Name: NOTES KEY
Date: _____

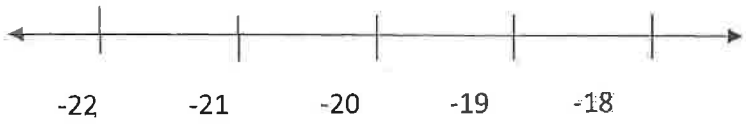
CHAPTER 3 NOTES – Rational Numbers

Calendar of Chapter: See the 'Homework' link on the webpage

What You'll Learn:

3.1 – Compare and order rational numbers
3.2/3.3 – Solve problems by adding & subtracting rational numbers
3.4A – Solve problems by multiplying & dividing rational numbers
3.4B – Explain and apply the order of operations with rational numbers

The label on a package of frozen cranberries says that it must be stored at a temperature between -18°C and -22°C. Name some possible temperatures and put them on the number line.



When in the 'real world' might negative fractions or decimals be needed?

3.0

1.1 – Number Systems

KEY

Learning Target: to examine and understand all the components of the real number system

Toolkit:	**Main Ideas:**
• Placing numbers on number lines • Anything you remember about classifying Real Numbers	

Definitions:

Natural Numbers – $\{1, 2, 3, \ldots\}$ The counting numbers

Whole Numbers – $\{0, 1, 2, 3, \ldots\}$ zero and the counting numbers

Integers – $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ neg. counting #'s, zero, pos. counting #'s

Rational Numbers – All numbers that can be written as a fraction $\frac{m}{n}$ ↯ integers $(n \neq 0)$

* Decimals: repeating ex. $\frac{1}{3} = 0.\overline{3}$

terminating ex $\frac{1}{4} = 0.25$

Irrational Numbers – All numbers that cannot be written as a fraction, a terminating decimal, or repeating decimal

Ex. $\pi, \sqrt{2}, \sqrt{7}, \ldots$

Real Numbers – All the rational numbers and irrational numbers combined

** If not "real", then a number is complex (imaginary)

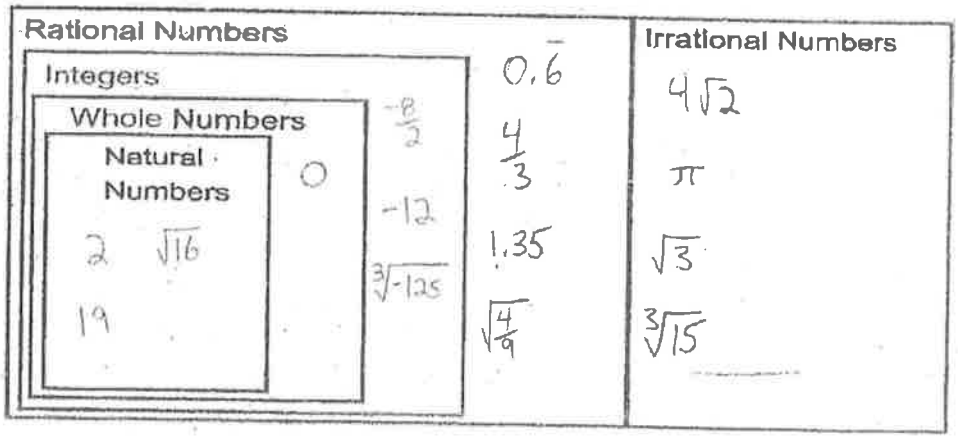
Classifying Real Numbers

Ex. 1) Where do these numbers belong in the diagram of Real Numbers?

$2 \quad 0.\overline{6} \quad 4\sqrt{2} \quad \frac{4}{3} \quad \frac{-8}{2} \quad -12 \quad \pi \quad 0 \quad \sqrt{16}$

$1.35 \quad \sqrt[3]{-125} \quad \sqrt{3} \quad \sqrt[3]{15} \quad 19 \quad \sqrt{\frac{4}{9}}$

Real Numbers:



True or False

Ex 2) State whether each statement is **true or false**.

a) Every integer is a natural number — False

-3 not natural

b) All whole numbers are integers — True

c) Every real number is a rational number — False

irrational is real, but not rational

$\sqrt{2}$

List of numbers

Ex 3) Consider the list of numbers: $0, -4, -1.3, 0.\overline{7}, \frac{3}{5}, \sqrt{17}, 13, -\sqrt{25}, 3.232232223\ldots$

