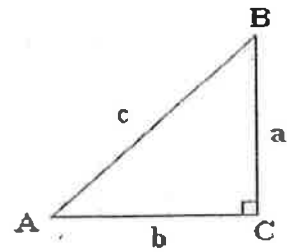
7.0 – Trigonometry Review

In trigonometry problems, all vertices (corners or angles) of the triangle are labeled with capital letters. The right angle is usually labeled *C*. Sides are usually labeled with lower case letters. The side opposite to <*A* will be labeled *a* and so on.

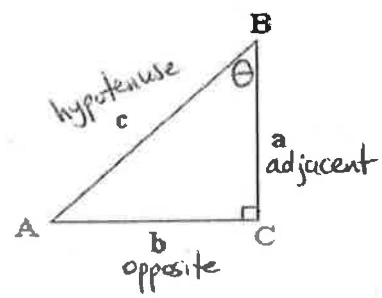


Whenever we do trigonometry problems on a right triangle, we focus on a target angle. The target angle can be any of the two angles that are **not** the right angle.

using triangle to the right, target angle could be <A and <B

Once we have a target angle, we can name each side of the triangle. Let's suppose *A* is the target angle. Then side *a* is called the ___OPPOSITE___ side because it is on the other side of the triangle. Side *c* is always called the ___HYPOTENUSE___ because it is the longest side, and side *b* is called the ___ADJACENT___ side as it is beside <*A*. If the target angle does not have an angle measurement on it, we represent it with the greek letter theta, θ.

Suppose *B* is the target angle in the triangle on the right. Label all appropriate parts.

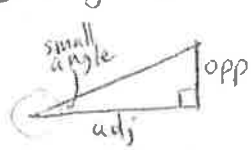


The three trigonometric ratios for right triangles are:

SINE	COSINE	TANGENT
$\sin\theta = \frac{opposite}{hypotenuse}$	$\cos\theta = \frac{adjacent}{hypotenuse}$	$\tan\theta = \frac{opposite}{adjacent}$
S O H	C A H	T O A

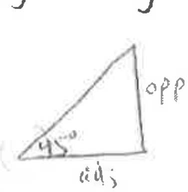
What is the point of the trigonometric ratios? To explain the relationship between the sides and angles of a right triangle

ex using tangent

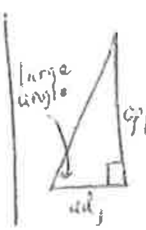


$\tan\theta = \frac{opp}{adj}$

if opp small and adj large, θ will be small, as tan tells us



tan 45° = 1 meaning opp = adj



if opp large and adj small, θ is large as tan tells us

Example 1 – Solve each to the nearest hundredth.

a) $\cos 42°$

$= 0.74$

is angle is 42°, adj is 0.74 as big as hyp.

b) $\tan 67° = \frac{x}{7}$

$x = 7(\tan 67°)$

$x = 16.49$

if 67° angle, opp more than twice as big as adj

c) $\sin\theta = \frac{5}{9}$

$\theta = \sin^{-1}\frac{5}{9}$

$\theta = 33.75°$

d) $\cos 35° = \frac{8}{x}$

$x = \frac{8}{\cos 35°}$

$x = 9.77$

In order to solve a right triangle, you must find the measurement of all three sides and all three angles.

Example 2 - Solve ΔABC to the nearest tenth.

$\angle A =$	$a =$
$\angle B = 33°$	$b = 7$
$\angle C = 90°$	$c =$

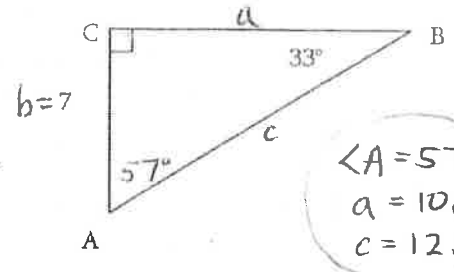
$\angle A = 180 - 90 - 33$
$= 57°$

side a: $\tan 33° = \frac{7}{a}$

$a = \frac{7}{\tan 33°} = 10.78$

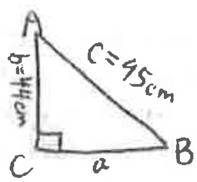
side c: $\sin 33° = \frac{7}{c}$

$c = \frac{7}{\sin 33°} = 12.85$



$\angle A = 57°$
$a = 10.8$
$c = 12.9$

Example 3 – Sketch & solve ΔABC to the nearest tenth where $\angle C=90°$, c =95cm & b =44cm



$\angle A =$	$a =$
$\angle B =$	$b = 44$cm
$\angle C = 90°$	$c = 95$cm

to find $\angle A$:
$\cos A = \frac{44}{95}$
$\angle A = \cos^{-1} \frac{44}{95}$
$\angle A = 62.4°$

to find $\angle B$:
$180 - 90 - 62.4$
$= 27.6°$

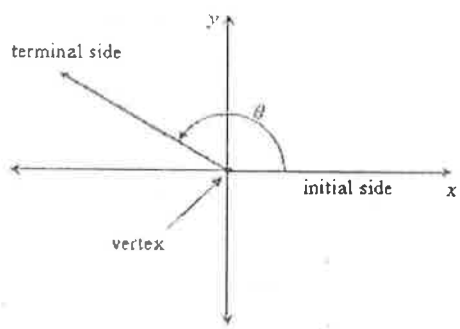
side a:
$\tan 62.4 = \frac{a}{44}$
$a = 44(\tan 62.4)$
$a = 84.2$

$\angle A = 62.4°$
$\angle B = 27.6°$
$a = 84.2$cm

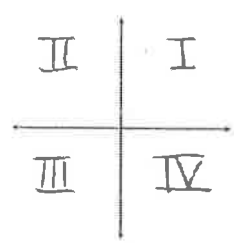
angles in standard position

An angle that is drawn in **standard position** must have its vertex at the origin of the Cartesian plane, and its initial arm must coincide with the positive x-axis.

