Name: NOTES KEY

Date: ______________________

CHAPTER 1 NOTES – Powers and Exponent Laws

Calendar of Chapter: See the 'Homework' link on the webpage

What You'll Learn:

1.4 – Use powers to represent repeated multiplication & use patterns to understand a power with exponent 0

Scientific Notation – Learn how to put numbers into and take them out of scientific notation

1.5 – Solve problems and perform operations (BEDMAS) involving powers

1.6/1.7 – Explain and apply exponent laws

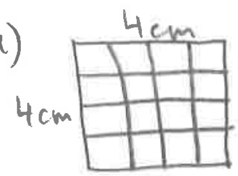
Negative Exponents - To understand and apply negative exponents in evaluating powers

When are powers needed in the 'real world'?

1.4 – Defining a Power

Focus: Use powers to represent repeated multiplication & understand the zero exponent law.

Warmup:
Suppose you have a square with length 4cm.
a) Draw a picture and calculate the area with proper units.
b) How did you calculate the area?
c) How could you write a calculation for the area using a power?

a) 

$A = 4cm \times 4cm = 16cm^2$

b) 4×4
or
count up little squares

c) 4^2 as $4^2 = 4 \times 4$

Name all of the parts of power form.

4^2
exponent
power
base

There are 3 ways to write 'power math':

Example:
POWER FORM: 2^3

REPEATED MULTIPLICATION: $2 \times 2 \times 2$

STANDARD FORM: 8

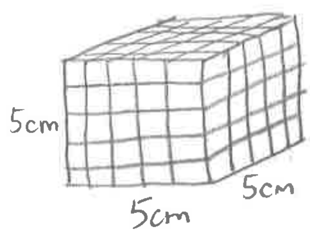
Ex1 - Write 16 using each form:

POWER FORM (two ways ☺): 4^2 and 2^4

REPEATED MULTIPLICATION (two ways ☺) 4×4 and $2 \times 2 \times 2 \times 2$

STANDARD FORM: 16

Ex2
Suppose you have a cube with side length 5cm. Draw the cube & write the volume in power form, as a repeated multiplication, & in standard form with correct units.



POWER: 5^3
$5 \times 5 \times 5$
$125cm^3$

What do you call a:
power with exponent 2?
'squared'
power with exponent 3?
'cubed'

Ex3: Write as a power & then in standard form:
a) 2 x 2 x 2 x 2 x 2
b) (3)(3)(3)
c) 7

a) $2^5 = 32$
b) $3^3 = 27$
c) $7^1 = 7$

Ex4: Write as a repeated multiplication & in standard form: 4^6

$4 \times 4 \times 4 \times 4 \times 4 \times 4$

$= 4096$

To do a power on your calculator, using the exponent button, which looks like:

^ or y^x

Ex5 Identify the base of each power, then evaluate the power

a) $(-3)^4$

b) -3^4

a) $(-3)^4$ base: -3

$-3 \times -3 \times -3 \times -3 = 81$

b) -3^4 base: 3

$-3 \times 3 \times 3 \times 3 = -81$

Can you write a summary sentence that explains when a negative is part of a base & when it isn't?

The base is whatever is inside the brackets. If no brackets, the base is the number immediately left of the exponent, and does not include the negative sign if there is one.

Ex6: Get a blank piece of paper from your binder. **Fill in the table:**

FOLDS	LAYERS	LAYERS IN POWER FORM
0	1	$= 2^0$
1	2	$= 2^1$
2	4	$= 2^2$
3	8	$= 2^3$
4	16	$= 2^4$

How many layers if 5 folds are made? How many layers if 26 folds are made?

$2^5 = 32$ layers

$2^{26} = 67\ 108\ 864$ layers

Ex7 Complete the table

Power	Standard Form
10^6	1 000 000
10^5	100 000
10^4	10 000
10^3	1 000
10^2	100
10^1	10
10^0	1

What is the shortcut for base 10 without a calc?

the exponent is how many zeros are after the 1

Using the pattern developed from the table, what is 10^0 ?

$10^0 = 1$ as there are zero zeros after the 1.

What is the Zero Exponent Law?

Any base to the zero exponent equals 1.

Ex8 Evaluate each expression:

a) 2^0

b) $(-2)^0$

c) -2^0

a) $2^0 = 1$

b) $(-2)^0 = 1$ (base is -2)

c) $-2^0 = -1$ (base is 2)

$-2^0 = -1$

Scientific Notation

Focus: To understand and apply scientific notation correctly.

