

$2x^2 - 5x - 1 = 0$

1 $2x^2 - 5x = 1$

2 $2(x^2 - \frac{5}{2}x) = 1$

3 $x^2 - \frac{5}{2}x = \frac{1}{2}$

$b = -\frac{5}{2}$; $-\frac{5}{4}$; $\frac{25}{16}$

4 $x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{1^{\times 8}}{2_{\times 8}} + \frac{25}{16}$

5 $(x - \frac{5}{4})^2 = \frac{33}{16}$

6 $x - \frac{5}{4} = \pm\frac{\sqrt{33}}{\sqrt{16}}$

$x = \pm\frac{\sqrt{33}}{4} + \frac{5}{4}$

$x = \frac{5 \pm \sqrt{33}}{4}$

Example 2 – Solve by completing the square. Answer b to the nearest hundredth.

An easy way to halve a fraction is to double the denominator.

a) $3x^2 - 2 = -4x$

$3x^2 + 4x = 2$

$3(x^2 + \frac{4}{3}x) = 2$

$x^2 + \frac{4}{3}x = \frac{2}{3}$

$b = \frac{4}{3}$; $\frac{4}{6} = \frac{2}{3}$; $\frac{4}{9}$

$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{2^{\times 3}}{3_{\times 3}} + \frac{4}{9}$

$(x + \frac{2}{3})^2 = \frac{10}{9}$

$x + \frac{2}{3} = \pm\frac{\sqrt{10}}{\sqrt{9}}$

$x = \pm\frac{\sqrt{10}}{3} - \frac{2}{3}$

$x = \frac{-2 \pm \sqrt{10}}{3}$

b) $-2x^2 - 3x + 7 = 0$

$7 = 2x^2 + 3x$

$2x^2 + 3x = 7$

$2(x^2 + \frac{3}{2}x) = 7$

$x^2 + \frac{3}{2}x = \frac{7}{2}$

$b = \frac{3}{2}$; $\frac{3}{4}$; $\frac{9}{16}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7^{\times 8}}{2_{\times 8}} + \frac{9}{16}$

$(x + \frac{3}{4})^2 = \frac{65}{16}$

$x + \frac{3}{4} = \pm\frac{\sqrt{65}}{\sqrt{16}}$

$x = \pm\frac{\sqrt{65}}{4} - \frac{3}{4}$

$x = \frac{-3 \pm \sqrt{65}}{4}$

$x = 1.27, -2.77$

Example 3 – Solve by completing the square: $3x^2 + 6x - 1 = 0$

$$3x^2+6x=1$$

$$\frac{3(x^2+2x)}{3}=\frac{1}{3}$$

$$x^2+2x=\frac{1}{3}$$

$$b=2\;;\;1\;;\;1$$

$$x^2+2x+1=\frac{1}{3}+\frac{1\times3}{1\times3}$$

$$(x+1)^2=\frac{4}{3}$$

$$x+1=\pm\sqrt{\frac{4}{3}}$$

$$x+1=\pm\frac{2\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

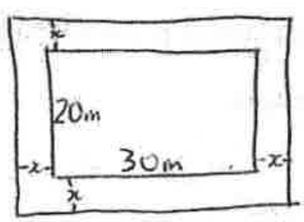
$$\underset{-1}{x+1}=\frac{\pm2\sqrt{3}}{3}\;-1$$

$$x=\frac{\pm2\sqrt{3}}{3}-\frac{3}{3}$$

$$x=\frac{-3\pm2\sqrt{3}}{3}$$

word problems

Example 4 - Butchart Gardens wants to build a pathway around its rose garden. The rose garden is currently 30m x 20m. The pathway will be built by extending each side by an equal amount. If the area of the garden and path together is 1.173 times larger than the area of just the rose garden, how wide will the new path be (to the nearest tenth)?



$$A_{\substack{rose\\garden}}=(20)(30)=600m^2$$

$$A_{\substack{garden+\\path}}=(600)(1.173)=703.8\,m^2$$

length of garden and path $= 30+2x$

width of garden and path $= 20+2x$

$$(30+2x)(20+2x)=703.8$$

$$600+60x+40x+4x^2=703.8$$

$$4x^2+100x+600=703.8$$

$$4x^2+100x=103.8$$

$$\frac{4(x^2+25x)}{4}=\frac{103.8}{4}$$

$$x^2+25x=25.95$$

$$b=25\;;\;12.5\;;\;156.25$$

$$x^2+25x+156.25=25.95+156.25$$

$$(x+12.5)^2=182.2$$

$$x+12.5=\pm13.5$$

$x=1.0$ $\quad$ $x=\cancel{-26}$ reject

The new path will be 1.0m wide.