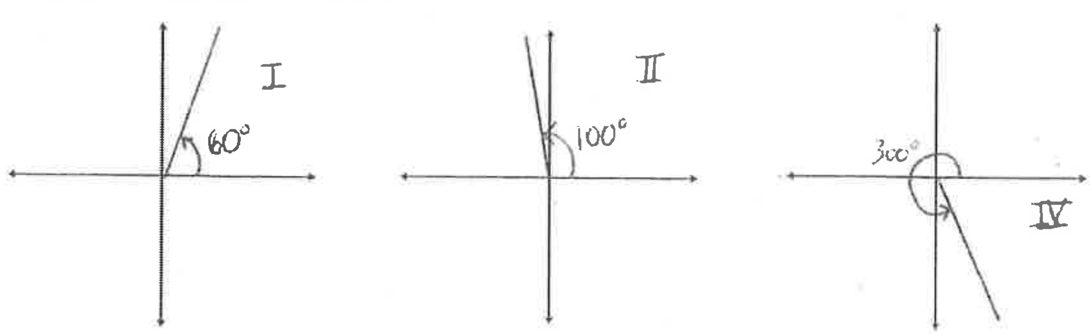
To draw angles in standard position, we use an **initial arm** (always the positive x-axis) and a **terminal arm** (the final position after a rotation). The angle is labeled **θ** (*theta*). The **vertex** of the angle must be at the origin (0, 0) of a Cartesian plane. Positive angles are measured in a counterclockwise direction. Here is an example:



Label the four quadrants of a Cartesian plane:



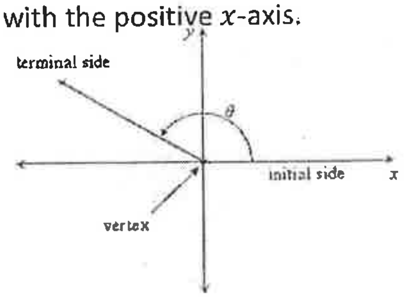
Example 4 – Draw each angle in standard position and identify the quadrant in which it lies: a) 60° b) 100° c) 300°



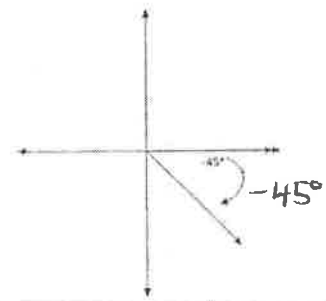
7.1 – Angles and Their Measure

angles in standard position

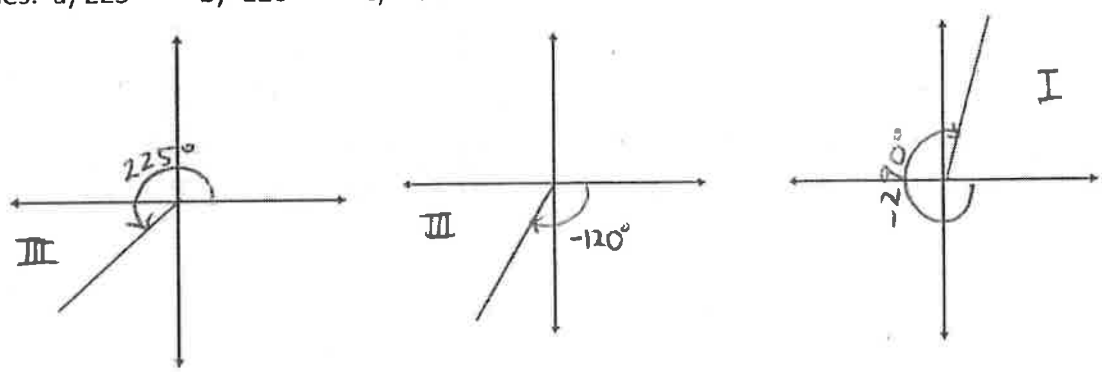
An angle that is drawn in **standard position** must have its vertex at the origin of the Cartesian plane, and its initial arm must coincide with the positive x-axis.



Clockwise angles have a negative measure:

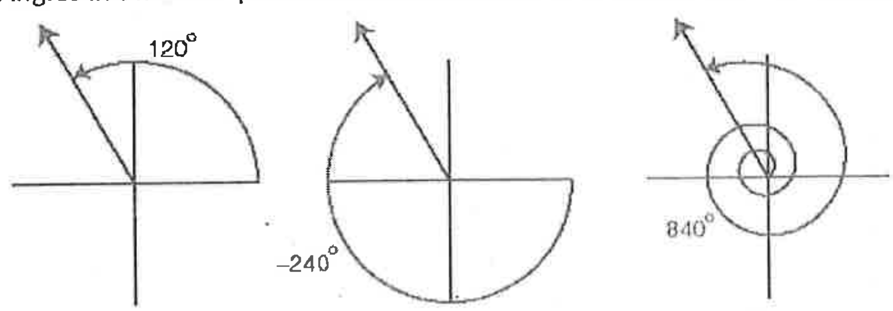


Example 1– Draw each angle in standard position and identify the quadrant in which it lies: a) 225° b) -120° c) -290°



coterminal angles

Angles in standard position that have the same terminal side are **coterminal**.



Example 2 – Find three coterminal angles (at least one negative) for:

a) 60°

$60° + 360° = 420°$

$60° + 360° + 360° = 780°$

$60° - 360° = -300°$

b) 225°

$225 + 360° = 585°$

$225 + 360(2) = 945°$

$225 - 360° = -135°$

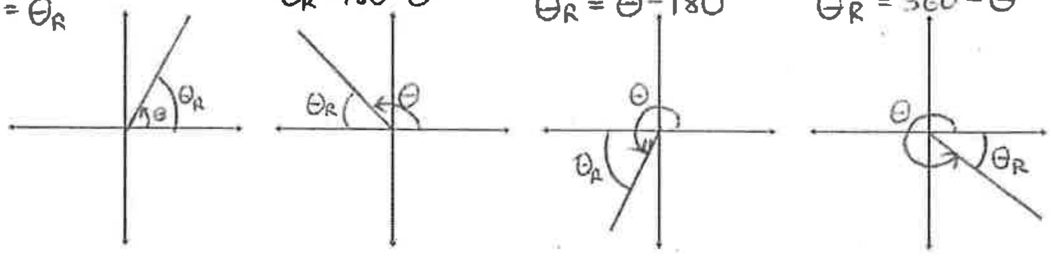
What is a general formula to find **coterminal** angles?

