Key.

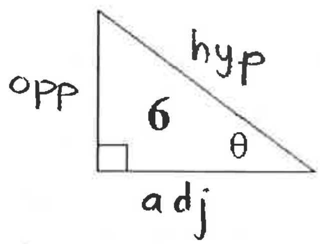
8.1A – Sine, Cosine and Tangent Right Triangles

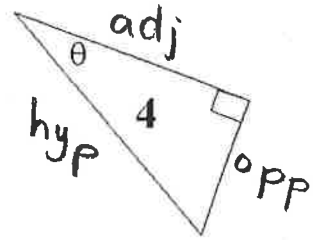
Learning Target: to use the ratio of the sides of a right triangle to solve for angles and sides.

Toolkit:
- Pythagorus
- Labeling the sides of a triangle
- Similar triangles

Warm up

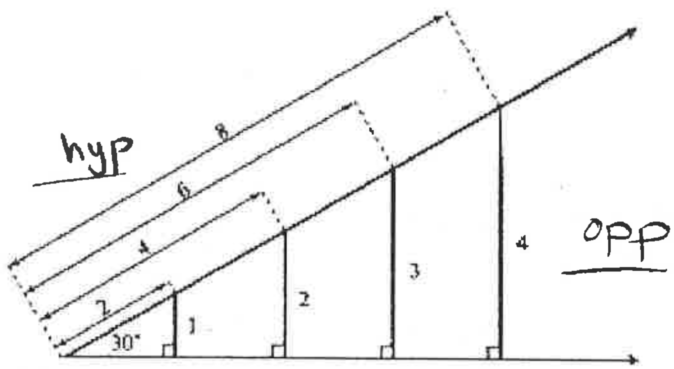
Label the following triangles with hypotenuse (**hyp**), opposite (**opp**) and adjacent (**adj**), with respect to the angle indicated:





Ratios in Right Triangles

In the four similar triangles below, consider the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.



Fundamental Trigonometric Ratios

Each triangle uses an angle of 30°.

The values for the $\frac{\text{opposite}}{\text{hypotenuse}}$ are $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$.

Each ratio is the same $(\frac{1}{2})$

This will always stay true if the angle is the same and the ratio between the sides are the same.

The three trigonometric ratios are as follows:

Sine of θ:
$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{\text{opp}}{\text{hyp}}$

Cosine of θ:
$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{\text{adj}}{\text{hyp}}$

Tangent of θ:
$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{\text{opp}}{\text{adj}}$

The best way to remember the three ratios is to use the phrase:

SOHCAHTOA or $S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

Example 1

Use a calculator to write the ratio in decimal form (to 4 decimals):
make sure your calculator is in degree mode

a) sin35 = 0.5736 b) cos35 = 0.8192 c) tan35 = 0.7002

Example 2

Use a calculator to solve for θ to one decimal place:
use the $\sin^{-1}$, $\cos^{-1}$ $\tan^{-1}$ buttons when we are missing the angles. These are called the inverse trigonometric functions.

a) $\sin\theta = 0.5687$

$\theta = \sin^{-1}(0.5687) = 34.7°$

b) $\cos\theta = 0.5687$

$\theta = \cos^{-1}(0.5687) = 55.3°$

c) $\tan\theta = 0.5687$

$\theta = \tan^{-1}(0.5687) = 29.6°$

Using the Trig Ratios

We can use the trigonometric relationships between angles and the sides of a triangle to find missing pieces of information from the triangles given.

To find a missing angle:

Step 1: Label the triangle using hyp, opp and adj using the missing angle as a reference.

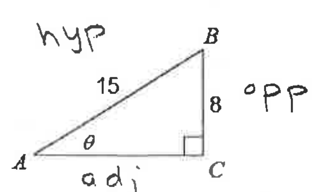
Step 2: Choose the trigonometric ratio that matches the sides given and set up the equation.

Step 3: use the inverse trigonometric function buttons to solve for θ.

