Period: _____ Name: Solutions KEY

***Chapter 4 Extra Practice Test**

Systems of Equations & Linear and Quadratic Inequalities

Show all of your work.

1) Solve the system by graphing : $y = \frac{1}{2}(x-1)^2 - 4$ and $y = 2x^2 - 4x - 8$

① $y = \frac{1}{2}(x-1)^2 - 4$

vertex $(1,-4)$, $a = \frac{1}{2}$

over 1, up $\frac{1}{2}$
over 2, up 2
over 3, up 4.5
over 4, up 8

② $y = 2x^2 - 4x - 8$

$y = 2(x^2 - 2x + 1 - 1) - 8$

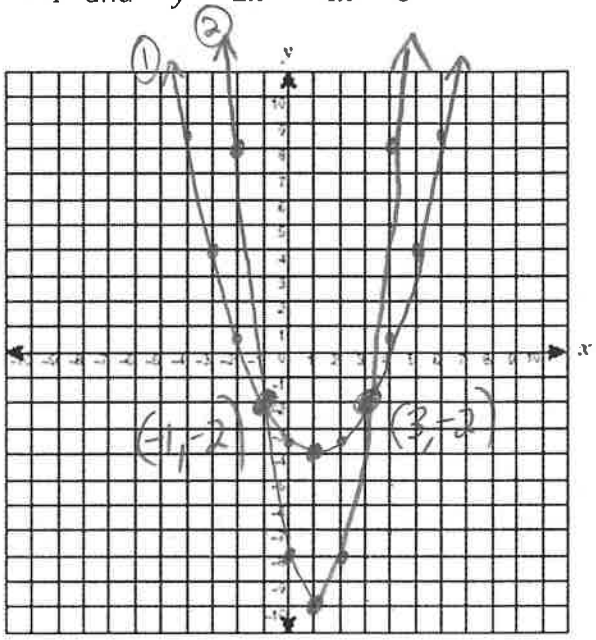
$b = -2$; (-1) save ; (1) use

$y = 2(x-1)^2 - 2 - 8$

$y = 2(x-1)^2 - 10$

vertex $(1,-10)$, $a = 2$

over 1, up 2
over 2, up 8
over 3, up 18



ANSWER(S): $(-1,-2)$ and $(3,-2)$

2) Solve the system by substitution: $x^2 - y = 3$ and $3x - y = -7$

$-y = -3x - 7$

$y = 3x + 7$

$x^2 - y = 3$

$x^2 - (3x+7) = 3$

$x^2 - 3x - 7 = 3$

$x^2 - 3x - 10 = 0$

$-5 \times 2 = -10$

$-5 + 2 = -3$

$(x-5)(x+2) = 0$

$x = 5$, $x = -2$

$x = 5$

$y = 3x + 7$

$y = 3(5) + 7$

$y = 15 + 7$

$y = 22$

$(5,22)$

$x = -2$

$y = 3x + 7$

$y = 3(-2) + 7$

$y = -6 + 7$

$y = 1$

$(-2,1)$

ANSWER(S): $(5,22)$ and $(-2,1)$

line up "like terms" in columns! Then, you need opposite coefficients on y-terms so multiply by −10 to get −10y and 10y!

3) Solve the system by elimination (3 marks): $3x^2 - 10y = 5$ and $x - y = -2$. Any solutions that are fractions should be answered as fractions in lowest terms, and not as decimals.

$x^2 + x + y = c$

$3x^2 - 10y = 5$

$(x - y = -2) \times -10$

$3x^2 - 10y = 5$

$+ \quad -10x + 10y = 20$

y's are eliminated!

