4.1

[redacted] - Slope                                                            KEY

**Learning Target:** to understand the slope of a line and how to calculate it using two points on a line from a graph.

**Toolkit:**
- points represented as ordered pairs
- interpreting graphs

**Definitions**

**Slope:** the slope of a linear equation describes the steepness and direction of the line.

Finding the slope from a graph

To find the slope of a line from a graph, divide the **vertical change** between two points by the **horizontal change** of the **same two points.**

Example 1)

Negative Slope:
The line decreases as we move from left to right

Positive Slope:
the line increases as we move from left to right

**Slope (*m*):**

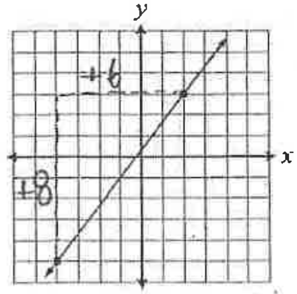
$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}$$

Example 2)

Use the graph to find the slope of the line:
- *pick two points on the graph that are easy to read
- *Always count from **left** to **right** ← left to right: positive change

up: positive change
down: negative change

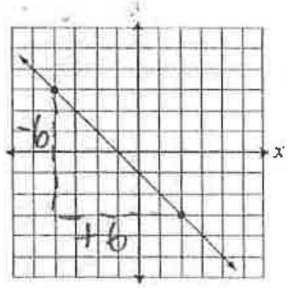
**a)**




$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{8}{6} = \frac{4}{3}$$

reduce, but leave as a fraction

**b)**




$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{-6}{6} = -1$$

**Finding slope from ordered pairs**

**Slope ($m$):**
The slope $m$ of a line segment between two ordered pairs $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example 3)**

Find the slope of the line containing each pair of points:

a) (4, 5) and (7, 1)

$x_1\ y_1 \quad x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{7 - 4} = \boxed{\frac{-4}{3}}$$

b) (0, -3) and (2, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{2 - 0} = \frac{5+3}{2} = \frac{8}{2} = 4$$

watch the negatives

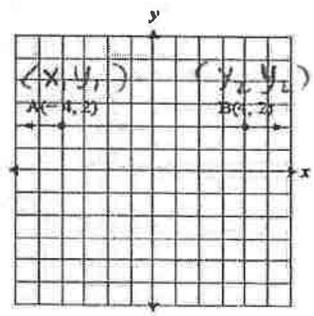
**Zero Slope**

If two points have the same y-value, the line joining the two points is horizontal, with a slope m= 0.

**Example 4)**

Let's use the slope equation to see how the slope is zero:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{4-(-4)} = \frac{0}{4+4} = \frac{0}{8} = 0$$




**Undefined Slope**

If two points have the same x-value, the line joining the two points is vertical, with a slope m= undefined

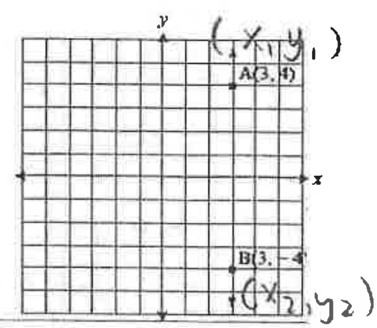
**Example 5)**

Let's us the slope equation to see how the slope is undefined:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 4}{3 - 3} = \frac{-8}{0}$$

cannot divide by zero

So we say the slope m= undefined



4.2

~~5.2~~ – Rate of Change Key.

**Learning Target:** to develop an understanding of rate of change and represent it using formulas and graphs.

**Toolkit:**

- Slope of a line on a graph
- Slope equation: $m = \frac{y_2-y_1}{x_2-x_1}$

**Definitions**

Δ - **Greek letter delta:** mathematical symbol used to represent change.

**Rate of change:** rates of change are special ratios comparing quantities with different units.

Rate of change formula

**Rate of change:**

$$\frac{\Delta y}{\Delta x} = \frac{\text{change in y}}{\text{change in x}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Notice that the rate of change formula is the exact same as the slope formula we looked at in 5.1.