Example 3

Find the missing angle:

a)

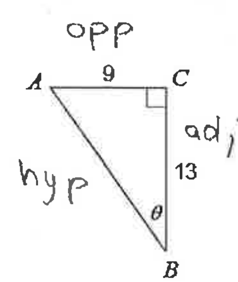


we have opp and hyp so

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\sin\theta = \dfrac{8}{15}$

$\theta = \sin^{-1}\left(\frac{8}{15}\right) = \boxed{32.2°}$

b)



we have opp and adj so

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan\theta = \dfrac{9}{13}$

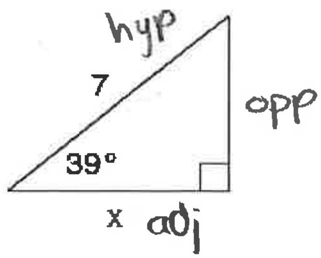
$\theta = \tan^{-1}\left(\frac{9}{13}\right)$

$= 34.7°$

To find a missing side length:
Step 1: Label the triangle using hyp, opp and adj using the angle given.
Step 2: Choose the trigonometric ratio that includes the angle given, the side given and the missing side.
Step 3: Set up the trigonometric equation.
Step 4: Use cross multiplication to solve for the missing side length.

Example 4 Find the missing side:

a)



we have hyp but need adj so

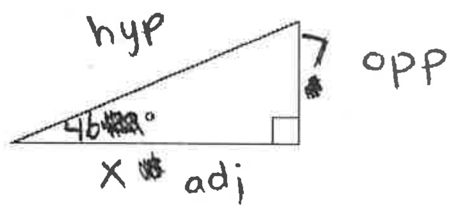
$\cos\theta = \frac{adj}{hyp}$

$7 \cdot \cos 39 = \frac{x}{7} \cdot 7$

$x = 7 \cdot \cos 39$

$\boxed{x = 5.44}$

b)



we have opp but need adj, so

$\tan\theta = \frac{opp}{adj}$

$x \cdot \tan 46 = \frac{7}{x} \cdot x$

$\frac{x \tan 46}{\tan 46} = \frac{7}{\tan 46}$

$x = \frac{7}{\tan 46}$

$\boxed{x = 6.76}$

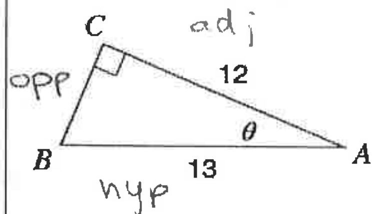
8.1B – Sine, Cosine and Tangent Right Triangles

Learning Target: to use the ratio of the sides of a right triangle to solve for all angles and sides of a triangle.

Toolkit:	
- Pythagorus - Labeling the sides of a triangle - SOHCAHTOA	

Warm up

Find the missing angle:

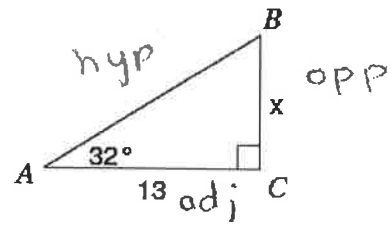


$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{12}{13}$ $\theta = \cos^{-1}\left(\frac{12}{13}\right)$

$= 22.6°$

Find the missing side:



$\tan\theta = \frac{opp}{adj}$

$\tan 32 = \frac{x}{13}$ $x = 13 \cdot \tan 32$

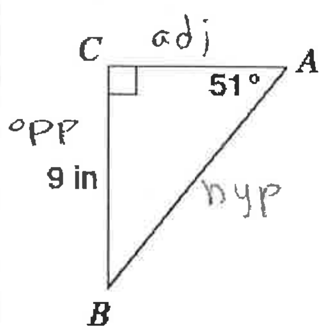
$\boxed{= 8.12}$

As long as we have a right triangle with 2 pieces of information, we can calculate all of other side lengths and angles. This is called "solving" the triangle. To solve a right triangle, we can use SOHCAHTOA, the Pythagorean theorem and the sum of three angles in a triangle are 180°. Our final answer will have all **three angles** and all **three sides**.

There are many ways to solve each right triangle.

Example 1

Solve the right triangle:



$\angle B = 90° - \angle A$

$= 90° - 51° = \boxed{39°}$

AC:

$\tan\theta = \frac{opp}{adj} \rightarrow \tan 51 = \frac{9}{AC}$

$AC = \frac{9}{\tan 51} = \boxed{7.3\text{in}}$

AB:

$\sin\theta = \frac{opp}{hyp}$

$\sin 51 = \frac{9}{AB}$

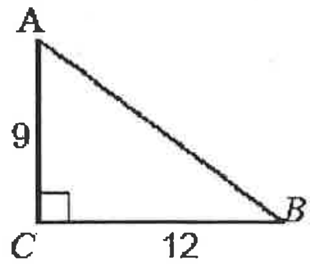
$A = \frac{9}{\sin 51} = \boxed{11.6\text{in}}$

