Name: NOTES KEY

Date: _____

# CHAPTER 4 NOTES – Linear Relations

***Calendar of Chapter: See the 'Homework' link on the webpage***

## What You'll Learn:

4.1 – Use expressions and equations to generalize patterns
4.1 – Verify a pattern by using substitution
4.2/4.3– Graph and analyze linear relations
4.4 – Match Equations to Graphs
6.6 - Interpolate and Extrapolate to solve problems

One definition of math is 'the study of patterns'. How can finding and studying patterns be important in our world?

What word is inside the word 'linear'? So what may a linear relation be? Can you think of a real world example of a linear relation?

## 4.1 –Patterns

Focus: Use tables, words, & equations to describe and solve problems involving patterns.

**Warmup:**
Can you describe the pattern in:
a) in words
b) a table
c) an equation where
$t$ = number of tables &
$p$ = number of people
d) How many people are at 25 tables? Use your equation to solve the problem.

A banquet hall has small square tables that seat 1 person on each side. The tables can be pushed together to form longer tables.

1 table 2 tables 3 tables

a) each time a new table is added, two more people are added

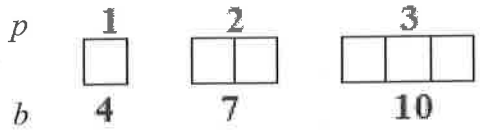
c) $p = 2t + 2$

d) $p = 2t + 2$ $p = 52$
$p = 2(25) + 2$ 52 people at 25 tables.
$p = 50 + 2$

b)

| t | p |
|---|---|
| | |
| 1 | 4 |
| 2 | 6 |
| 3 | 8 |
| 4 | 10 |
| 5 | 12 |
| 6 | 14 |

Ex1 – When building garden plots at a communal garden, The number of boards, $b$, is related to the number of plots, $p$. Describe this relation in words, a table, and an equation.




Words: Every time a new plot is added, three more boards are used

$b = 3p + 1$

| p | b |
|---|---|
| | |
| 1 | 4 |
| 2 | 7 |
| 3 | 10 |
| 4 | 13 |
| 5 | 16 |
| 6 | 19 |

How do you know if A pattern is '*linear*'?

A pattern is linear if both variables change by constant values.

What are the 3 steps to building a linear equation using a table?

① The variable in the second column of the table is all by itself on the left side of the equation
② Look for zero in the first column (or create it), and the number beside it in the second column is the constant and is written at the end (far right side) of the equation.
③ If the first column is increasing by one, however the second column is changing is the coefficient.

Ex2 You are at the store to rent a carpet cleaner machine. It costs $30 to rent, plus an additional $18 for each day you have it at home.
a) Construct a table (*d* for days & *C* for cost)
b) Develop an equation for this situation.
c) Use the equation to find the cost of renting for 3 days.

a)

| d | C |
|---|---|
| 0 | 30 |
| 1 | 48 |
| 2 | 66 |
| 3 | 84 |

b) ① $C =$

② $C = \quad +30$

③ $C = 18d + 30$

c) $C = 18(3) + 30$

$= 54 + 30$

$= \$84$

Ex3 – Build an equation for the relationship, and test it to make sure it's correct.

① $n =$

② $n = \quad +1$

③ $n = 2m + 1$

test: $n = 2m + 1$

$n = 2(2) + 1$

$= 5$ ✓

$n = 2(3) + 1$

$= 7$ ✓

| m | n |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |

Ex4 –
a) Build an equation for the relationship, and test it to make sure it's correct.
b) Using your equation, if $a = 26$, what is $b$?

a) ① $b =$

② $b = \quad +9$

③ $b = -3a + 9$

$b = -3(4) + 9$

$= -12 + 9$

$= -3$ ✓

$b = -3(26) + 9$

$b = -78 + 9$

$b = -69$

| a | b |
|---|---|
| 2 | 3 |
| 3 | 0 |
| 4 | -3 |
| 5 | -6 |
| 6 | -9 |

Ex4
A cab company charges $3.75 to start a cab ride, and an additional $1.25 per km.
a) if $d$ = distance (number of km) and $C$ is cost, construct a table with at least 5 entries.
b) Write an equation for the relation.
b) Use the equation to calculate the fare for a 19 km ride.

a)

| d | C |
|---|---|
| 0 | 3.75 |
| 1 | 5.00 |
| 2 | 6.25 |
| 3 | 7.50 |
| 4 | 8.75 |
| 5 | 10.00 |