We see rate of change every day:

a) kilometres per hour: km/hr
b) litres per kilometer: L/km
c) dollars per hour: \$/hr

Example 1) If you work for 20 hours and make \$230, there is a rate of change of:

$\frac{\Delta y}{\Delta x} = \frac{\$230}{20} = \$11.50$ per hour.

Example 2) Simon rents a car with the gas tank full. The odometer registered 31 720 km. He used it for five days. When the car was returned the odometer reading was 32 421 km and it needed 63 litres to fill up. The cost of renting the car was \$96 plus gas, which cost 90¢ per litre.

a) Determine the rate of gas consumption for the car.
b) Determine the average rate of travel per day.
c) Determine the cost of renting the car per day.

$\frac{\Delta km}{\Delta L}$ a) $\frac{32421km - 31720km}{63L - 0L} = \frac{701km}{63L} = 11.13$ km/L "km per L"

$\frac{\Delta km}{\Delta days}$ b) $\frac{32421km - 31720km}{5 - 0} = \frac{701km}{5days} =$ 140.2 ~~233.7~~ km/day "km per day"

c) car + gas

$\frac{96-0}{5-0} + \frac{63(0.90)}{5} = \frac{96}{5} + \frac{56.70}{5} = \$$ 30.54 ~~50.40~~ per day.

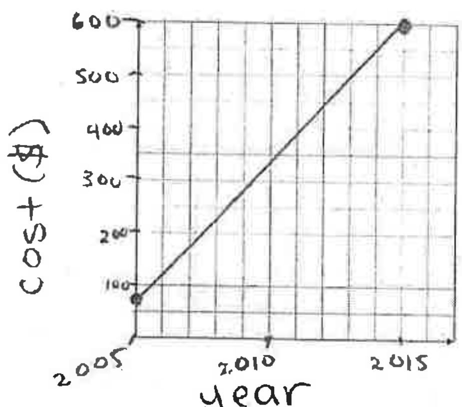
**Rate of Change on a graph**

The rate of change on a graph is represented by the **slope**. In general, we put the values in the denominator along the x-axis, and the values in the numerator along the y-axis.

(ind var)

dep var.

**Example 3)** Between 2005 and 2015, the cost of a cell phone increased from \$75 to \$650. Graph this result, and determine the average cost increase per year.




$$\frac{\Delta y}{\Delta x} = \frac{\text{change in cost}}{\text{change in year}}$$

$$= \frac{600 - 75}{2015 - 2005} = \frac{525}{10}$$

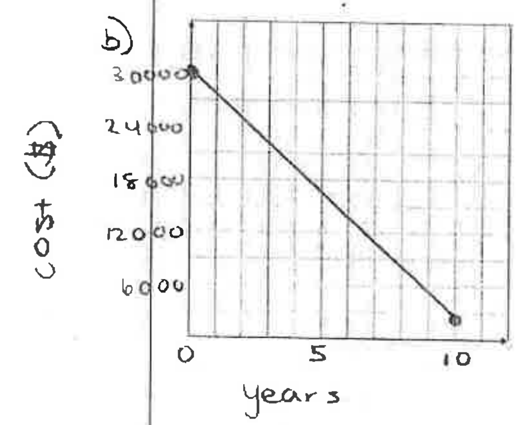
$$= \$52.50$$

cell phones have been increasing cost at an average rate of \$52.50 per year

**Example 4)** Most cars depreciate as they age. A car costing \$30 000 will have a value of \$2500 at the end of 10 years.

a) Write a formula for its value V, when it is t years old. 0 # t # 10
b) Draw a graph of this linear function.
c) Determine the car's value after 4.5 years.
d) When is the car's value ~~between~~ \$12 000? ~~and \$15 000?~~
e) How much value does the car lose every 2.5 years?
f) What is the rate of change of the car's value with respect to time?