$\angle B = 39°$

$AB = 11.6$

$AC = 7.3\text{in}$

Example 2	Solve the right triangle:



AB: Pythagorus

$a^2 + b^2 = c^2$

$9^2 + 12^2 = c^2$

$81 + 144 = c^2$

$\sqrt{225} = \sqrt{c^2}$

$c = \boxed{15}$

∠A:

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{12}{9}$

$\theta = \tan^{-1}\left(\frac{12}{9}\right)$

$\boxed{= 53.1^\circ}$

∠B:

$\angle B = 90^\circ - \angle A$

$= 90^\circ - 53.1^\circ$

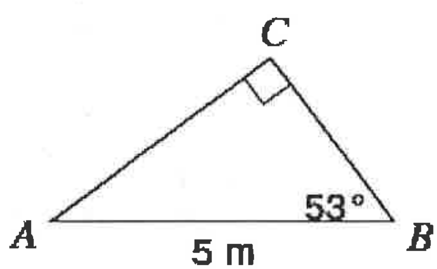
$\boxed{= 36.9^\circ}$

$\angle A = \underline{53.1^\circ}$

$\angle B = \underline{36.9^\circ}$

$AB = \underline{15}$

Example 3	Solve the right triangle:



∠A:

$\angle A = 90^\circ - \angle B$

$= 90^\circ - 53^\circ$

$\boxed{= 37^\circ}$

AC:

$\sin\theta = \frac{opp}{hyp}$

$\sin 53 = \frac{AC}{5}$

$AC = 5 \cdot \sin 53$

$\boxed{= 4.0}$

CB:

$\cos\theta = \frac{adj}{hyp}$

$\cos 53 = \frac{BC}{5}$

$BC = 5 \cdot \cos 53$

$\boxed{= 3.0}$

$\angle A = \underline{37^\circ}$

$AC = \underline{4.0}$

$BC\ \underline{3.0}$

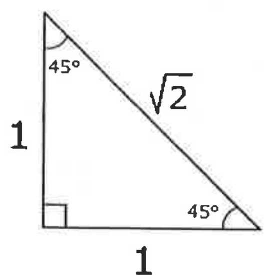
8.3 – Special Angles

Learning Target: to use the ratio of the sides of a right triangle to solve for angles and sides.

Toolkit:
- Pythagorus
- Labeling the sides of a triangle
- SOHCAHTOA

There are two triangles that are often seen in mathematics (especially Pre-Calculus 11/12). We can use these triangles to shoe the exact value for the trigonometric functions.

45° – 45° – 90° Triangle



The 45° – 45° – 90° right triangle is an isosceles triangle with two equal sides of 1.

Use The Pythagorean Theorem to show the length of the hypotenuse is $\sqrt{2}$:

$a^2 + b^2 = c^2$

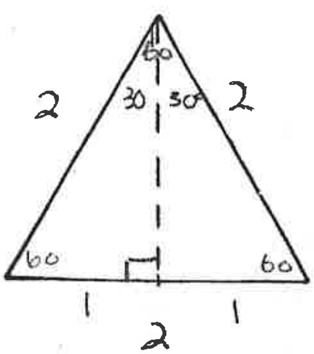
$1^2 + 1^2 = c^2$

$1 + 1 = c^2$

$\sqrt{2} = \sqrt{c^2} \longrightarrow c = \sqrt{2}$

$\sin 45 = \frac{opp}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \cos 45 = \frac{adj}{hyp} = \boxed{\frac{1}{\sqrt{2}}}$ $\quad \tan 45 = \frac{opp}{adj} = \frac{1}{1} = \boxed{1}$

30° – 60° – 90° Triangle



We start with an equilateral triangle with all side lengths of 2. Each angle in an equilateral triangle is 60°.

By drawing a line down the middle of the triangle, the base is divided into two equal parts.

Use The Pythagorean Theorem to show the length of the line drawn:

$a^2 + b^2 = c^2$

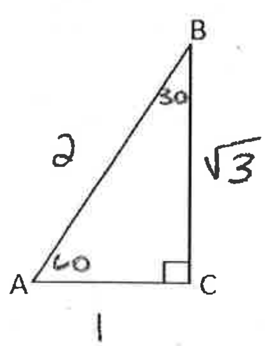
$a^2 = c^2 - b^2$

$a^2 = 2^2 - 1^2$

$a^2 = 4 - 1$

$\sqrt{a^2} = \sqrt{3}$

$a = \sqrt{3}$



This creates the 30° – 60° – 90° triangle.