$= -5$

List all:

a) Natural Numbers: 13

b) Whole Numbers: 0, 13

c) Integers: $0, -4, 13, -\sqrt{25}$

d) Rational numbers: $0, -4, -1.3, 0.\overline{7}, \frac{3}{5}, 13, -\sqrt{25}$

e) Irrational Numbers: $\sqrt{17}$, 3.232232223...

f) Real numbers: All of them

State the number System

Ex 4) State the number systems each of the following belong to:

a) $\sqrt[3]{125}$

$= 5$

natural, whole, integer, rational, real

b) $7.\overline{59}$

rational, real

c) $\sqrt{27}$

$= 5.196152423\ldots$

Irrational, Real

3.1 – What is a Rational Number?

Focus: Compare and order rational numbers.

INTERNET SEARCH:
What is a rational number?

A number that can be written as a fraction. (any terminating (ending) or repeating decimal)

Are integers rational?

Yes (ex) $2 = \frac{2}{1}$ ← written as a fraction

Write down 5 random fractions (pick at least one with a deonominator of 9 or 11), and then divide each one. What do you notice?

$\frac{3}{5} = 0.6$, $\frac{2}{9} = 0.\overline{2}$, $\frac{5}{11} = 0.\overline{45}$, $\frac{37}{100} = 0.37$, $\frac{7}{12} = 0.58\overline{3}$

Which decimals are rational?

Decimals that terminate (and) or repeat

What is an irrational number, and give two examples.

A decimal that neither terminates or repeats
(ex) π, $\sqrt{2}$, $\sqrt{3}$

Comparing & Ordering Rational Numbers

How do you order decimals?

By place value

hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	units	. decimal	tenths	hundredths	thousandths	ten thousandths
								0	.	3			
								0	.	3	1		
								0	.	0	3		
								0	.	2	9		

Ex1 – Order from least to greatest:
0.3, 0.31, 0.03, 0.29
0.39, 0.039, $0.\overline{3}$

0.03, 0.039, 0.29, 0.3, 0.31, $0.\overline{3}$, 0.39

Ex2 – Order from Least to greatest:
-0.05, 0.55, $-0.\overline{5}$, 0.5, 0.05, -0.55, -0.56

negs: -0.05, $-0.\overline{5}$, -0.55, -056
pos: 0.55, 0.5, 0.05

-0.56, $-0.\overline{5}$, -0.55, -0.05, 0.05, 0.5, 0.55

↑ furthest left on number line

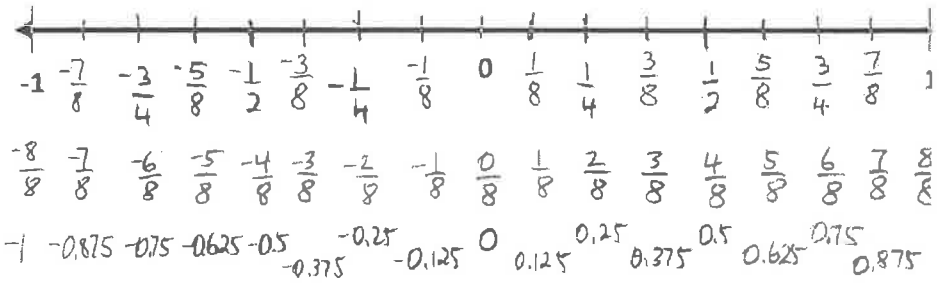
0 . 3 9
0 . 0 3 9
0 . 3 3 3

Ex3- On the # line:

a) Label: $\frac{1}{2}, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{4}, \frac{3}{4}, -\frac{3}{4}, \frac{1}{8}, -\frac{1}{8}, \frac{3}{8}, -\frac{3}{8}, \frac{5}{8}, -\frac{5}{8}, \frac{7}{8}, -\frac{7}{8}$

b) Rewrite each fraction with a denominator of 8

c) underneath each fraction, change to decimal



Ex4

Order these numbers from least to greatest

$-\frac{2}{3}$, -0.5, -1.31, $1\frac{1}{8}$, $\frac{3}{2}$, -.7, $-1\frac{3}{10}$, $-0.\overline{5}$

$-\frac{2}{3}$, -0.5, -1.31, $1\frac{1}{8}$, $\frac{3}{2}$, -.7, $-1\frac{3}{10}$, $-0.\overline{5}$