$\theta + 360n$, n is an integer

reference angles

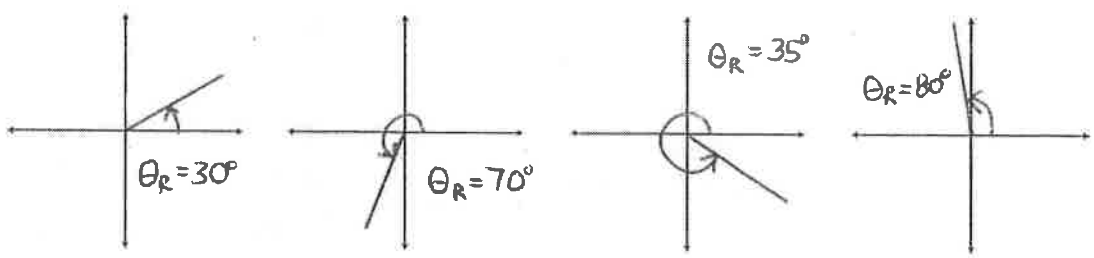
For each **angle in standard position**, there is a corresponding acute angle called the **reference angle**, which is the <u>acute</u> angle between the terminal arm and the (nearest) x-axis. Thus, any reference angle is between 0° and 90°

Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
$\theta = \theta_R$	$\theta_R = 180 - \theta$	$\theta_R = \theta - 180$	$\theta_R = 360 - \theta$



Example 3 – Draw each angle in standard position, and find the reference angle.

a) 30° b) 250° c) 325° d) 100°



Example 4 – Find the reference angle for:

a) 1450°

$1450 - 360(4) = 10°$

$\theta_R = 10°$

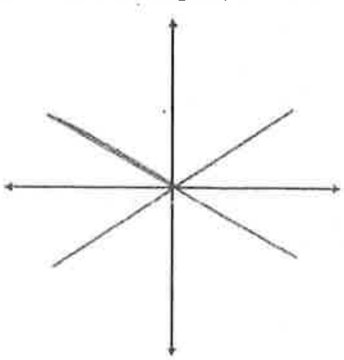
b) -870°

$-870 + 360(3) = 210°$

$\theta_R = 30°$

Example 5 – Find all angles, $0° \leq \theta \leq 360°$, that have reference angles of 30°. Do a sketch.

30°
150°
210°
330°



Find $\sin\theta$ for each with your calculator:

$\sin 30° = 0.5$
$\sin 150° = 0.5$
$\sin 210° = -0.5$
$\sin 330° = -0.5$

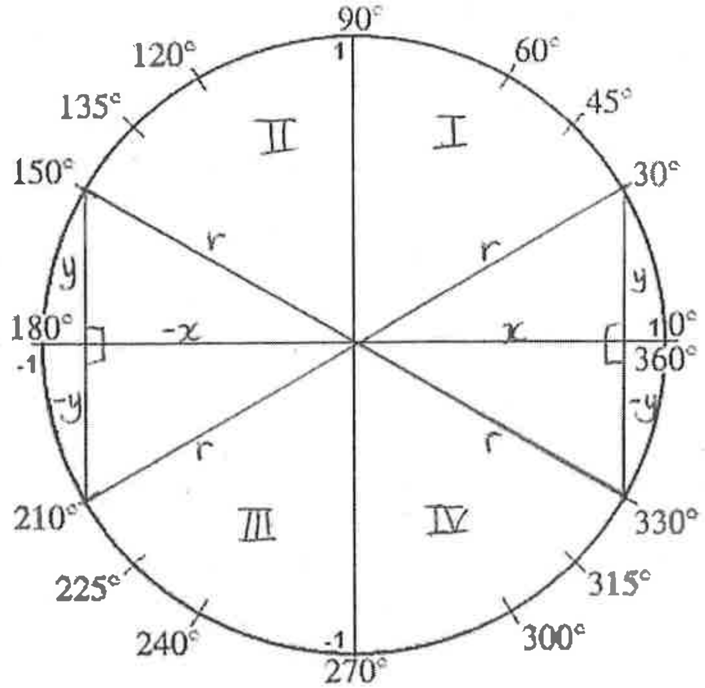
Find $\cos\theta$ for each with your calculator:

$\cos 30° = 0.866$ $\cos 210° = -0.866$
$\cos 150° = -0.866$ $\cos 330° = 0.866$

What do you notice? Why are some results positive and some negative?

Because coordinates can be negative depending on quadrants.

As a class, let's complete the diagram to help explain the results in Example 5:



- A point in Quadrant I is $(\overset{x}{pos}, \overset{y}{pos})$

  so $\sin\theta = \frac{y}{r}$ is positive and $\cos\theta = \frac{x}{r}$ is positive.

  (ex) $\sin 30° = 0.5$ and $\cos 30° \approx 0.866$

- A point in Quadrant II is $(\overset{-x}{neg}, \overset{y}{pos})$

  so $\sin\theta = \frac{y}{r}$ is positive and $\cos\theta = \frac{-x}{r}$ is negative

  (ex) $\sin 150° = 0.5$ and $\cos 150° = -0.866$

- A point in Quadrant III is $(\overset{-x}{neg}, \overset{-y}{neg})$

  so $\sin\theta = \frac{-y}{r}$ is negative and $\cos\theta = \frac{-x}{r}$ is negative

  (ex) $\sin 210° = -0.5$ and $\cos 210° = -0.866$

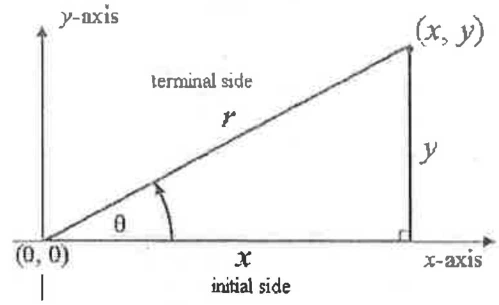
- A point in Quadrant IV is $(\overset{x}{pos}, \overset{-y}{neg})$

  so $\sin\theta = \frac{-y}{r}$ is negative and $\cos\theta = \frac{x}{r}$ is positive

  (ex) $\sin 330° = -0.5$ and $\cos 330° = 0.866$.