b)




a/f) For the formula, we need to know how much the car loses value each year

$$\Rightarrow \frac{\Delta y}{\Delta x} = \frac{\text{change in cost}}{\text{change in year}}$$

$$= \frac{30000 - 2500}{0 - 10} = -2750$$

Every year the car drops \$2750.0

so $V = 30000 - 2750t$

c) Use the Formula $\Rightarrow t = 4.5$

$$V = 30000 - 2750(4.5)$$
$$= 30000 - 12375$$
$$= \$17625.00$$

d) $12000 = 30000 - 2750t$

$-30000 \quad -30000$

$$\frac{-18000}{-2750} = \frac{-2750t}{-2750}$$

$t = 6.5$

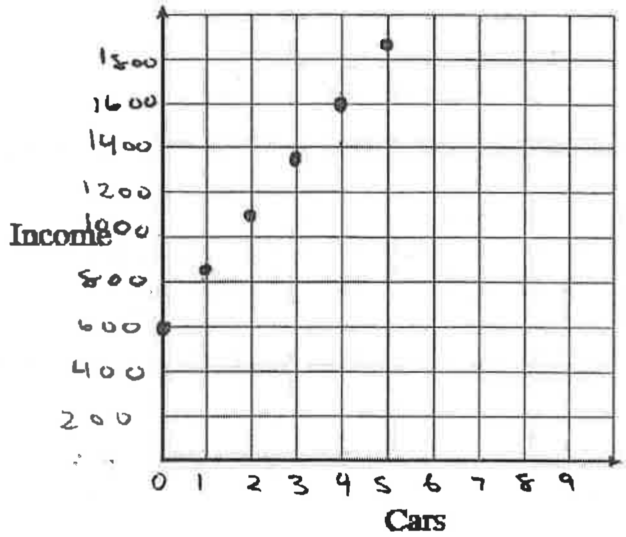
e) Since it loses \$2750 in one year,

\$2750 × 2.5 = \$6875

the car lose \$6875 in value in 2.5 years

Example 5)

**Draw a graph to describe the income of a car sales person who earns $600 per month plus $250 for every car sold.**




cannot connect the dots because you cannot sell 1.5 cars; 2.3 cars, etc.

4.3

## [redacted] – Graphing Linear Functions

**Learning Target:** Graph a linear function using the slope and a point on the line.

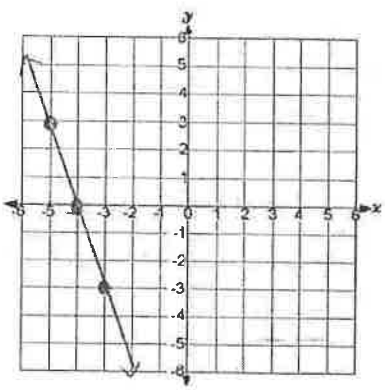
**Toolkit:**

- Slope $= m = \frac{rise}{run}$
- X-intercepts and y-intercepts

If we are given the slope of a line, and one point on the line, we can graph the line.

**Step 1:** Plot the point on the graph.
**Step 2:** Use the rise and the run of the slope to plot a second point.
**Step 3:** Connect the points using a straight line.

Example 1) Graph a line with slope -3, going through the point (-5, 3).



Slope $= \frac{-3}{1} = \frac{rise}{run}$

$\frac{-3}{1}$ ← down 3, ← right 1

Intercepts

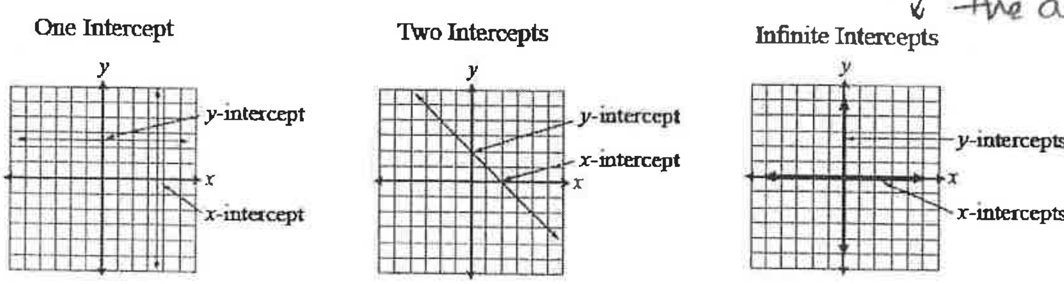
**X-Intercepts:**

- The point at which the line crosses over the X – axis.
- The x-intercept is represented by the point (a, 0).