$\sin 30 = \frac{1}{2}$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\sin 60 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$ $\tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

Example 1

Find the exact value of $\sin\frac{\theta}{2}$ if $\theta = 90$

$$\sin\left(\frac{90}{2}\right) = \sin 45 = \boxed{\frac{1}{\sqrt{2}}}$$

simplify first

Now use special triangles

Example 2

Find the exact value of $\cos 2\theta$ if $\theta = 15$

$$\cos(2\cdot 15) = \cos 30 = \boxed{\frac{\sqrt{3}}{2}}$$

simplify

Special triangles

Example 3

Find the exact value of $\tan 30 \times \cos 45 +$ ~~sin 60~~ tan 45

* Remember bedmos *

$$\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} + 1$$

$$\frac{1}{\sqrt{6}} + \frac{1\cdot\sqrt{6}}{1\cdot\sqrt{6}}$$

Need common denominator

$$\frac{1}{\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{1+\sqrt{6}}{\sqrt{6}}}$$

8.4 – Applications of Trigonometry

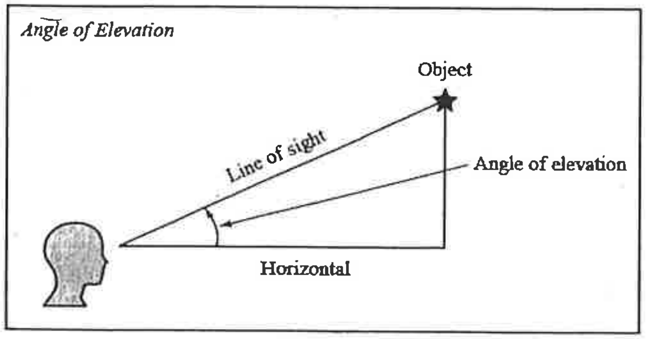
Learning Target: to apply trigonometric concepts to solve word problems

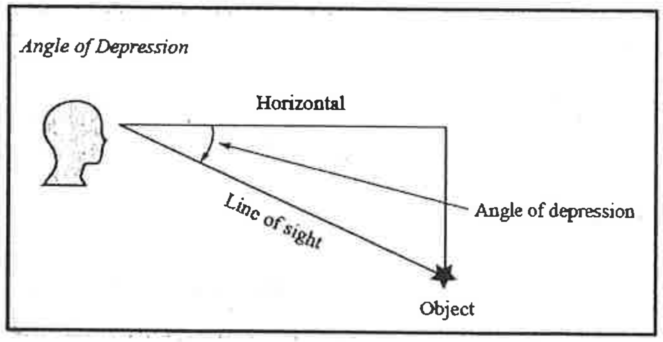
Toolkit:

- SOHCAHTOA
- Equilateral, isosceles, scalene
  - Equilateral → All sides equal; isosceles → two sides equal; scalene → no sides equal
- Horizontal vs. Vertical ⟷ ↕

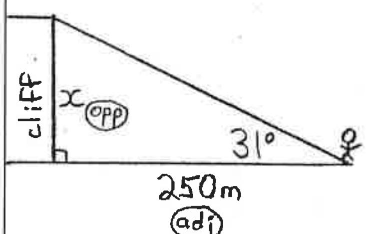
* 3 angles in any triangle add to 180°

Terminology:





Ex 1) Standing 250 metres from the base of a cliff, there is a 31° angle from your feet to the top of the cliff. How tall is the cliff? round to nearest tenth



TOA, so use tan!

$$(\tan 31°)\times 250 = \left(\frac{x}{\cancel{250}}\right)\times \cancel{250}$$

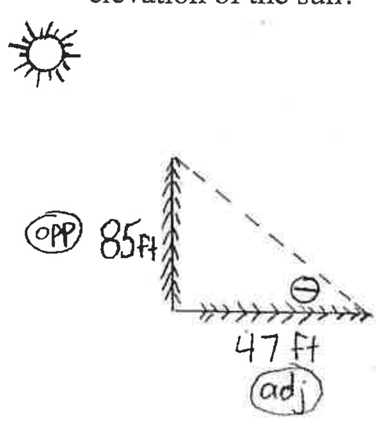
$$x = (\tan 31°)\times 250$$

$$x = 150.2$$

$\boxed{\text{The cliff is 150.2m tall}}$

Ex 2) A Douglas Fir tree 85 feet high casts a shadow of 47 feet. What is the angle of elevation of the sun? round to nearest tenth.