Warmup:
Write the following as a power of ten:
a) 7000

b) 60

a) $7 \times 1000 = 7 \times 10^3$

b) $60 = 6 \times 10 = 6 \times 10^1$

What is Scientific Notation?

It's a way of expressing very large or very small numbers using base 10 powers so you don't have to write numbers in standard form.

Ex1
Put 80 000 into scientific notation and indicate the coefficient

80 000
8×10000
8×10^4

OR

80,000.
4 jumps
$= 8 \times 10^4$

What must be true of the coefficient in order to be in scientific notation?

It must be between 1 and 10, not including 10. (only one digit to the left of the decimal)

Ex2
Put 22 000 000 into scientific notation

22,000,000. $= 2.2 \times 10^7$
7 jumps

Ex3
Put 360 000 000 into scientific notation

360,000,000 $= 3.6 \times 10^8$
8 jumps

Remember: *Scientific notation is just a different way to write the same number!*

Ex4
Watch '**Act 1**' of 'the flashlight'

Take a guess: Do you think the light travels to the moon like the top picture, or the bottom one?

Ask two classmates and record their guess here:

What do we need to know to figure this out?

distance to moon:
speed of light:

Put each into scientific notation with units:

dist to moon: 3.844×10^8 m
speed of light: 3×10^8 m/s

Ex5
Put 0.0000088 into scientific notation

$\underbrace{0.0000088}_{\text{6 jumps}} = 8.8 \times 10^{-6}$

What does a negative exponent mean?

It means you are working with a small number (less than 1)

Ex6
Put into scientific notation:
a) 0.00956

b) 0.000014

a) 0.00956
9.56×10^{-3}

(b) 0.000014
1.4×10^{-5}

How do you put numbers that are in scientific notation back into standard form?

Look at the exponent. If positive, your number is big (bigger than 1), so jump decimal right. If exponent is negative, your number is small (smaller than 1), so jump decimal left.

Ex7
Put into standard form:
a) 2.65×10^{-3}

b) 7×10^{6}

c) 8.3×10^{-5}

d) 1×10^{0}

a) $2.65 \times 10^{-3} \leftarrow$ jump decimal LEFT $2.65 \times 10^{-3} = 0.00265$

b) $7 \times 10^{6} = 7 \times 10^{6} = 7\,000\,000$

c) $8.3 \times 10^{-5} = 0.000083$

d) $1 \times 10^{0} \leftarrow$ no jumps $= 1$

Ex8
Use your calculator & answer in sci not:
a) $(3.56 \times 10^{-3})(7.4 \times 10^{6})$

2.6×10^{4} or 2.6344×10^{4}

To put 3.56×10^{-3} into your calculator, type in: 3.56
Then, press the:

EXP	or	EE	or	x10$^{\square}$

Then press the negative button, then 3 (or 3, then the neg button)

b) Use your values from Ex4 to finish 'the flashlight'.

Time = $\dfrac{\text{Distance}}{\text{Speed}}$

How long does it take for light to reach the moon?

$\text{time} = \dfrac{3.844 \times 10^{8}\ m}{3 \times 10^{8}\ m/s} = 1.3$ seconds

Was your guess correct? How do you know?

1.5 – Order of Operations (DISCOVERY LESSON)

Focus: Explain and apply the order of operations with exponents while working in a group.

Group Warmup:
You win the big prize in the Thrifty's sweepstakes, but can only claim top prize if you get the skill testing question correct:
6 x (3 + 2) – 10 ÷ 2
What is the answer?

6 x (3 + 2) – 10 ÷ 2
6 x (5) – 10 ÷ 2
30 – 10 ÷ 2
30 – 5
= 25

Teacher led method for solving an order of operations question:

6 x (3 + 2) – 10 ÷ 2 circle each step!
6 x 5 – 10 ÷ 2
30 – 10 ÷ 2
30 – 5
25

What is the key word for order of operations, and what does each letter mean?

B E $\boxed{D\ M}$ $\boxed{A\ S}$

B: brackets
E: exponents
D: division
M: multiplication
A: addition
S: subtraction

Are there any letters that have the same ranking when answering a question?