**Y-Intercepts:**

- The point at which the line crosses over the y – axis.
- The y-intercept is represented by the point (0, b).

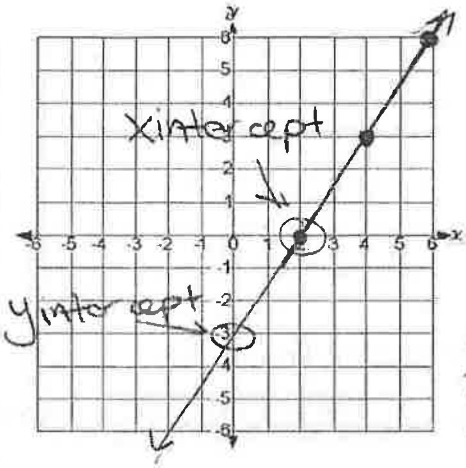
when the line lays directly over the axes




We can use the graph of a line to find the x-intercept(s) and y-intercept(s).

Example 2) Determine the x-intercept and y-intercept of the linear equation with a slope of $\frac{3}{2}$, going through (4, 3).

① Graph the line as above.
② Use the graph to find intercepts




$m = \frac{rise}{run} = \frac{3}{2}$ ← up 3, ← right 2

*If you want to plot a point in the other direction, change both directions of slope
⇒ down 3, left 2

x-intercept: (2, 0)
y-intercept: (0, -3)

Example 3) Determine the slope of a line that has an x-intercept of 4 and a y-intercept of -1.

x-intercept of 4 ⟶ the point $(\overset{x_1}{4}, \overset{y_1}{0})$
y intercept of −1 ⟶ the point $(\underset{x_2}{0}, \underset{y_2}{-1})$

use slope formula.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-0}{0-(4)} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$

Example 4) Find a number $n$ so that the line passing through the points $(\overset{x_1}{3}, \overset{y_1}{2})$ and $(\overset{x_2}{n}, \overset{y_2}{8})$ has a slope of $-\frac{2}{3}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{-2}{3} = \frac{8-2}{n-3}$ ⟶ $\frac{-2}{3} = \frac{6}{n-3}$

*cross multiply*

$-2(n-3) = 3 \times 6$

$-2n + 6 = 18$
$\quad\ \ -6 \quad -6$

$\frac{-2n}{-2} = \frac{12}{-2}$ $\boxed{n = -6}$

4.4A

KEY

**– Parallel and Perpendicular Lines**

**Learning Target:** To determine if lines are parallel or perpendicular, and investigate slopes of parallel and perpendicular lines

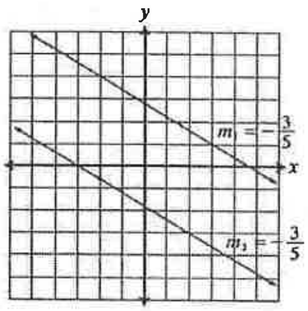
**Toolkit:**
- Finding the slope of lines $m = \frac{rise}{run}$
- Rise over run using graph
- Slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

**Definitions:**

**Parallel Lines** – lines in a coordinate system that never intersect.

* They have identical slopes because they rise or fall at the same rate

**Ex.**




**Perpendicular lines** – lines that form right (90°) angles when they intersect.

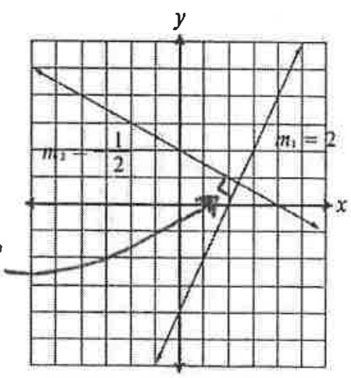
* If the slope of one line is $\frac{a}{b}$, the slope of a line that is perpendicular to it has slope $-\frac{b}{a}$.

* The slopes of perpendicular lines are **NEGATIVE RECIPROCALS.**

* The **product** (×) of the slopes of perpendicular lines is $-1$.