7.2 – The Three Trigonometric Functions

Suppose θ is an angle in standard position. Suppose the point at the end of the terminal arm is labeled $P(x, y)$, at a distance r from the origin.

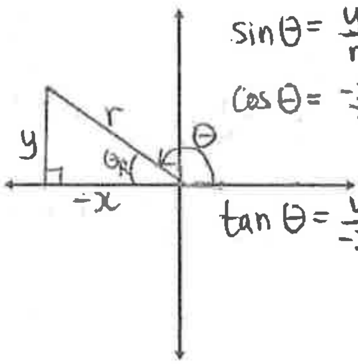


You can use a reference angle to determine the three trigonometric ratios in terms of x, y, and r.

$sin\theta = \frac{y}{r}$ $\quad cos\theta = \frac{x}{r}$ $\quad tan\theta = \frac{y}{x}$

Trigonometry ratios in the four quadrants:

Quadrant 2 $\quad 90° < \theta < 180°$

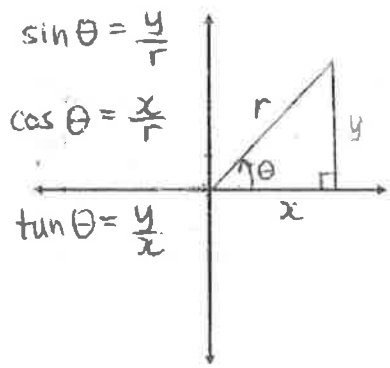


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{y}{-x}$

Quadrant 1 $\quad 0° < \theta < 90°$

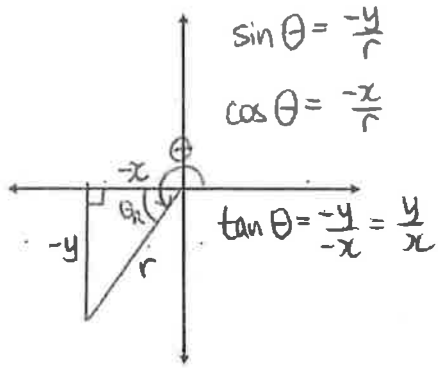


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Quadrant 3 $\quad 180° < \theta < 270°$

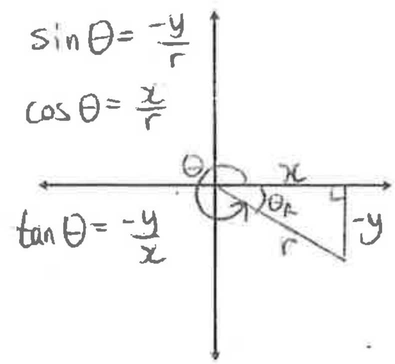


$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

Quadrant 4 $\quad 270° < \theta < 360°$



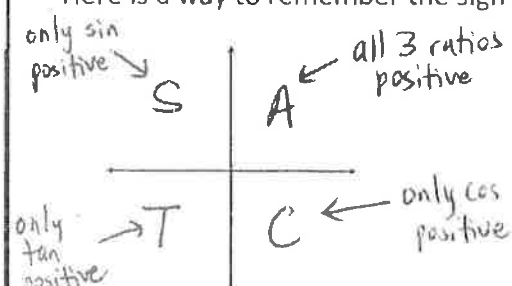
$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{-y}{x}$

CAST

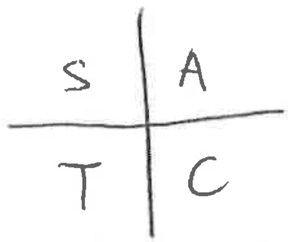
Here is a way to remember the sign of the trigonometric ratios in each quadrant:



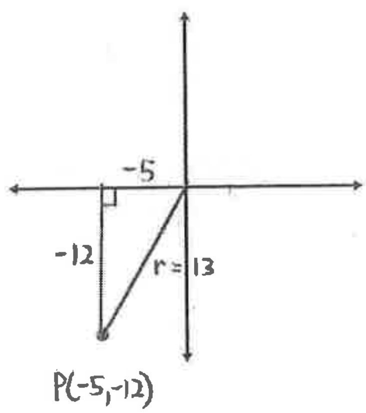
Example 1 – Identify the quadrant(s) for the angles satisfying the following conditions:

a) $\sin\theta < 0, \cos\theta > 0$ Quadrant IV

b) $\tan\theta < 0, \cos\theta < 0$ Quadrant II



Example 2 – The point *P*(-5, -12) lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$$(-5)^2 + (-12)^2 = r^2$$
$$25 + 144 = r^2$$
$$169 = r^2$$
$$r = 13$$

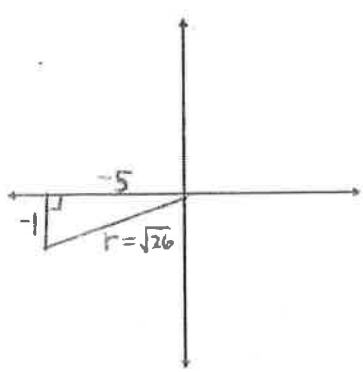
$$\sin\theta = \frac{-12}{13}$$

$$\cos\theta = \frac{-5}{13}$$

$$\tan\theta = \frac{-12}{-5} = \frac{12}{5}$$

Example 3 – Suppose θ is an angle in standard position with terminal arm in quadrant III, and $tan\theta = \frac{1}{5}$. Determine the exact values of $\sin\theta$ and $\cos\theta$.

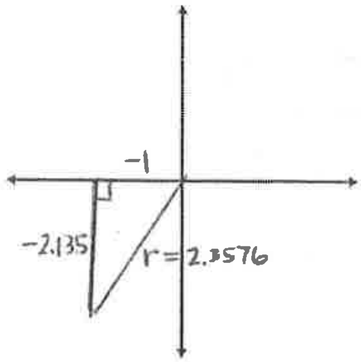
$$\tan\theta = \frac{1}{5} = \frac{-1}{-5} \begin{matrix} \leftarrow \text{opp} \\ \leftarrow \text{adj} \end{matrix}$$



$$(-1)^2 + (-5)^2 = r^2$$
$$1 + 25 = r^2$$
$$26 = r^2$$
$$r = \sqrt{26}$$

$$\sin\theta = \frac{-1}{\sqrt{26}} \qquad \cos\theta = \frac{-5}{\sqrt{26}}$$

Example 4 – Find $\sin \alpha$ if $\tan \alpha = 2.135$ with α in Quadrant III.