$3x^2 - 10x = 25$

$-25 \quad -25$

$3x^2 - 10x - 25 = 0$

$3x^2 - 15x + 5x - 25 = 0$

$(3x - 25)$

$-15 \times 5 = -75$

$-15 + 5 = -10$

$3x(x-5) + 5(x-5) = 0$

$(x-5)(3x+5) = 0$

$x = 5, \quad x = \frac{-5}{3}$

$x = 5$

$x - y = -2$

$5 - y = -2$

$-5 \quad -5$

$-y = -7$

$y = 7$

$(5, 7)$

$x = \frac{-5}{3}$

$x - y = -2$

$-\frac{5}{3} - y = -2$

$+\frac{5}{3} \quad +\frac{5}{3}$

$-y = \frac{-6}{3} + \frac{5}{3}$

$-y = -\frac{1}{3}$

$y = \frac{1}{3}$

$\left(\frac{-5}{3}, \frac{1}{3}\right)$

$\frac{-6}{3}$!

ANSWER(S):

$(5, 7)$ and $\left(\frac{-5}{3}, \frac{1}{3}\right)$

4) Solve the inequality by graphing: $x - 3y \leq 3$

$-x \quad -x$

$x - 3y \leq 3$

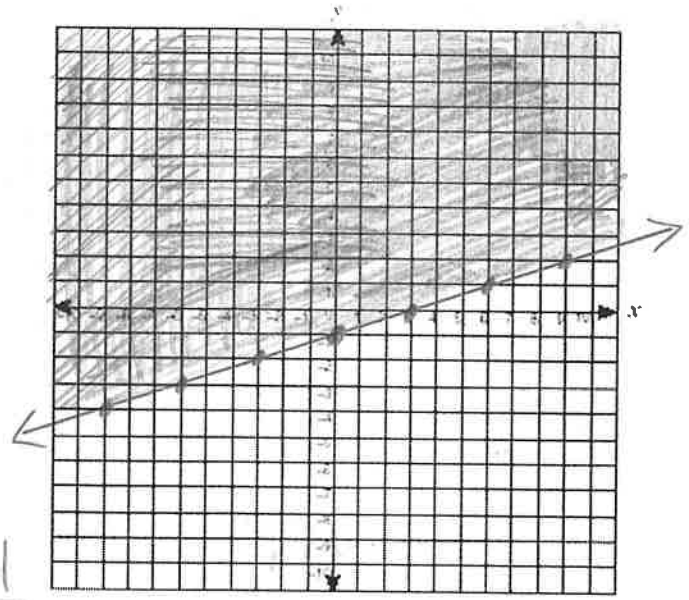
$\frac{-3y}{-3} \leq \frac{-x}{-3} + \frac{3}{-3}$

÷ by negative, flip inequality!

$y \geqslant \frac{1}{3}x - 1$

y-int @ −1, slope $\frac{1}{3} = \frac{\text{up 1}}{\text{right 3}}$

Solid line, shade ABOVE!



$-17x+12$ $-17x+12$

5) Solve $-5x^2 \leq 17x - 12$ and graph the solution on a number line.

$\frac{-5x^2}{-1} \quad \frac{-17x}{-1} \quad \frac{+12}{-1} \leq \frac{0}{-1}$ ↓ flip!

$5x^2 + 17x - 12 \geq 0$

$5x^2 + 20x - 3x - 12 \geq 0$ ($-3 \times 20 = -60$, $-3 + 20 = 17$)

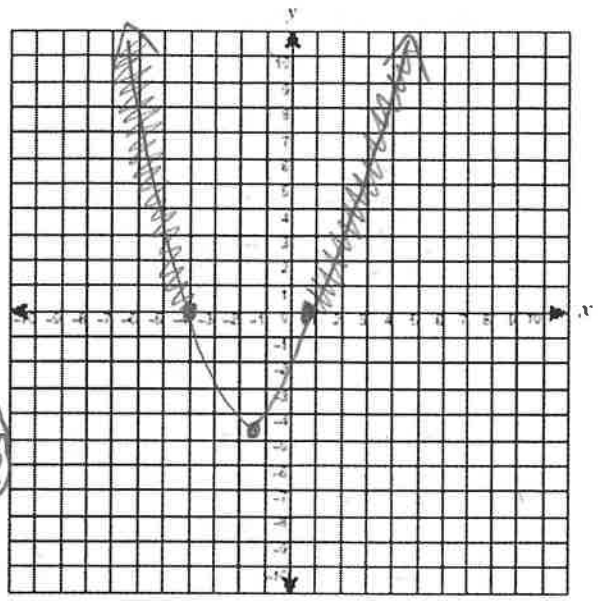
$5x(x+4) - 3(x+4) \geq 0$

$(x+4)(5x-3) \geq 0$

critical points (x-int's) → $x = -4, \; x = \frac{3}{5}$

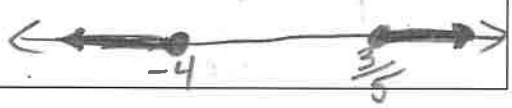
where is parab. ABOVE (or on) x-axis?