$-0.\overline{6}$, -0.5, -1.31, 1.125, 1.5, -0.7, -1.3, $-0.\overline{5}$

neg: $-0.\overline{6}$, -0.5, -1.31, -0.7, -1.3, $-0.\overline{5}$

pos: 1.125, 1.5

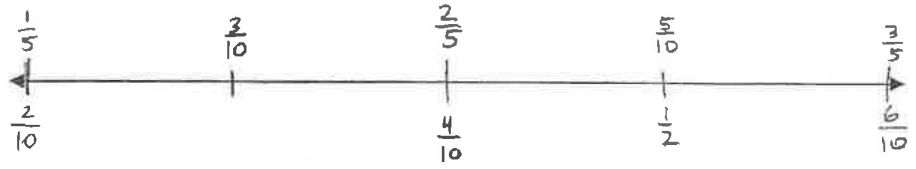
↑ furthest left on number line

-1.31, $-1\frac{3}{10}$, -0.7, $-\frac{2}{3}$, $-0.\overline{5}$, -0.5, $1\frac{1}{8}$, $\frac{3}{2}$

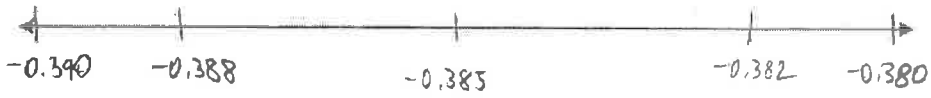
Ex5

Write 3 rational #s (fraction or decimal) between each pair of numbers on the number line.

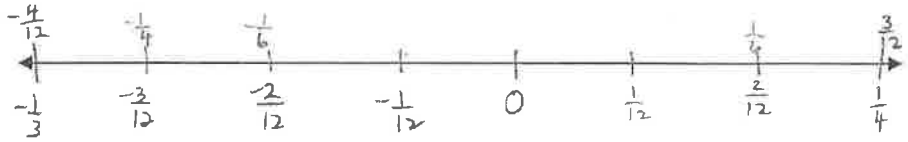
a) $\frac{1}{5}$ *and* $\frac{3}{5}$



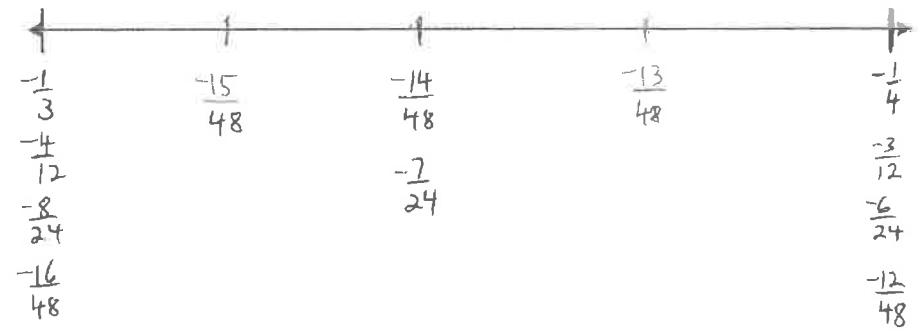
b) -0.38 and -0.39



c) $-\frac{1}{3}$ and $\frac{1}{4}$



d) $-\frac{1}{3}$ and $-\frac{1}{4}$



3.2/3.3 – Adding & Subtracting Rational Numbers

Focus: Solve problems that require adding & subtracting rational numbers.

Warmup:
Find each answer using strategies you've learned in the past.

a) -3 + 5
b) 4 + -3
c) -1 + -5
d) 3 – 7
e) -2 – 6
f) 3 – (-6)

Jot down your Strategies for adding & subtracting integers.

a) 2 b) 1 c) -6 d) -4
e) -8 f) 9

money strategy: negative is money owed
positive is money gained

if you see 4 – (-7)
turns into +

ex) 3 – 7
gain 3, lose 7 = lost 4 = -4

Be sure that you can add and subtract integers and get the correct answer consistently. If this is something you struggle with, work through section 3.2 and/or seek help from the teacher to learn any helpful strategies!