**Ex.**

90°




$m_1 \times m_2 = 2 \times \left(-\frac{1}{2}\right) = -1$

Ex. 1) Determine if the slopes are **parallel** ( ∥ ), **perpendicular** ( ⊥ ) , or **neither**

a) $m_1 = 6,\ m_2 = \frac{18}{3} = 6$

same slopes, ∴ $\boxed{\text{parallel}}$

b) $m_1 = 3,\ m_2 = \frac{1}{3}$

not equal, not neg. recips, ∴ $\boxed{\text{neither}}$

c) $m_1 = 7,\ m_2 = -\frac{1}{7}$

flipped, and sign changed

negative reciprocals ∴ $\boxed{\text{perpendicular}}$

d) $m_1 = \frac{1}{4},\ m_2 = 0.25$

$m_1 = 0.25$

same slopes, ∴ $\boxed{\text{parallel}}$

e) $m_1 = undefined,\ m_2 = 0$

vertical line — horizontal line

where vertical meets horizontal is 90°, ∴ $\boxed{\text{perpendicular}}$

Ex. 2)

**Complete the table.**

**Line $l_1$ has slope $m_1$, line $l_2$ has slope $m_2$, line $l_3$ has slope $m_3$, with $l_1 \parallel l_2$, and $l_1 \perp l_3$**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $m_1$ | $\frac{2}{5}$ | $-\frac{4}{3}$ | $-7$ | 0 | undefined | undefined | 0 |
| $m_2$ (parallel to $m_1$) | $\frac{2}{5}$ | $-\frac{4}{3}$ | $-7$ | 0 | undefined | undefined | 0 |
| $m_3$ (perpendicular to $m_1$) | $-\frac{5}{2}$ | $\frac{3}{4}$ | $\frac{1}{7}$ | undefined | 0 | 0 | undefined |

Ex. 3) Determine whether the line passing through the first pair of points is **parallel**, **perpendicular**, or **neither** to the line passing through the second pair of points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

a) $(6, 5)$ *and* $(11, 3)$; $(-2, 1)$ *and* $(3, -2)$

$m = \frac{3-5}{11-6} = \boxed{\frac{-2}{5}}$ $\qquad \frac{-2-1}{3-(-2)} = \boxed{\frac{-3}{5}}$

diff slopes, not neg. reciprocals, $\boxed{\therefore \text{ neither}}$

b) $(-3, -6)$ *and* $(3, 1)$; $(0, -2)$ *and* $(7, -8)$

$m = \frac{1-(-6)}{3-(-3)} = \boxed{\frac{7}{6}}$ $\qquad m = \frac{-8-(-2)}{7-0} = \boxed{\frac{-6}{7}}$

neg. reciprocals $\boxed{\therefore \text{ perpendicular}}$

Flipped, AND sign changed

4.4B

**– Parallel and Perpendicular Lines**

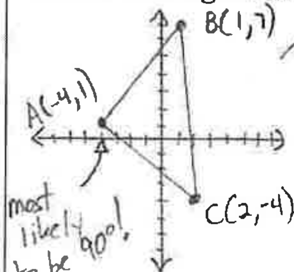
**Learning Target:** Apply concepts of parallel and perpendicular lines to solve problems

**Toolkit:**

- Slopes of parallel lines are equal (∥)
- Slopes of perpendicular lines are negative reciprocals (flipped AND sign changed) (⊥)

**Ex. 1)**
Show that the points $A\ (-4,\ 1)$, $B\ (1,\ 7)$, *and* $C\ (2, -4)$ are the vertices of a **right** triangle.

sketch:




so show that $AB \perp$ to $AC$!

slope AB: $\overset{x_1\ y_1}{A(-4,1)}\ \overset{x_2\ y_2}{B(1,7)}$

$$\frac{y_2-y_1}{x_2-x_1} = \frac{7-1}{1-(-4)} = \boxed{\frac{6}{5}}$$

slope AC: $\overset{x_1\ y_1}{A(-4,1)}\ \overset{x_2\ y_2}{C(2,-4)}$

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-4-1}{2-(-4)} = \boxed{\frac{-5}{6}}$$