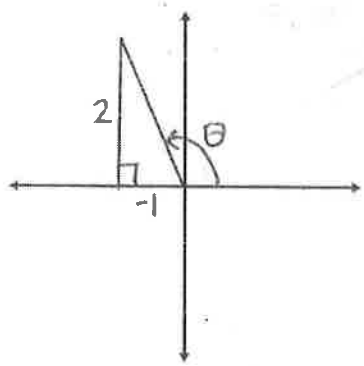
$$\tan\alpha = \frac{2.135}{1} = \frac{-2.135}{-1}$$

$$(-1)^2 + (-2.135)^2 = r^2$$
$$1 + 4.558225 = r^2$$
$$r^2 = 5.558225$$
$$r = 2.3576$$

$$\sin\alpha = \frac{-2.135}{2.3576} = -0.906$$

$$\cos\alpha = \frac{-1}{2.3576} = -0.424$$

Example 5 - $y = -2x,\ x \leq 0$ is the equation of the terminal side of an angle θ in standard position. Sketch the smallest positive angle θ, and determine $\sin\theta$, $\cos\theta$, and $\tan\theta$.



$y = -2x$ slope is $\frac{-2}{1}$ ← down 2, ← right 1

$x \leq 0$ so line is in Quadrant II

$$(-1)^2 + (2)^2 = r^2$$
$$1 + 4 = r^2$$
$$5 = r^2$$
$$r = \sqrt{5}$$

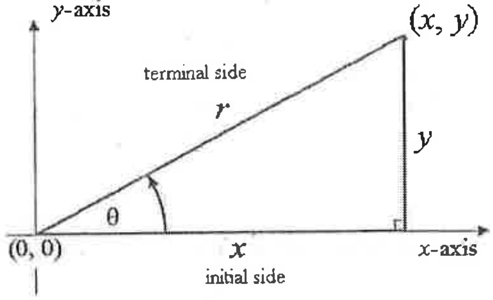
$$\sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$$

$$\tan\theta = \frac{2}{-1} = -2$$

7.3A – Special Angles Part 1

Suppose θ is an angle in standard position. Suppose the point at the end of the terminal arm is labeled $P(x, y)$, at a distance r from the origin.

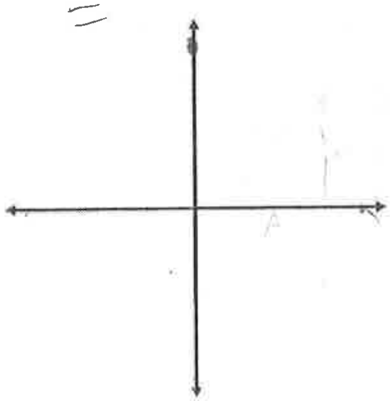


You can use a reference angle to determine the three trigonometric ratios in terms of x, y, and r.

$sin\theta = \frac{y}{r}$ $\quad cos\theta = \frac{x}{r}$ $\quad tan\theta = \frac{y}{x}$

A **quadrantal angle** is an angle in standard position whose terminal arm lies on one of the axes. It's easiest to suppose the terminal arm, r, has a length of 1.

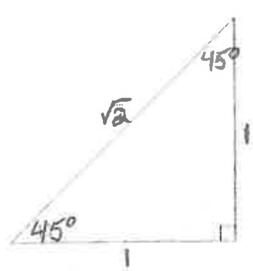
Example – Find the values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each quadrantal angle on the Cartesian plane.



	0°	90°	180°	270°
$\sin\theta$ $\frac{y}{r}$	$\frac{0}{1} = 0$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$
$\cos\theta$ $\frac{x}{r}$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$	$\frac{0}{1} = 0$
$\tan\theta$ $\frac{y}{x}$	$\frac{0}{1} = 0$	$\frac{1}{0}$ = undefined	$\frac{0}{-1} = 0$	$\frac{-1}{0}$ = undefined

There are two right triangles in trigonometry that are especially significant because of their frequent occurrence.

A **45°-45°-90° triangle** with legs of each 1 unit has a hypotenuse of $\sqrt{2}$.



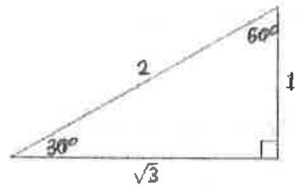
$sin\theta = \frac{opposite}{hypotenuse}$ $\quad cos\theta = \frac{adjacent}{hypotenuse}$ $\quad tan\theta = \frac{opposite}{adjacent}$

S O H C A H T O A

$sin45° = \frac{1}{\sqrt{2}}$ $\quad cos45° = \frac{1}{\sqrt{2}}$ $\quad tan45° = \frac{1}{1} = 1$

or $\sin 45° = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \frac{\sqrt{2}}{2}$

A **30°-60°-90° triangle** has legs of 1 unit and $\sqrt{3}$ units, with hypotenuse 2 units.

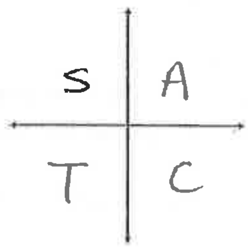


$$\sin 30^\circ = \frac{1}{2} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \tan 30^\circ = \frac{1}{\sqrt{3}}$$

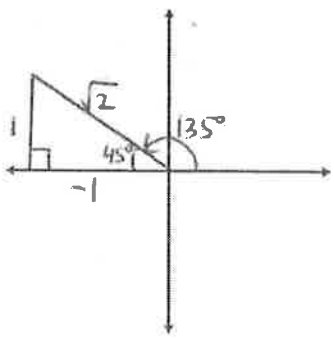
$$\sin 60^\circ = \frac{\sqrt{3}}{2} \qquad \cos 60^\circ = \frac{1}{2} \qquad \tan 60^\circ = \frac{\sqrt{3}}{1}$$

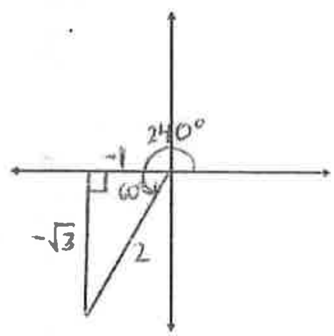
The trigonometric ratios are given as **exact values** (in fraction/radical form as opposed to an approximated decimal).