Ex1
Let's add using fraction pies:

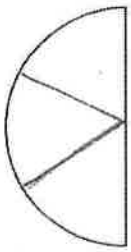
$$\frac{1}{2}+\frac{1}{3}$$

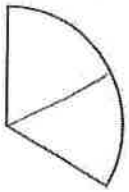
What is the problem with trying to combine the pie pieces?

What could we do to the pie pieces in order to be able to add them together?

Now, let's compare it to the method you've learned for adding fractions in middle school.

don't have same size pieces





$\frac{1}{3} = \frac{2}{6}$

cut them up to make all the pieces the same size

$\frac{1}{2} = \frac{3}{6}$

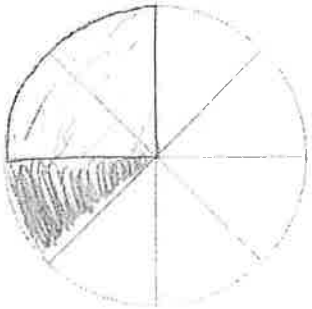
$\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

getting common denominators = getting same size pieces!

$$\frac{1}{2}\times\frac{3}{3} + \frac{1}{3}\times\frac{2}{2} = \frac{3}{6}+\frac{2}{6} = \frac{5}{6}$$

Ex2 – Subtract

$$\frac{3}{8}-\frac{1}{4}$$



$$\frac{3}{8}-\frac{1}{4}$$
$$=\frac{3}{8}-\frac{2}{8}$$
$$=\frac{1}{8}$$

Watch 'Act 1' of **'The Recipe'**

Find your solution to **'The Recipe'**

Watch 'Act 3'

How do you deal with negative fractions?

What are the **steps** involved with adding & subtracting fractions?

Ex3 – Simplify:

a) $-\frac{2}{3}+\frac{1}{6}$

b) $-\frac{3}{4}-\frac{5}{6}$

c) $-\frac{1}{3}+\left(-\frac{7}{8}\right)$

d) $-3\frac{1}{3}+2\frac{5}{6}$

e) $\frac{5}{-4}-\left(-3\frac{1}{5}\right)$

What questions come to mind? What do you need to know?

what types of quantities must be mixed in the measuring cup? Will it exceed 1 cup?

$\frac{1^{\times 6}}{2_{\times 6}}+\frac{1^{\times 4}}{3_{\times 4}}+\frac{1^{\times 3}}{4_{\times 3}}=\frac{6}{12}+\frac{4}{12}+\frac{3}{12}=\frac{13}{12}$

always put the negative on top $\frac{1}{-3}$ becomes $\frac{-1}{3}$ $-\frac{1}{3}$ becomes $\frac{-1}{3}$

1. Change any mixed to improper
2. Put all negatives on top
3. Get common denominators
4. Add or subtract numerators
5. Reduce if necessary.

a) $-\frac{2^{\times 2}}{3_{\times 2}}+\frac{1}{6}$

$=\frac{-4}{6}+\frac{1}{6}$

$=\frac{-3}{6}=\boxed{\frac{-1}{2}}$

b) $\frac{-3^{\times 3}}{4_{\times 3}}-\frac{5^{\times 2}}{6_{\times 2}}$

$\frac{-9}{12}-\frac{10}{12}$

$=\boxed{\frac{-19}{12}}$

c) $\frac{-1^{\times 8}}{3_{\times 8}}+\left(\frac{-7^{\times 3}}{8_{\times 3}}\right)$

$\frac{-8}{24}+\frac{-21}{24}$

$=\boxed{\frac{-29}{24}}$

d) $-\left(3\frac{1}{3}\right)+2\frac{5}{6}$

$-\frac{10^{\times 2}}{3_{\times 2}}+\frac{17}{6}$

$\frac{-20}{6}+\frac{17}{6}$

$\frac{-3}{6}=\boxed{\frac{-1}{2}}$

e) $\frac{-5}{4}+3\frac{1}{5}$

$\frac{-5^{\times 5}}{4}+\frac{16^{\times 4}}{5_{\times 4}}$

$\frac{-25}{20}+\frac{64}{20}$

$=\boxed{\frac{39}{20}}$

3.4A - Multiplying & Dividing Rational Numbers

Focus: Solve problems that require multiplying and dividing rational numbers

Warmup:
Multiply:
a) -9 x 4
b) 3 x -3
c) -24 ÷ -3
d) -3 x 2 ÷ -1
e) -1 x -1 x -1

a) -36 (b) -9 (c) 8 (d) 6 (e) -1

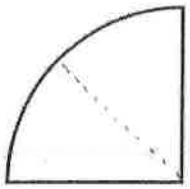
Explain how you know whether the answer is positive or negative:

when multiplying and dividing integers, if there are an even number of integers the answer is positive. If there are an odd number of integers the answer is negative.