3.3 – The Quadratic Formula

Quadratic equations can be solved by graphing, factoring (6.1), the square root property (6.2) and/or completing the square (6.2). Each of these methods have advantages and limitations.

Any quadratic equation can be solved using something called the *quadratic formula*. If the quadratic equation is in standard form ($ax^2 \pm bx \pm c = 0$), the quadratic formula is:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example 1 - Solve $3x^2 + 5x - 2 = 0$ using the quadratic formula

$a = 3 \quad b = 5 \quad c = -2$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-5 \pm \sqrt{25 + 24}}{6}$$

$$x = \frac{-5 \pm \sqrt{49}}{6}$$

$$x = \frac{-5 \pm 7}{6}$$

$$x = \frac{2}{6} = \frac{1}{3}$$

$$x = \frac{-12}{6} = -2$$

$$x = -2, \frac{1}{3}$$

discriminant

There could be 0, 1, or 2 resulting roots, depending on the **discriminant**, the expression under the square root ($b^2 - 4ac$). Solutions can be written in simplest radical form, or decimal form.

How would the discriminant determine the *nature of the roots* (the number of roots)?

If $b^2 - 4ac > 0$, there are 2 x-intercepts.

If $b^2 - 4ac = 0$, there is 1 x-intercept

If $b^2 - 4ac < 0$, there are no x-intercepts (cannot square root a negative number)

quadratic formula

Example 2 – Determine the nature of the roots, and then solve $3x^2 + 2x - 4 = 0$ using the quadratic formula

discriminant: $b^2 - 4ac$

$= 2^2 - 4(3)(-4)$

$= 4 + 48$

$= 52$

discrim > 0, so <u>2 roots</u>

$a = 3 \quad b = 2 \quad c = -4$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{52}}{6}$$

$$x = \frac{-2 \pm 2\sqrt{13}}{6}$$

$$x = \frac{-1 \pm \sqrt{13}}{3}$$

Example 3 – Determine the nature of the roots for $\frac{1}{4}x^2 - 3x + 9 = 0$

$b^2 - 4ac$ $\quad$ $a = 0.25$ $b = -3$ $c = 9$

$= (-3)^2 - 4(0.25)(9)$

$= 9 - 9$

$= 0$ so $\underline{1}$ root

Example 4 – Solve using the quadratic formula. Leave answers in exact form.

a) $x^2 = 2x + 1$

$x^2 - 2x - 1 = 0$

$a = 1,\ b = -2,\ c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4 + 4}}{2}$

$x = \frac{2 \pm \sqrt{8}}{2}$

$x = \frac{2 \pm 2\sqrt{2}}{2}$

$x = 1 \pm \sqrt{2}$

b) $\frac{x^2}{2} - \frac{5x}{6} = \frac{-3}{2}$ $\quad$ $\frac{1}{2}x^2 - \frac{5}{6}x + \frac{3}{2} = 0$

$a = \frac{1}{2}$ $b = -\frac{5}{6}$ $c = \frac{3}{2}$

$x = \frac{\frac{5}{6} \pm \sqrt{(-\frac{5}{6})^2 - 4(\frac{1}{2})(\frac{3}{2})}}{2(\frac{1}{2})}$

$x = \frac{\frac{5}{6} \pm \sqrt{\frac{25}{36} - \frac{3}{1}\,{}^{\times 36}_{\times 36}}}{1}$

$x = \frac{5}{6} \pm \sqrt{-\frac{83}{36}}$

No solutions

derivation

Derive the quadratic formula:

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

$a(x^2 + \frac{b}{a}x) = -c$

$x^2 + \frac{b}{a}x = \frac{-c}{a}$

$\frac{b}{a}$; $\frac{b}{2a}$; $\frac{b^2}{4a^2}$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c^{\times 4a}}{a_{\times 4a}} + \frac{b^2}{4a^2}$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$

$(x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

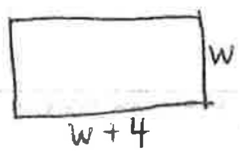
$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

$x = \frac{\pm\sqrt{b^2 - 4ac}}{2a} - \frac{b}{2a}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3.5 – Applications of Quadratic Equations

Example 1 – The area of a regulation Ping Pong table is $45ft^2$. The length is $4ft$ more than the width. What are the dimensions of the table?