not to scale



TOA, so use tan!

$$\tan\theta = \frac{85}{47}$$

$$\angle\theta = \tan^{-1}\left(\frac{85}{47}\right)$$

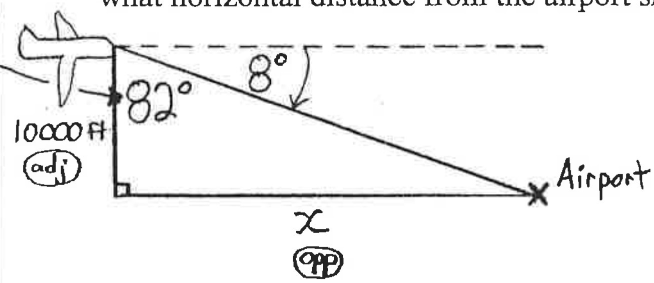
$$\angle\theta = \tan^{-1}(1.80851)$$

$$\angle\theta = 61.1°$$

$\boxed{\text{The angle of elevation of the sun is } 61.1°}$

$90° - 8°!$

Ex 3) A pilot is required to approach Vancouver airport at an 8° angle of descent (angle of depression). If the plane is travelling at an altitude of 10 000 ft, at what horizontal distance from the airport should the descent begin? round to nearest tenth



$\frac{T}{A}\frac{O}{}$, so use tan!

$$\left(\tan 82°\right) \times 10000 = \left(\frac{x}{10000}\right) \times 10000$$

$$x = (\tan 82°) \times 10000$$

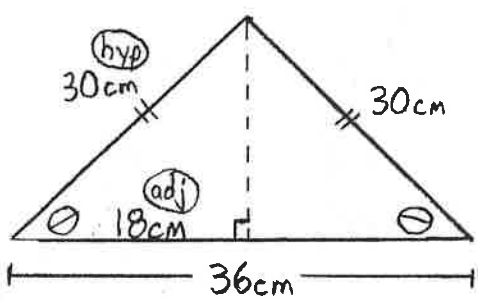
$$x = 71153.7 \text{ ft}$$

The descent should begin when the horizontal distance to the airport is 71153.7 ft

Ex 4) The equal sides of an isosceles triangle are 30cm, and the third side is 36 cm. Determine the measure of the interior angles of the triangle. round to nearest tenth

(* angles opposite equal sides are equ.)

$\frac{36}{2} = 18!$



$\frac{C}{}\frac{A}{H}$, so use cos!

$$\cos\theta = \frac{18}{30}$$

$$\angle\theta = \cos^{-1}\left(\frac{18}{30}\right)$$

$$\angle\theta = \cos^{-1}(0.6)$$

$$\angle\theta = 53.1°$$

top angle is $180° - 53.1° - 53.1°$
$= 73.8°$

$\therefore$ The three interior angles are 53.1°, 53.1°, and 73.8°

bottom left and bottom right angles are both 53.1°!

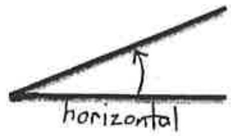
8.5 – Compound Trigonometry Applications

Learning Target: to apply trigonometry to solve problems, sometimes with two right triangles

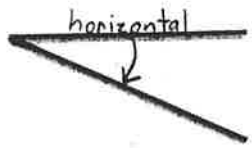
Toolkit:

- $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$
- Making a PLAN to solve the problem
- $a^2+b^2=c^2$

Angle of elevation:

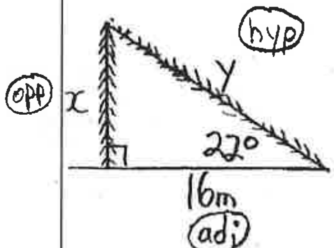


Angle of depression:



Ex 1) The top of an arbutus tree broken in the wind hits the ground 16 metres from the base of the tree. If the top of the tree now makes an angle of 22° with the ground, what was the original height of the arbutus tree?

Round to nearest tenth



∗ original height of tree is $x + y$!

Find x:

$T\frac{O}{A}$, use tan!