$a = 5$, opens up!



ANSWER: $x \leq -4, \; x \geq \frac{3}{5}$

NUMBER LINE:



6) Solve the inequality by graphing: $y \geq -x^2 + 6x - 5$

$y \geq -x^2 + 6x - 5$

$y \geq -(x^2 - 6x + 9 - 9) - 5$

-6; (-3) save; (9) use

$y \geq -(x-3)^2 + 9 - 5$

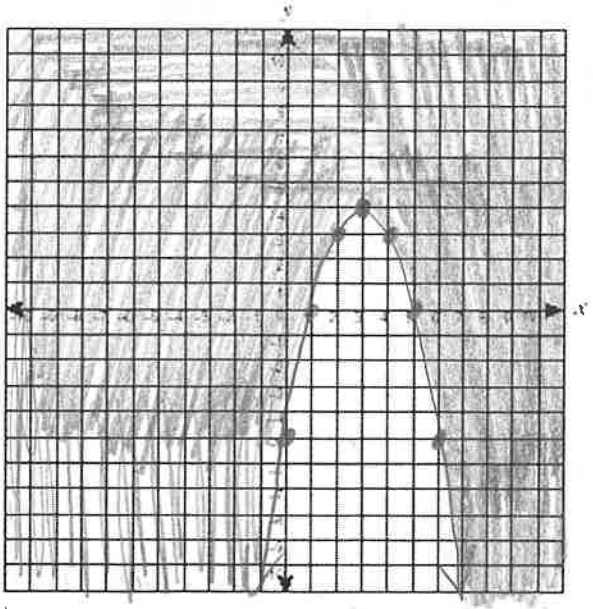
$y \geq -(x-3)^2 + 4$

vertex $(3, 4)$, $a = -1$

over 1, down 1

over 2, down 4

Solid parabola, shade ABOVE!