Flipped, AND sign changed

slope AB is $\frac{6}{5}$, slope AC is $\frac{-5}{6}$, neg. reciprocals, ∴ perpendicular (90°)

∴ we have a right triangle

**Ex. 2)**
The line through $(6, y)$ and $(8, -8)$ is **perpendicular** to a line with a slope of $\frac{1}{3}$. Find the value of $y$.

slope of line perp. to slope $\frac{1}{3} \xRightarrow{\text{flip, change sign}} -3$!

so... $\overset{x_1\ y_1}{(6,y)}\ \overset{x_2\ y_2}{(8,-8)}$

$$\frac{y_2-y_1}{x_2-x_1} = \frac{-8-y}{8-6} = -3$$

now, solve for y

$$\left(\frac{-8-y}{2}\right)^{\times 2} = (-3)^{\times 2}$$

$$-8-y = -6 \quad (+8 \text{ both sides})$$

$$\frac{-y}{-1} = \frac{2}{-1} \longrightarrow \boxed{y=-2}$$

**Ex. 3)**
A line through $(-4, y)$ and $(-2, -5)$ is **parallel** to a line with slope $-4$. Find the value of $y$.

parallel means equal slopes

so... slope between $\underset{x_1\ y_1}{(-4, y)}$ and $\underset{x_2\ y_2}{(-2, -5)} = -4$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{-5-y}{-2-(-4)} = -4$$

$$\left(\frac{-5-y}{2}\right)^{\times 2} = (-4)^{\times 2}$$

$$-5^{+5} - y = -8^{+5}$$

$$-y = -3$$

$$\boxed{y = 3}$$

**Ex. 4)**
The line through $(x, -1)$ and $(11, 2)$ is **perpendicular** to a line with slope $-3$. Find the value of $x$ .

slope of line perpendicular to $-3 \longrightarrow \frac{1}{3}$

$\underset{x_1\ y_1}{(x, -1)}\ \underset{x_2\ y_2}{(11, 2)}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\frac{2-(-1)}{11-x} = \frac{1}{3}$$

fraction = fraction
cross multiply!

$$\frac{3}{11-x} \times \frac{1}{3}$$

$$11 - x(1) = 3(3)$$

$$11^{-11} - x = 9^{-11}$$

$$-x = -2$$

$$\boxed{x = 2}$$

4.5A

~~8.5A~~ – **Applications of Linear Relations**

**Learning Target:** To focus on problems involving intercepts, slope, and domain and range, of linear relations.

**Toolkit:**

- Slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Domain and Range
- X-intercept: where $y = 0$
- Y-intercept: where $x = 0$

**Ex. 1)**

A Computer repair company charges a fixed amount, plus an hourly rate for a service call. A two hour service call is \$80, and a four hour service call is \$140.

$h_2$ $T_2$ $h_4$ $T_4$

a) Write the equation that shows how the total cost, *T*, depends on the number of hours, *h*, and the fixed cost, *C*. Use *R* for hourly rate.

$$T = Rh + C$$

b) Find the hourly rate.

$$R = \frac{\Delta T}{\Delta h} = \frac{\text{change in cost}}{\text{change in hours}} = \frac{T_4 - T_2}{h_4 - h_2} = \frac{140-80}{4-2} = \frac{60}{2} = \boxed{30\ \$/hr}$$

c) Find the fixed amount cost. (C)

Fixed cost will be the T when $h = 0$.

To find C, use $R = 30$, and either point $(2, 80)$ or $(4, 140)$ (h, T)

ex. $T = Rh + C \Rightarrow 80 = 30(2) + C$

$80 = 60 + C$

$-60 \quad -60$

$20 = C$ $\boxed{\text{Fixed cost is \$20}}$

$\boxed{T = 30h + 20}$

d) Find the domain and range of this relation.

domain: $\{0, 1, 2, 3, \ldots\}$

range: $\{20, 50, 80, 110, \ldots\}$

**Ex. 2)**

A collectable baseball card (Sammy Sosa's rookie card), increases in value \$50 per year. The baseball card is worth \$600 now.