Ex1 - When multiplying fractions such as

$$\frac{1}{4} \times \frac{1}{2}$$

you may want to think: **'What's *half* of a *quarter*?'** Use the fraction pie.



← here is $\frac{1}{4}$. What is half of that?

$\frac{1}{8}$

Same

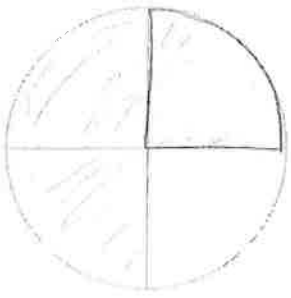
What are the **steps** You learned in middle school to multiply fractions?

- change mixed to improper
- multiply tops
- multiply bottoms
- reduce

$$\frac{1}{4} \times \frac{1}{2} = \frac{1 \times 1}{4 \times 2} = \frac{1}{8}$$

Ex2 – Use fraction pies to find:

$$\frac{3}{4} x \frac{1}{3}$$



What's $\frac{1}{3}$ of $\frac{3}{4}$?

$\frac{1}{4}$ ← Same

$$\frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{3^{\div 3}}{12_{\div 3}} = \frac{1}{4}$$

Ex3 – Multiply

a) $-\frac{3}{2} \times \frac{1}{-5}$

***Remember*: Always put negatives on top!**

$$\frac{-3}{2} \times \frac{-1}{5} = \frac{3}{10}$$

b) $\frac{4}{7}(-3)$

*Remember: Always turn a whole number into a fraction first!

$\frac{4}{7} \times \frac{-3}{1} = \frac{-12}{7}$

Ex4 – Multiply

***You can REDUCE fractions before multiplying them by '<u>cross reducing</u>'. (can only do this process when MULTIPLYING fractions)**

a) $\left(\frac{-11}{7}\right)\left(-\frac{21}{44}\right)$

$\frac{-11}{7}\left(\frac{-21}{44}\right) = \frac{-1}{1}\left(\frac{-3}{4}\right) = \frac{3}{4}$

b) $\left(2\frac{2}{3}\right)\left(-1\frac{3}{4}\right)$

b) $\frac{8}{3}\left(\frac{-7}{4}\right) = \frac{2}{3}\left(\frac{-7}{1}\right) = \frac{-14}{3}$

An Investigation:

a) $\frac{1}{4} \times \frac{1}{2}$

think: what's half of a quarter?

a) $\frac{1}{8}$

$\frac{1}{4} \div 2$

think: what's a quarter divided by 2?

$\frac{1}{8}$

What can you conclude about $\times \frac{1}{2}$ and $\div 2$?

they are the same!

If they are the same, then their opposites must also be the same.

What's the opposite of $\times \frac{1}{2}$? $\div \frac{1}{2}$ What's the opposite of $\div 2$? $\times 2$

Therefore, $\div \frac{1}{2}$ & $\times 2$ are the same things!

$\div \frac{1}{2}$ same as $\times 2$

Ex5 - Divide

$\frac{1}{4} \div \frac{1}{2}$

$\frac{1}{4} \div \frac{1}{2} = \frac{1}{4} \times \frac{2}{1} = \frac{2}{4} = \frac{1}{2}$

Notice how this is exactly what we figured out above!

What are the steps to dividing fractions that you learned in middle school?

1. change mixed to improper
2. Flip 2nd fraction and change ÷ to ×
3. Put negatives on top
4. Multiply numerators (tops)
5. Multiply denominators (bottoms)
6. Reduce if necessary

What is a fraction that has been flipped called?

reciprocal

Ex6 – Divide

a) $\left(-\frac{4}{3}\right)\div\left(\frac{-8}{3}\right)$

b) $-4\frac{2}{3}\div 1\frac{4}{5}$

Ex7 – Simplify

$\left(-\frac{1}{3}\right)\times\frac{4}{3}\div\left(\frac{5}{-6}\right)$

a) $\frac{-4}{3}\div\frac{-8}{3}=\frac{-4}{3}\times\frac{-3}{8}=\frac{-1}{1}\times\frac{-1}{2}=\frac{1}{2}$