$A = w(w+4)$

$45 = w(w+4)$

$45 = w^2 + 4w$

$w^2 + 4w - 45 = 0$

$(w+9)(w-5) = 0$

$w = \cancel{-9}, 5$

reject

$w = 5$

$w + 4$

$5 + 4$

$l = 9$

The dimensions are $9ft \times 5ft$.

Example 2 – The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number.

Let x = the number

$x + 2\left(\frac{1}{x}\right) = \frac{9}{2}$

$x + \frac{2}{x} = \frac{9}{2}$ $\boxed{x \neq 0}$

$x(2x) + \frac{2(2x)}{x} = \frac{9}{2}(2x)$

$2x^2 + 4 = 9x$

$2x^2 - 9x + 4 = 0$

$2x^2 - 8x - x + 4 = 0$

$2x(x-4) - 1(x-4) = 0$

$(x-4)(2x-1) = 0$

$x = 4, \frac{1}{2}$

Check: $x = 4$

$4 + 2\left(\frac{1}{4}\right) = \frac{9}{2}$

$4 + \frac{1}{2} = \frac{9}{2}$ ✓

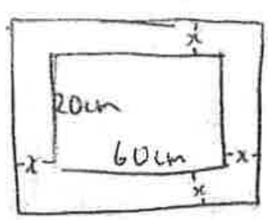
Check: $x = \frac{1}{2}$

$\frac{1}{2} + 2\left(\frac{1}{\frac{1}{2}}\right) = \frac{9}{2}$

$\frac{1}{2} + 4 = \frac{9}{2}$ ✓

The number can be 4 or $\frac{1}{2}$.

Example 3 – A 20cm by 60cm painting has a frame surrounding it. If the frame is the same width all around, and the total area of the frame is 516cm², how wide is the frame?



$A_{painting} = (20)(60) = 1200cm^2$

$A_{frame} = 516$

$A_{total} = 1200 + 516 = 1716\ cm^2$

$(60+2x)(20+2x) = 1716$

$1200 + 160x + 4x^2 = 1716$

$4x^2 + 160x - 516 = 0$

$4(x^2 + 40x - 129) = 0$

43, -3

$4(x+43)(x-3) = 0$

$x = \cancel{-43}, 3$

$x = 3cm$

The frame is 3cm wide.

Example 4 – Sally biked from Mt. Douglas to Stelly's, a distance of 25km, on two consecutive days. On Day 1, she rode 3km/h faster so her ride took 20 minutes less. Calculate her speed on both days and round to the nearest tenth.

Let x = speed on Day 2

	d (km)	s (km/hr)	time (hr)
Day 1	25	$x+3$	$\frac{25}{x+3}$
Day 2	25	x	$\frac{25}{x}$

20 mins = $\frac{1}{3}$ hr

$$\frac{25}{x+3} + \frac{1}{3} = \frac{25}{x} \quad \boxed{x \neq 0, -3}$$

LCD: $3x(x+3)$

$$75x + x(x+3) = 75(x+3)$$

$$75x + x^2 + 3x = 75x + 225$$

$$x^2 + 3x - 225 = 0$$

$$a=1,\ b=3,\ c=-225$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-225)}}{2}$$

$$x = \frac{-3 \pm \sqrt{909}}{2}$$

$$x = \frac{-3 \pm 30.15}{2}$$

$$x = \cancel{-16.6},\ 13.6$$

Day 2 speed: 13.6 km/h

Day 1 speed: 16.6 km/h

Example 5 – The cold water tap can fill a container two hours faster than the hot water tap. The two taps together can fill the container in 80 minutes. How long does it take each tap to fill the container on its own?

Think back to the pattern of the equation for these types of questions:

$$\frac{\text{time it takes together}}{\text{time for solo}} + \frac{\text{time it takes together}}{\text{time for solo}} = 1$$

in minutes:

$$\frac{80}{x} + \frac{80}{x+120} = 1$$

time for cold water solo (x); time for hot water solo ($x+120$)

$$80(x+120) + 80x = 1x(x+120)$$

$$80x + 9600 + 80x = x^2 + 120x$$

$$x^2 - 40x - 9600 = 0$$

$$-120, 80$$

$$(x-120)(x+80) = 0$$

$$x = 120,\ \cancel{-80} \text{ reject}$$

$$x = 120 \text{ minutes}$$

$$x + 120 = 120 + 120 = 240 \text{ mins}$$

Cold water solo takes 120 mins or 2 hrs

Hot water solo takes 240 mins or 4 hrs.