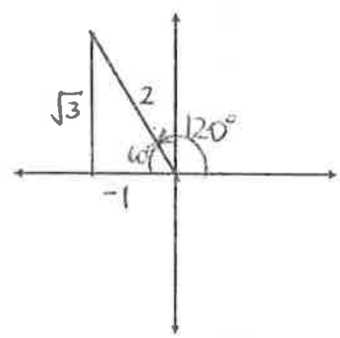
What is the CAST rule again?



Example 1 - Determine the exact values of: a) cos 135° b) sin 240° c) tan 120°

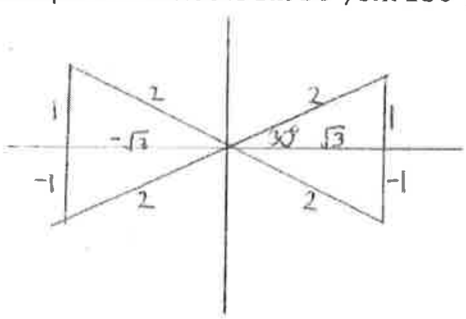






$$\cos 135^\circ = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \qquad \sin 240^\circ = -\frac{\sqrt{3}}{2} \qquad \tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

Example 2 – Evaluate sin 30°, sin 150°, sin 210°, and sin 330°.

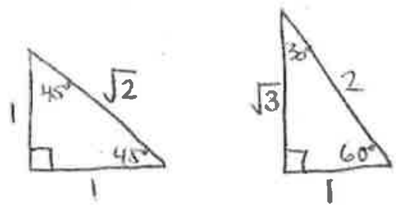


$$\sin 30^\circ = \frac{1}{2} \qquad \sin 210^\circ = \frac{-1}{2}$$

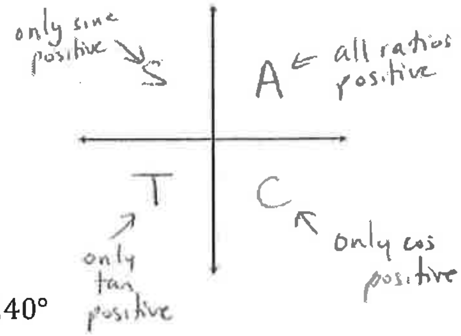
$$\sin 150^\circ = \frac{1}{2} \qquad \sin 330^\circ = \frac{-1}{2}$$

7.3B – Special Angles Part 2

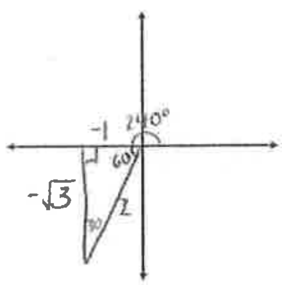
Warmup 1 – Draw the 45°-45°-90° triangle and the 30°-60°-90° triangle below:



Warmup 2 – Quickly draw and explain the 'CAST' rule:



Example 1 – Find the exact value of cos 240°



$\cos 240° = \frac{-1}{2}$

solving for angles

Steps for solving for angles given their sine, cosine, or tangent ratio:

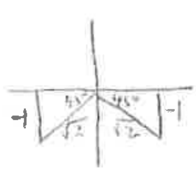
1. Use the sign (+ or -) to determine the quadrant the solution is in.
2. Solve for the reference angle.
3. Draw a diagram and use the reference angle to find the angle in standard position.

Example 2 – Solve for θ.

a) $\sin\theta = -\frac{1}{\sqrt{2}}, 0° \leq \theta < 360°$

Sine is negative in Quads III and IV

$\frac{-1}{\sqrt{2}}$ ← this ratio is from a 45°, 45°, 90° triangle



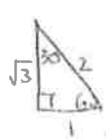
$\theta_R = 45°$ in Quads III and IV

So $\theta = 180 + 45 = 225°$
and $\theta = 360 - 45 = 315°$

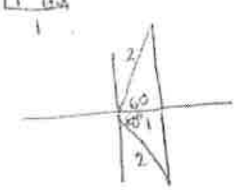
b) $\cos\theta = \frac{1}{2}, 0° \leq \theta < 360°$

Cos is positive in quads I and IV.

$\frac{1}{2}$ ← this ratio is from 30°, 60°, 90° triangle



1 is adjacent to 60° so 60° is θ_R



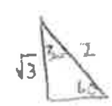
$\theta = 60°$
and $\theta = 360° - 60° = 300°$

$\theta = 60°, 300°$

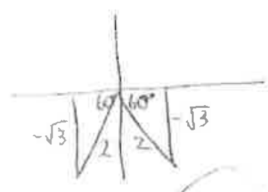
c) $\sin\theta = -\frac{\sqrt{3}}{2}, 0° \leq \theta \leq 360°$

Sine neg in Quads III and IV

$\frac{-\sqrt{3}}{2}$ } ratios from 30, 60, 90°



$\sqrt{3}$ is opposite 60° so 60° is θ_R



$\theta = 180° + 60° = 240°$
$\theta = 360 - 60 = 300°$

Example 3 – Determine the measure of θ, to the nearest degree, given

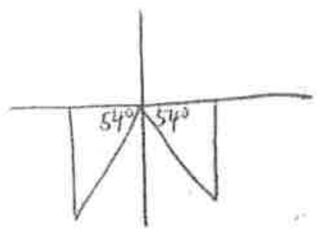
a) $\sin\theta = -0.8090$, where $0° \le \theta < 360°$

Sine is negative in Quads III and IV

θ_R is always acute, so to find it.