*cross reduce

b) $-4\frac{2}{3}\div 1\frac{4}{5}$

$-\frac{14}{3}\div\frac{9}{5}=\frac{-14}{3}\times\frac{5}{9}=\frac{-70}{27}$

$\frac{-1}{3}\times\frac{4}{3}\div\frac{-5}{6}$

$\frac{-4}{9}\div\frac{-5}{6}$

$=\frac{-4}{9}\times\frac{-6}{5}$

$=\frac{-4}{3}\times\frac{-2}{5}$

$=\frac{8}{15}$

3.4B - Order of Operations with Rational Numbers (DISCOVERY LESSON)

Focus: Explain and apply the order of operations with rational numbers.

Warmup:
a) Find (circle) and explain the error that each student made
b) Simplify the expression correctly.

Two students were asked to evaluate: $-8 - 2(24 \div (-8))^2$

Student 1:

$-8 - 2(24 \div (-8))^2$
$-10(24 \div (-8))^2$ subtracted first
$-10(-3)^2$
$-10(9)$
-90

Student 2:

$-8 - 2(24 \div (-8))^2$
$-8 - 2(-3)^2$
$-8 - (-6)^2$ multiplied before exponents
$-8 - 36$
-44

$-8 - 2(24 \div (-8))^2$
$-8 - 2(-3)^2$
$-8 - 2(9)$
$-8 - 18$
-26

What is the key word for order of operations?

B Brackets
E Exponents
D Division
M Multiply
A Addition
S Subtract

* for divide/multiply, do whichever comes first
* for add/subtract, do whichever comes first

Ex1 – Simplify

$$3\frac{2}{3} - \left(-1\frac{1}{3}\right) \div \frac{5}{3} + \left(-2\frac{1}{4}\right)$$

$$\frac{11}{3} - \left(-\frac{4}{3}\right) \div \frac{5}{3} + \left(-\frac{9}{4}\right)$$

$$\frac{11}{3} - \left(-\frac{4}{3}\right) \times \frac{3}{5} + \left(-\frac{9}{4}\right)$$

$$\frac{11}{3_{\times 5}} - \left(-\frac{4}{5}\right)_{\times 3} + \left(-\frac{9}{4}\right)$$

$$\frac{55}{15} + \frac{12}{15} + \frac{-9}{4}$$

$$\frac{4 \times 67}{4 \times 15} + \frac{-9 \times 15}{4 \times 15} = \frac{268}{60} + \frac{-135}{60} = \frac{133}{60}$$

Ex2 –Simplify

$$\left(\frac{8}{3}\right)\left(\frac{(-1)^3}{2^3}\right) - 1\frac{1}{2} \div \frac{4}{-3}$$

$$\frac{8}{3}\left(\frac{(-1)^3}{2^3}\right) - \frac{3}{2} \div \frac{-4}{3}$$

$$\frac{8^1}{3}\left(\frac{-1}{8_1}\right) - \frac{3}{2} \div \frac{-4}{3}$$

$$\frac{-1}{3} - \left(\frac{3}{2} \div \frac{-4}{3}\right)$$

$$\frac{-1}{3} - \frac{3}{2} \times \frac{-3}{4}$$

$$\frac{-1^{\times 8}}{3_{\times 8}} + \frac{9^{\times 3}}{8_{\times 3}} = \frac{-8}{24} + \frac{27}{24} = \frac{19}{24}$$

Ex3 – Evaluate

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)-\left(-\frac{2}{3}\right)\div\left[\frac{1}{3}+\left(-\frac{3}{12}\right)\right]$$

$$\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)+\tfrac{2}{3}\div\left[\tfrac{4}{12}+\tfrac{-3}{12}\right]$$

$$\left(-\tfrac{1}{2}\right)\left(-\tfrac{1}{2}\right)+\tfrac{2}{3}\div\tfrac{1}{12}$$

$$\tfrac{1}{4}+\tfrac{2}{3}\div\tfrac{1}{12}$$

$$\tfrac{1}{4}+\tfrac{2}{3}\times\tfrac{12}{1}$$

$$\tfrac{1}{4}+\tfrac{24}{3}$$

$$\tfrac{1}{4}+8$$

$$8\tfrac{1}{4} \text{ or } \tfrac{33}{4}$$