Yes. D & M Do whichever comes first in the question
Yes. A & S " " " " " "

Ex1 – Evaluate by circling each step:
a) $3^3 + 2^3$

b) $3 - 2^3$

c) $(3 + 2)^3$

a) $3^3 + 2^3$
27 + 8
= 35

(b) $3 - 2^3$
3 – 8
= –5

(c) $(3+2)^3$
5^3
= 125

Ex2 – Evaluate by circling each step:
a) $[2 \times (-3)^3 - 6]^2$

b) $3 + 2^4 - 3 \times (2^2 - 1)$

c) $(18^2 + 5^0)^2 \div (-5)^3$

a) $[2 \times (-3)^3 - 6]^2$
$[2 \times -27 - 6]^2$
$[-54 - 6]^2$
$[-60]^2$
$= -60 \times -60$
$= 3600$

(b) $3 + 2^4 - 3 \times (2^2 - 1)$
$3 + 2^4 - 3 \times (4 - 1)$
$3 + 2^4 - 3 \times 3$
$3 + 16 - 3 \times 3$
$3 + 16 - 9$
$19 - 9$
10

(c) $(18^2 + 5^0)^2 \div (-5)^3$
$(324 + 1)^2 \div (-5)^3$
$(325)^2 \div (-5)^3$
$105625 \div -125$
$= -845$

Ex3 – Evaluate to one decimal place
$$\frac{690}{2 \times 4^2 + \pi \times 1^3}$$

$\frac{690}{2 \times 4^3 + \pi \times 1^3} = \frac{690}{2 \times 16 + \pi \times 1} = \frac{690}{32 + \pi} = \frac{690}{35.1416} = 19.6$

*Note: When you have a long division bar in your question, then do the division LAST.

Have your teacher check over all examples to make sure they are correct before moving onto your assignment, which you can also do as a group!

1.6 – Exponent Laws

Focus: Understand and apply the exponent laws for products and quotients of powers.

Warmup:
Complete the table and see if you can find a pattern:

Product of Powers	Product as Repeated Multiplication	Product as a Power
$5^4 \times 5^2$	5x5x5x5 x 5x5	5^6
$3^5 \times 3^4$	$3\times3\times3\times3\times3 \times 3\times3\times3\times3$	3^9
$2^3 \times 2^3$	$2\times2\times2 \times 2\times2\times2$	2^6
$4^6 \times 4$	$4\times4\times4\times4\times4\times4 \times 4$	4^7

Can you describe what the pattern is, so that you do not have to write the middle step?

add exponents

Will the pattern work for the following?
$2^3 \times 3^2$
Why or why not?

No, because the bases are not the same!

What is the **Product Rule**?

When multiplying powers with the same base, ADD exponents $x^a \times x^b = x^{a+b}$

Ex1
Write each expression as a single power:
a) $3^5 \times 3^2$

b) $6 \times 6^3 \times 6^4$

a) $3^5 \times 3^2$
$= 3^{5+2}$
$= 3^7$

b) $6^1 \times 6^3 \times 6^4$
$= 6^{1+3+4}$
$= 6^8$

Quotient of Powers	Quotient as Repeated Multiplication	Quotient as a Power
$8^7 \div 8^4$	$\dfrac{\cancel{8}\times\cancel{8}\times\cancel{8}\times\cancel{8}\times 8\times 8\times 8}{\cancel{8}\times\cancel{8}\times\cancel{8}\times\cancel{8}}$	$8^{\boxed{3}}$

Let's investigate how to find $8^7 \div 8^4$

Can you describe what the pattern is, so that you don't have to write the middle step?

subtract exponents

What is the **Quotient Rule**?

When dividing powers with the same base, SUBTRACT exponents.

$x^a \div x^b = x^{a-b}$

Ex2
Write each expression as a single power:
a) $4^8 \div 4^3$

b) $\frac{(-5)^6}{(-5)^4}$

a) $4^8 \div 4^3$
4^{8-3}
4^5

(b) $\frac{(-5)^6}{(-5)^4}$
$= (-5)^{6-4}$
$= (-5)^2$

Ex3 – Write as a single power:
a) $3^2 \times 3^4 \div 3^3$

b) $\frac{(-4)^3 \times (-4)^6}{(-4) \times (-4)^4}$

a) $3^2 \times 3^4 \div 3^3$
$3^6 \div 3^3$
$= 3^3$

(b) $\frac{(-4)^3 \times (-4)^6}{(-4)^1 \times (-4)^4}$
$\frac{(-4)^9}{(-4)^5} = (-4)^4$

Ex4 $\frac{2^3}{2^3}$
a) Write as a single power
b) Simplify using repeated multiplication.
c) Why is any base to the zero exponent equa to 1?

a) $\frac{2^3}{2^3}$
$= 2^{3-3}$
$= 2^0$

(b) $\frac{2 \times 2 \times 2}{2 \times 2 \times 2}$
$= \frac{8}{8}$
$= 1$

(c) Therefore, $2^0 = 1$
To end up with a base to the zero exponent, you must have a fraction where the top is an identical power as the bottom, so the fraction equals 1.