we need to find $\theta_R = \sin^{-1} 0.8090$

$\theta_R = 54°$



$\theta = 180° + 54° = 234°$

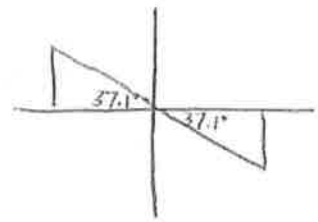
$\theta = 360 - 54° = 306°$

b) $\tan\theta = -0.7565$, where $0° \le \theta < 360°$

tan is neg in quads II and IV

$\theta_R = \tan^{-1} 0.7565$

$\theta_R = 37.1°$



$\theta = 180 - 37.1 = 142.9° = 143°$

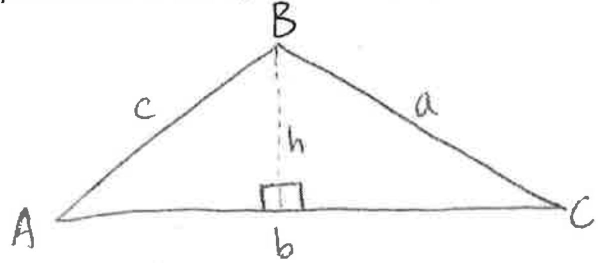
$\theta = 360 - 37.1 = 322.9 = 323°$

7.5 – The Sine Law

So far, you have learned how to use trigonometry when working with right triangles. Now, you will learn how to use trigonometry for **oblique triangles** (non-right triangles).

developing the sine law

Draw an oblique triangle *ABC* and label the sides *a*, *b*, & *c* (opposite the respective corresponding angles). Then, draw a line (call it *h*) from *B* to *b*, so that it is perpendicular to line *b*.



Write a ratio for sin*A*, and then for sin*C*. Then, solve each for *h*.

$\sin A = \frac{h}{c}$ $\quad h = c\sin A$

$\sin C = \frac{h}{a}$ $\quad h = a\sin C$

Since each ratio is equal to *h*, they must also equal one another.

$c\sin A = a\sin C$ so $\frac{\sin A}{a} = \frac{\sin C}{c}$

OR

$\frac{a}{\sin A} = \frac{c}{\sin C}$

By using similar steps, you can also show the same for *b* and sin*B*.

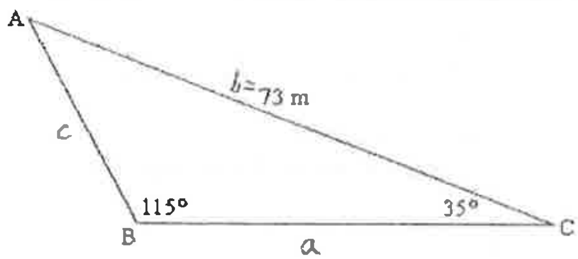
sine law

For any triangle, the sine law states that the sides of a triangle are proportional to the sines of the opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad OR \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Example 1 – Solve for side AB and side BC to the nearest tenth.

c a



$\angle A$:

$\angle A = 180° - 115° - 35°$

$\angle A = 30°$

To find side a (BC):

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 115°}{73} = \frac{\sin 30°}{a}$$

$$a = \frac{73(\sin 30°)}{\sin 115°} = 40.3\text{ m}$$

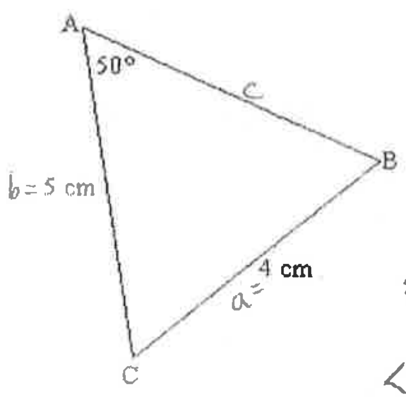
To find side c (AB):

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 115°}{73} = \frac{\sin 35°}{c}$$

$$c = \frac{73(\sin 35°)}{\sin 115°} = 46.2\text{m}$$

Example 2 – Solve for angle B to the nearest degree. Then find angle C to the nearest degree and side AB to the nearest tenth.



$\angle B$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50°}{4} = \frac{\sin B}{5}$$

$$\sin B = \frac{5(\sin 50°)}{4} = 0.9576$$

$$\angle B = \sin^{-1} 0.9576 = 73.2°$$

$\angle C = 180° - 50° - 73.247$

$\angle C = 56.753$

$\angle C = 57°$

Side AB is side c:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50°}{4} = \frac{\sin 56.753°}{c}$$

$$c = \frac{4(\sin 56.753°)}{\sin 50°}$$

$c = 4.4$ cm

information necessary to use the sine law

For oblique triangles, what is the minimum information needed in order to use the sine law to find new information?

Think of $\angle A$ and side a as partners. Same with $\angle B$ and b, and $\angle C$ and c.

To use sine law, you must know everything about one set of partners (angle and side) and at least half of another set of partners.

solving a triangle

When **solving a triangle**, you must find all of the unknown angles and sides.

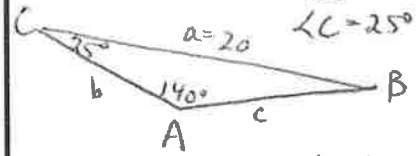
Example 3 – Sketch and solve the triangle (each answer to the nearest tenth).

$$< A = 140°, < C = 25°, a = 20$$

inventory:

$\angle A = 140°$ | $a = 20$
$\angle B =$ | $b =$
$\angle C = 25°$ | $c =$

$\angle B = 15°$
$b = 8.1$
$c = 13.1$



Find $\angle B$:

$180 - 140 - 25$

$\angle B = 15°$

side b:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 140°}{20} = \frac{\sin 15°}{b}$$

$$b = \frac{20(\sin 15°)}{\sin 140°}$$

$$b = 8.05 = 8.1$$

side c:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 140°}{20} = \frac{\sin 25°}{c}$$

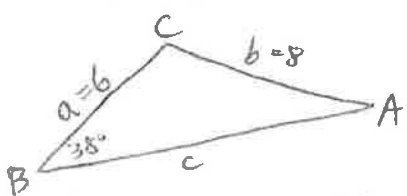
$$c = \frac{20(\sin 25°)}{\sin 140°}$$

$$c = 13.149 = 13.1$$

Example 4 – Solve the triangle (round to the nearest whole number).

$$< B = 38°, b = 8, a = 6$$

$\angle A = 27°$
$\angle C = 115°$
side $c = 12$



inventory:

$\angle A =$ $a = 6$
$\angle B = 38°$ $b = 8$
$\angle C =$ $c =$

$\angle A$:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{6} = \frac{\sin 38°}{8}$$

$$\sin A = \frac{6(\sin 38°)}{8}$$

$$\sin A = 0.461746$$

$$\angle A = \sin^{-1} 0.461746$$

$$\angle A = 27.499$$

$$\angle A = 27°$$

$\angle C$:

$180° - 38° - 27.499$

$\angle C = 114.501$

$\angle C = 115°$

side c

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 38°}{8} = \frac{\sin 114.501°}{c}$$

$$c = \frac{8(\sin 114.501)}{\sin 38°}$$

$$c = 11.8 = 12$$

7.6 – The Cosine Law

For right triangles, the trigonometric ratios sine, cosine, and tangent can be used to find unknown sides and angles. For oblique triangles, **sine law** and **cosine law** must be used.