$2 \div \frac{1}{2} = 2 \times \frac{2}{1} = 4!$

7) Solve the inequality by graphing: $y \geq \frac{1}{2}x^2 + 2x - 1$

*(hint: the **solution** to this is the correct boundary and shading!)*

$y \geq \left(\frac{1}{2}x^2 + 2x\right) - 1$ *factor out $\frac{1}{2}$

$y \geq \frac{1}{2}(x^2 + 4x + 4 - 4) - 1$

$b = 4$; (2) save ; (4) use

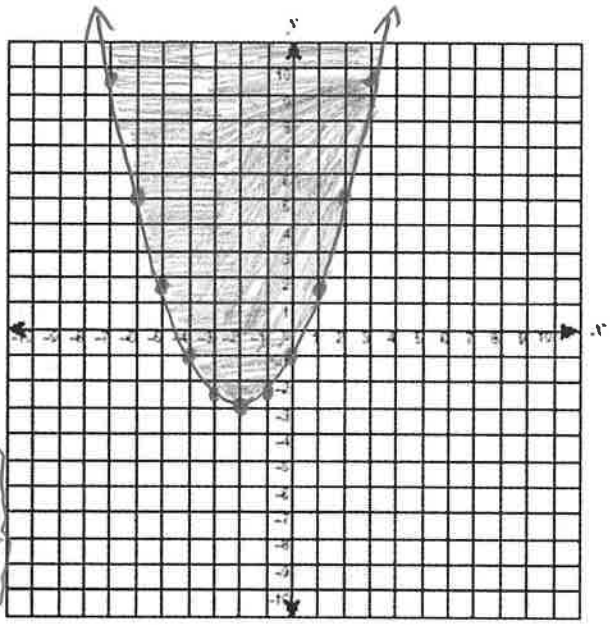
$y \geq \frac{1}{2}(x+2)^2 - 2 - 1$

$y \geq \frac{1}{2}(x+2)^2 - 3$

vertex $(-2, -3)$ $a = \frac{1}{2}$

over 1, up $\frac{1}{2}$
over 2, up 2
over 3, up 4.5
over 4, up 8

solid, shade ABOVE



8) The (1) length of a rectangle is 2 metres more than the width, and the (2) area is **less than 63** m^2.
What is the range of values for the **width** of the rectangle?

* width can't be negative... or 0!

let w = width ℓ = length

w+2
A<63 w

① $\ell = w + 2$

② $w\ell < 63$

$w(w+2) < 63$

$w^2 + 2w < 63$
$-63 \quad -63$

need x-ints!

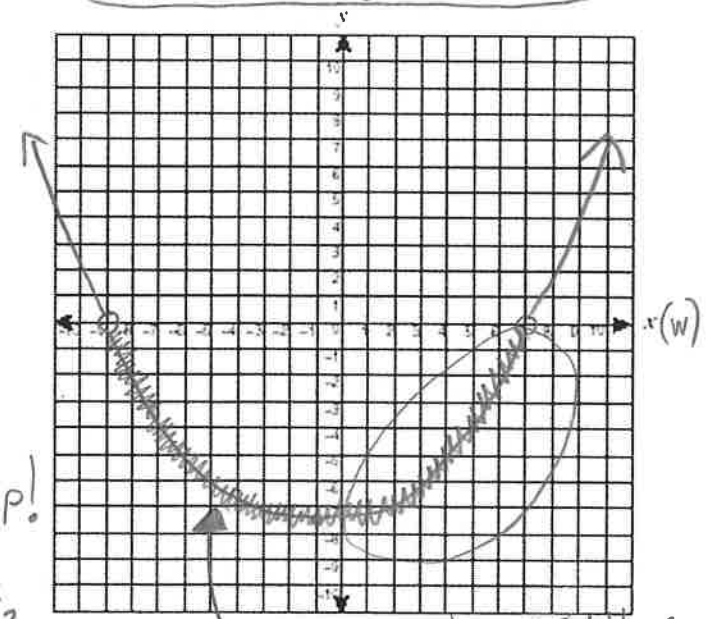
$w^2 + 2w - 63 < 0$

$(w+9)(w-7) < 0$

$w = -9, \; w = 7$

where is parabola BELOW x-axis?

$a = 1$, opens UP!



reject region where width is negative, or 0

SENTENCE ANSWER:

$0 < w < 7$; The width is between 0 and 7 meters.
(not including 0 or 7)

9) The height in metres of a ball thrown upward from the ground is $h(t) = -5t^2 + 30t$, where t is the time in seconds after releasing the ball. During what time interval will the ball be **above 35 meters**? (3 marks)

parabola opens down ($a = -5$), where is parab ABOVE x-axis?

height! $h(t) = -5t^2 + 30t$

we want... where is the parabola > 35 m?

$-5t^2 + 30t > 35$

$-35 \quad -35$

$-5t^2 + 30t - 35 > 0$

$\frac{-5t^2}{-5} + \frac{30t}{-5} - \frac{35}{-5} > 0$

$-5(t^2 - 6t + 7) > 0$ ☺!

doesn't factor... quadratic formula!

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $(a=1, b=-6, c=7)$

$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(7)}}{2(1)}$

$t = \frac{6 \pm \sqrt{36 - 28}}{2}$

$t = \frac{6 \pm \sqrt{8}}{2}$

$t = \frac{6 \pm 2.828}{2}$

pos: $t = \frac{6 + 2.828}{2}$ $t = 4.4$ s

neg: $t = \frac{6 - 2.828}{2}$ $t = 1.6$ s

these are the x-ints!

SENTENCE ANSWER:

The ball will be above 35 m between 1.6 s and 4.4 s