Ex5 – Evaluate

a) $2^3 \times 3^2$

b)
$(-10)^4[(-10)^6 \div (-10)^4] - 10^5$

a) $2^3 \times 3^2$
no exponent law, as bases are different, so BEDMAS
$2^3 \times 3^2$
8×9
72

b) $(-10)^4[(-10)^6 \div (-10)^4] - 10^5$
$(-10)^4[(-10)^2] - 10^5$
$(-10)^6 - 10^5$
base is -10 base is 10
$1\,000\,000 - 100\,000$
$= 900\,000$

1.7 – Power Rules

Focus: Understand and apply exponent laws for powers of: powers, products, & quotients.

Warmup:
Complete the table and see if you can find a pattern:

Power	Repeated Multiplication	Expanded Form	Power
$(2^3)^2$	$2^3 \times 2^3$	2x2x2 x 2x2x2	2^6
$(4^2)^4$	$4^2 \times 4^2 \times 4^2 \times 4^2$	4x4 x 4x4 x 4x4 x 4x4	4^8
$(5^3)^3$	$5^3 \times 5^3 \times 5^3$	5x5x5 x 5x5x5 x 5x5x5	5^9
$[(-3)^2]^3$	$(-3)^2 \times (-3)^2 \times (-3)^2$	-3x-3 x -3x-3 x -3x-3	$(-3)^6$

Can you describe what the pattern is, so that you don't have to write the two middle steps?

multiply exponents

What is the **Power Rule**?

When you have a power raised to an exponent, multiply the exponents $(x^m)^n = x^{mn}$

Ex1 – Simplify as a power:
a) $(9^5)^6$
b) $[(-1)^3]^4$
c) $-(3^7)^2$

a) $(9^5)^6$
$= 9^{5(6)}$
$= 9^{30}$

(b) $[(-1)^3]^4$
$= (-1)^{3(4)}$
$= (-1)^{12}$

(c) $-(3^7)^2$
$= -3^{7(2)}$
$= -3^{14}$

Evaluate $(3 \times 4)^5$ using BEDMAS

$(3\times4)^5$
$= 12^5$
$= 248\ 832$

Can you find another way to evaluate the question above to get the same answer?

$(3\times4)^5$
$3^5 \times 4^5$
$= 243 \times 1024$
$= 248\ 832$

$(3\times4)(3\times4)(3\times4)(3\times4)(3\times4)$
$= 3\times4\times3\times4\times3\times4\times3\times4\times3\times4$
$= 3\times3\times3\times3\times3\times4\times4\times4\times4\times4$
$= 3^5 \times 4^5$

What is the Exponent Law for **Raising a Product to a Power**?

$(ab)^m = a^m b^m$

Ex2 – Simplify as a power:
a) $(2 \times 3)^6$
b) $[(-8) \times 4]^2$

(a) $(2\times3)^6$
$= 2^6 3^6$

(b) $[(-8)\times4]^2$
$= (-8)^2 4^2$

Ex3 - Use your newest exponent law to evaluate:

a) $(2m)^3$

b) $(-3x)^2$

c) $-(2w)^4$

(a) $(2m)^3$
$= 2^3 m^3$
$= 8m^3$

(b) $(-3x)^2$
$= (-3)^2 x^2$
$= 9x^2$

(c) $-(2w)^4$
$= -2^4 w^4$
$= -16w^4$

Evaluate $(3 \div 4)^3$ using BEDMAS and a calculator

$(3 \div 4)^3$
0.75^3
$= 0.422$

$(3 \div 4)^3 = \left(\frac{3}{4}\right)^3 = \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$
$= \frac{3^3}{4^3} = 3^3 \div 4^3$

Can you use an exponent law instead, similar to the last exponent law you learned? Test your answer with a calc.

$(3 \div 4)^3$
$= 3^3 \div 4^3$
$= 27 \div 64$
$= 0.422$

Common Mistake:
$(2 + 4)^2 \neq 2^2 + 4^2$
The correct answer is $(2 + 4)^2 = 6^2 = 36$
The operation inside the brackets must be multiplication or division.