An effective way to work with oblique triangles is to imagine the angle and its opposite side as 'partners'. Thus, angle *A* and side *a* are partners, <*B* and *b* are partners, and <*C* and *c* are partners.

In order to use the sine law, you must know one full set of partners and half of another set. If you know only half of each set of the three partners, at least two of which are sides, you must use **cosine law**.

Example – For each oblique triangle, state which law you would use.

a) *x* =30cm, *y* =28cm, *z* =32cm (b) <*C* =27°, *a* =17m, *c* =13m (c) <*J* =41°, *k* =16cm, *p* =14cm

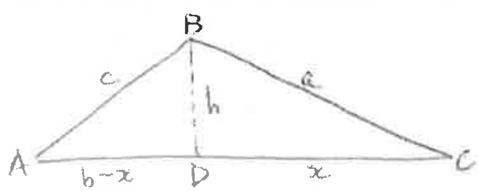
a) 3 half partners, at least two of which are sides
∴ COSINE LAW

b) full set of partners
∴ SINE LAW

c) 3 half partners, at least two of which are sides
∴ COSINE LAW

deriving cosine law

1. The **cosine law** can be developed by starting with oblique ΔABC and drawing vertical line *h* from <*B* to side *b*. Where *h* meets side *b*, call that vertex *D*. Side *CD* can then be labeled x, and side *DA* can be labeled $b - x$.



2. For ΔBCD, find cos*C* and rearrange the equation to isolate x. Then write a Pythagorean equation for ΔBCD.

$\cos C = \frac{x}{a} \Rightarrow x = a\cos C$

$x^2 + h^2 = a^2$

3. Next, for ΔABD, write a Pythagorean equation. Then FOIL $(b - x)^2$. Can you see where a^2 can now replace a part of the equation? What can you replace for x?

$h^2 + (b-x)^2 = c^2$

$h^2 + b^2 - 2bx + x^2 = c^2$

$\underbrace{h^2 + x^2} + b^2 - 2bx = c^2$

$a^2 + b^2 - 2bx = c^2$

$a^2 + b^2 - 2ba\cos C = c^2$

$c^2 = a^2 + b^2 - 2ab\cos C$

↑
COSINE LAW!

cosine law

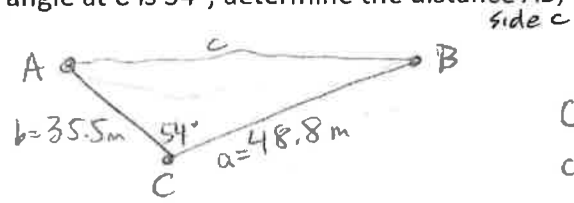
The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Cosine law can also be written as $a^2 = b^2 + c^2 - 2bc\cos A$ **OR**

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Example 1 – Kohl wants to find the distance between two points, *A* and *B*, on opposite sides of a pond. She locates a point *C* that is 35.5m from *A* and 48.8m from *B*. If the angle at *C* is 54°, determine the distance *AB*, to the nearest tenth of a metre.



$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (48.8)^2 + (35.5)^2 - 2(48.8)(35.5)\cos 54°$

$c^2 = 2381.44 + 1260.25 - 2036.56$

$c^2 = 1605.13$

$c = \sqrt{1605.13}$

$c = \underline{40.1\text{m}}$

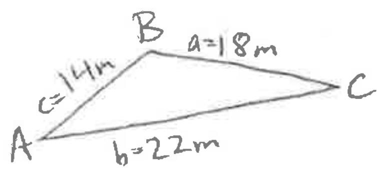
inventory:

$\angle A =$	$a = 48.8$m
$\angle B =$	$b = 35.5$m
$\angle C = 54°$	$c =$

3 half partners, at least two of which are sides

$\therefore$ COSINE LAW

Example 2 – A triangular brace has side lengths 14m, 18m, and 22m. Determine the measure of the angle opposite the 18m side, to the nearest degree.



3 half partners, all three are sides $\therefore$ COSINE LAW

Need to solve for $\angle A$.

$a^2 = b^2 + c^2 - 2bc\cos A$

$18^2 = 22^2 + 14^2 - 2(22)(14)\cos A$

$324 = 484 + 196 - 616\cos A$

$324 = 680 - 616\cos A$

$-680 \quad -680$

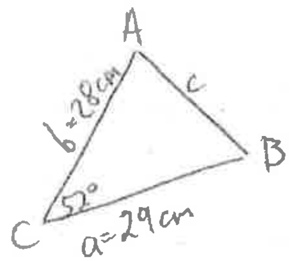
$\frac{-356}{-616} = \frac{-616\cos A}{-616}$

$0.57792 = \cos A$

$\angle A = \cos^{-1} 0.57792 = \underline{55°}$

using cosine law & sine law

Example 3 – In ΔABC, a = 29cm, b = 28cm, and <C = 52°. Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.



$\angle A =$ $\quad a = 29$ cm

$\angle B =$ $\quad b = 28$ cm

$\angle C = 52°$ $\quad c =$

3 half partners, two are sides

∴ COSINE LAW

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (29)^2 + (28)^2 - 2(29)(28)\cos 52°$

$c^2 = 841 + 784 - 999.834$

$c^2 = 625.166$

$c = \sqrt{625.166}$

$c = 25$ cm

To find $\angle A$:

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{29} = \frac{\sin 52°}{25}$

$\sin A = \frac{29(\sin 52°)}{25}$

$\sin A = 0.914$

$\angle A = \sin^{-1} 0.914$

$\angle A = 66.1°$

$\angle B$:

$180° - 52° - 66.1°$

$\angle B = 61.9°$