a) Write the equation that shows how the current worth of the baseball, *W*, depends on the number of years, *t*.

$$W = 600 + 50t$$

b) What price was paid for the baseball card if it was bought 3 years ago?

set $t = -3 \rightarrow$ 3 years AGO!

$$W = 600 + 50(-3)$$
$$= 600 - 150$$
$$\boxed{= \$450 \text{ was the price paid 3 yrs ago.}}$$

c) What will the value of the baseball card be in five years?

set $t = 5$

$$W = 600 + 50(5)$$
$$= 600 + 250$$
$$= 850$$

$\boxed{\text{The card will be worth \$850 in 5 yrs.}}$

d) In how many years from now will the card be worth \$1750?

$W = 1750$, sub in $W = 1750$ and solve for $t$.

$$W = 600 + 50t$$
$$1750 = 600 + 50t$$
$$-600 \qquad -600$$
$$1150 = 50t$$
$$\frac{1150}{50} = \frac{50t}{50}$$
$$t = 23$$

$\boxed{\text{The card will be worth \$1750 in 23 years}}$

e) Determine the domain and range of this relation.

domain: $\{-3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$

range: $\{450, 500, 550, 600, 650, \ldots\}$

4.5B

~~5.5B~~ – **Applications of Linear Relations**

**Learning Target:** More word problems with linear relations

**Toolkit:**

- Slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Domain and Range
- Substitution values into an equation

**Ex. 1)**

A movie production company charges $19,000 for seven hours of work, and $15,000 for five hours of work.

a) Find the hourly rate

$$\text{hourly rate} = \frac{\Delta \text{charge}}{\Delta \text{hours}} = \frac{19000-15000}{7-5} = \frac{4000}{2} = \boxed{\$2000/hr}$$

b) Find the fixed amount cost

let f = fixed amount

$$C = 2000h + f$$
$$19000 = 2000(7) + f$$
$$19000 = 14000 + f$$
$$5000 = f$$

$\boxed{\text{\$5000 is fixed amount cost}}$

c) Write the equation that shows the charge of production, $C$, depends on the number of hours of work, $h$.

$$\boxed{C = 2000h + 5000}$$

d) How much will the production company charge for 29 hours of work?

$$C = 2000(29) + 5000$$
$$C = 58000 + 5000$$
$$C = 63,000$$

$\boxed{\text{They will charge \$63,000 for 29 hrs work}}$

e) If the charge was $51,000, how many hours did they work?

$$C = 2000h + 5000$$
$$51000 = 2000h + 5000 \quad -5000$$
$$-5000$$
$$\frac{46000}{2000} = \frac{2000h}{2000}$$

$$h = 23 \text{ hours}$$

$\boxed{\text{if charge was \$51,000, they worked for 23 hours.}}$

**Ex. 2)**

Each semester at the University of Victoria, a student must pay tuition costs (per course), plus a student service fee. To take five courses in a year costs \$3270, and to take four courses in a year costs \$2640.

a) Find the cost per course

$$\frac{\Delta \text{cost}}{\Delta \text{courses}} = \frac{3270-2640}{5-4} = \frac{630}{1}$$

$\boxed{\text{cost per course is } \$630}$

b) Find the student service fee

C = total cost

n = # of courses

f = fee

$$C = 630n + f \Rightarrow 3270 = 630(5) + f$$

$$3270 = 3150 + f$$

$$-3150 \quad -3150$$

$$120 = f$$

c) Write the equation that shows the cost for the year, *C*, depends on the number of courses taken, *n*.

$$C = 630n + 120$$

d) If a student paid \$4530 one year, how many courses did they take?

$C = 4530$

$$C = 630n + 120$$

$$\overset{-120}{4530} = 630n + 120^{-120}$$

$$\frac{4410}{630} = \frac{630n}{630}$$

$$n = 7$$

$\boxed{\text{They took 7 courses}}$

e) How much is the cost of taking 9 courses in one year?

$n = 9$

$$C = 630n + 120$$

$$C = 630(9) + 120$$

$$C = 5670 + 120$$

$$C = \$5790$$

$\boxed{\text{cost of taking 9 courses is } \$5790}$