What is the Exponent Law for **Raising a Quotient to a Power**?

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Ex4 – Simplify as a power

$\left(\frac{2}{3}\right)^3$

$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$

Ex5 – Simplify, then evaluate

a) $\left(\frac{2x}{5}\right)^2$

b) $(3^5 \times 3)^2 \div (3^3 \times 3^2)^2$

(a) $\left(\frac{2x}{5}\right)^2 = \frac{2^2 x^2}{5^2} = \frac{4x^2}{25}$

(b) $(3^5 \times 3^1)^2 \div (3^3 \times 3^2)^2$
$(3^6)^2 \div (3^5)^2$
$= 3^{12} \div 3^{10}$
$= 3^2 = 9$

Can you write a general rule for the 6 exponent laws you've learned so far?

1. $x^m \times x^n = x^{m+n}$
2. $x^m \div x^n = x^{m-n}$
3. $(x^m)^n = x^{mn}$
4. $(a \times b)^m = a^m \times b^m$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. $x^{-m} = \frac{1}{x^m}$

Negative Exponents

Focus: To understand and apply negative exponents in evaluating powers

Warmup: Can you write a general rule for the five exponent law you've learned so far?	① $x^m \times x^n = x^{m+n}$ ② $x^m \div x^n = x^{m-n}$ ③ $(x^m)^n = x^{mn}$ ④ $(ab)^m = a^m b^m$ ⑤ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
Simplify as a single power: $\frac{2^3}{2^5}$ Now, expand as a repeated multiplication, cancel, and evaluate.	$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$; $\frac{\cancel{2}\times\cancel{2}\times\cancel{2}}{\cancel{2}\times\cancel{2}\times\cancel{2}\times 2\times 2} = \frac{1}{2\times 2} = \frac{1}{2^2}$
What do you notice?	2^{-2} must be equal to $\frac{1}{2^2}$
What is an effective way to think about what a negative exponent is?	A negative exponent means that the base has been 'trapped' in the denominator.
What is the exponent law for negative exponents?	$a^{-m} = \frac{1}{a^m}$
Ex1: Simplify, then evaluate. Answer as a fraction in lowest terms: a) 4^{-2} b) 2^{-5} c) 76^{-1} d) $(-3)^{-4}$	a) $4^{-2} = \frac{1}{4^2} = \frac{1}{4\times 4} = \frac{1}{16}$ (b) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$ (c) $76^{-1} = \frac{1}{76^1} = \frac{1}{76}$ (d) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{81}$

Ex2 – Simplify using exponent laws.

$$\frac{2^{-5}}{2^{-3}}$$

Now simplify using expanded form.

$\frac{2^{-5}}{2^{-3}}$

$= 2^{-5-(-3)}$

$= 2^{-5+3}$

$= 2^{-2}$

$= \frac{1}{2^2}$

$= \frac{1}{4}$

*Note:
An answer can never be left in negative exponent form!!

Ex3: Simplify, then evaluate, as a fraction in lowest terms:

$$\left(\frac{3}{-2}\right)^{-2}$$

$\left(\frac{3}{-2}\right)^{-2} = \left(\frac{-2}{3}\right)^{2} = \frac{(-2)^2}{3^2} = \frac{4}{9}$

Ex4 – Simplify as a fraction in lowest terms using only exponent laws:

a) $[(-2)^2]^{-3} \times (-2)^2$

b) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{2^7 \times 2^{-5}}{2^3}\right)$

(a) $[(-2)^2]^{-3} \times (-2)^2$

$(-2)^{-6} \times (-2)^2$

$= (-2)^{-4}$

$= \frac{1}{(-2)^4}$

$= \frac{1}{16}$

(b) $\left(\frac{1}{4}\right)^{-2} - \left(\frac{2^7 \times 2^{-5}}{2^3}\right)$

$\left(\frac{1}{4}\right)^{-2} - \left(\frac{2^2}{2^3}\right)$

$\left(\frac{4}{1}\right)^{2} - 2^{-1}$

$\frac{4^2}{1^2} - 2^{-1}$

$= \frac{16}{1} - \frac{1}{2}$

$= 16 - \frac{1}{2} \Rightarrow \frac{16^{\times 2}}{1_{\times 2}} - \frac{1}{2}$

$= 15\frac{1}{2}$

$\frac{32}{2} - \frac{1}{2}$

$= \frac{31}{2} = 15\frac{1}{2}$