$(\tan 22°)\times 16 = \left(\frac{x}{16}\right)\times 16$

$x = (\tan 22°)\times 16$

$x = 6.46$

find y: (or pythag!)

$C\frac{A}{H}$, use cos!

$\cos 22° = \frac{16}{y}$

$y = \frac{16}{\cos 22°}$

$y = 17.26$

height $= x + y$

$= 6.46 + 17.26$

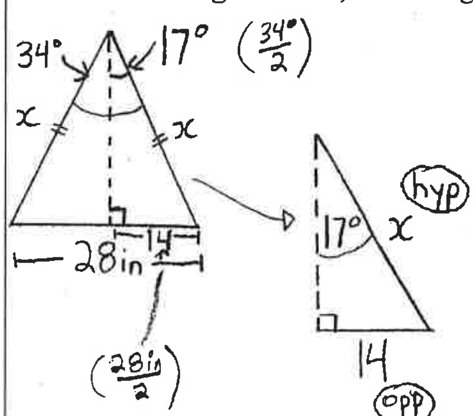
$\boxed{= 23.7\,m}$

The original height was 23.7 m

Ex 2) An isosceles triangle has a base of 28 in. If the **legs** (the two equal sides) meet at an angle of 34°, how long are they?

(x)

Round to nearest tenth



$S\frac{O}{H}$, so use sin!

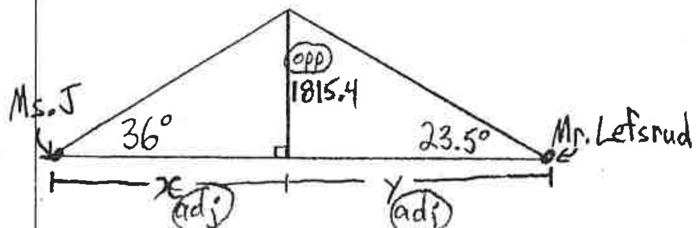
$\sin 17° = \frac{14}{x}$

$x = \frac{14}{\sin 17°}$

$\boxed{x = 47.9\,in}$

Each leg is 47.9 in

Ex 3) The CN Tower is 1815.4 ft high. Ms. J measures the angle of elevation to the top of the tower at 36°. Mr. Lefsrud, directly opposite the tower, measures the angle of elevation to the top of the tower at 23.5°. How far apart are Ms. J and Mr. Lefsrud? *Round to nearest tenth*



distance apart is $x + y$!

$= 2498.68 + 4175.13$

$= 6673.8$ ft

find x: TOA, use tan!

$\tan 36° = \frac{1815.4}{x}$

$x = \frac{1815.4}{\tan 36°}$

$x = 2498.68$ ft

find y: TOA, use tan!

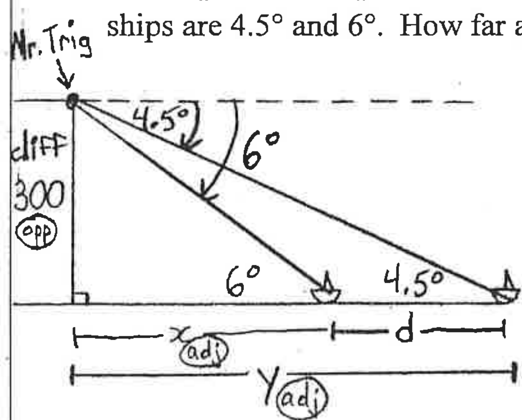
$\tan 23.5° = \frac{1815.4}{y}$

$y = \frac{1815.4}{\tan 23.5°}$

$y = 4175.13$ ft

Ms. J and Mr. Lefsrud are 6673.8 ft apart

Ex 4) A lighthouse keeper, Mr. Trig, who is at the top of a cliff 300 m above sea level, spots two ships directly off shore. The angles of depression of the ships are 4.5° and 6°. How far apart are the ships? *Round to nearest tenth*



distance between ships (d):

$d = y - x$

$d = 3811.86 - 2854.31$

$d = 957.6$ m

$\boxed{\text{The ships are 957.6m apart}}$

Find y using large right triangle:

TOA, use tan!

$\tan 4.5° = \frac{300}{y}$

$y = \frac{300}{\tan 4.5°}$

$y = 3811.86$ m

Find x using small right triangle:

TOA, use tan!

$\tan 6° = \frac{300}{x}$

$x = \frac{300}{\tan 6°}$

$x = 2854.31$