b) $C =$

$C = \quad +3.75$

$C = 1.25d + 3.75$

c) $C = 1.25(19) + 3.75$

$C = 23.75 + 3.75$

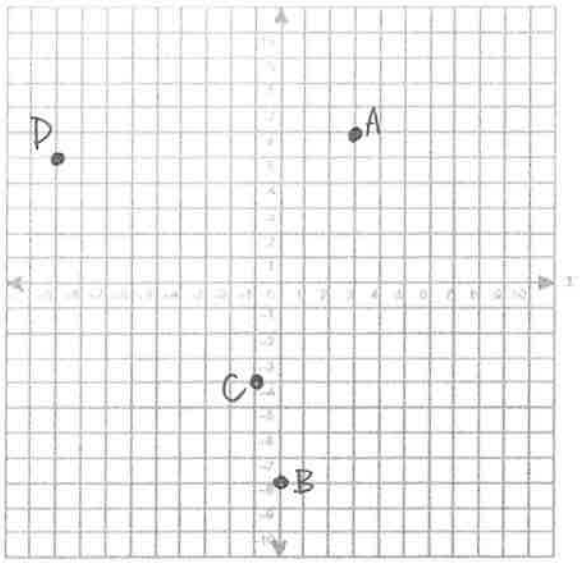
$C = \$27.50$

## 4.2A – Linear Systems: Graphing using a Table of Values

Focus: Draw and analyze the graph of a linear relation

**Warmup:** With a partner, discuss & try to recall how to plot the following points on a coordinate grid:
A(3, 6)  B(0, -8)

C(-1, -4)  D(-9, 5)




Coordinates for a point to plot are always written as **(*x*, *y*)**.
Always start at the **origin**, which is (0, 0) (the middle).

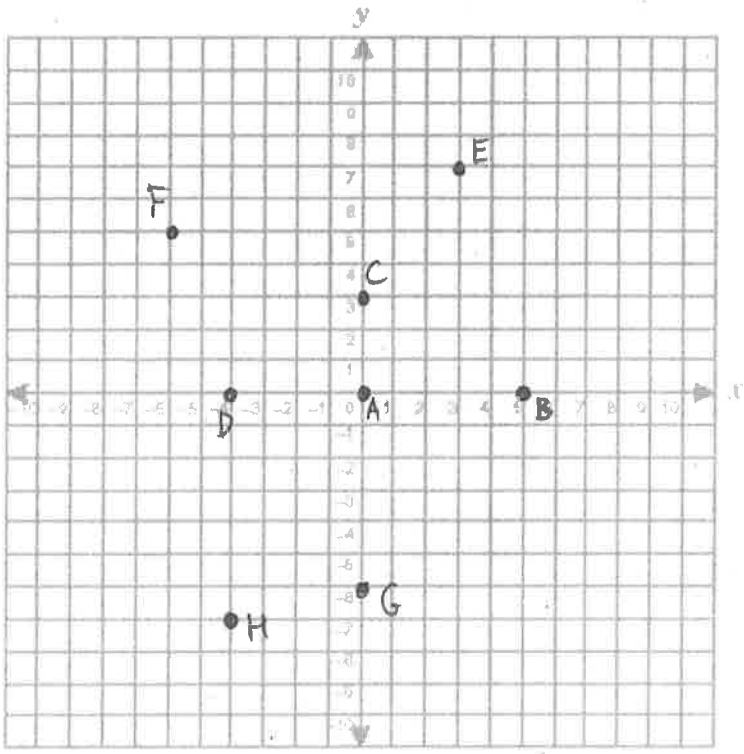
***x*** coordinate:
if positive, go RIGHT

if negative, go LEFT

***y*** coordinate:
if positive, go UP

if negative, go DOWN

Ex 1 - Plot and label the points on the coordinate plane.
A(0, 0)  B(5, 0)
C(0, 3)  D(-4, 0)
E(3, 7)  F(-8, 5)
G(0, -6)  H(-4, -7)



***On a table, *x* is always listed first, then *y*.**

Ex2 – 3 minutes ago, you were 7 km under the ocean. You travel upwards 3 km per min in your vehicle and continue into the sky
a) Complete the table.
b) Graph the points & draw a line.
c) Is the relation linear? How do you know from the table? How do you know from the graph?
d) What is the equation?

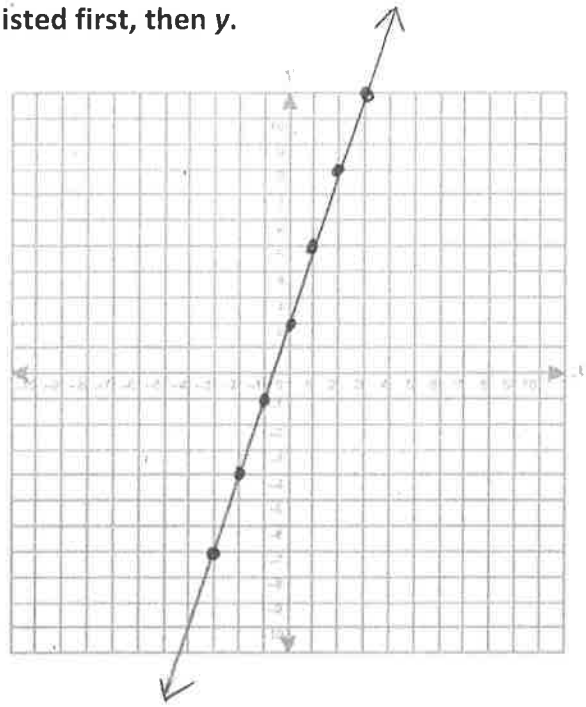
a)

| min (*x*) | km (*y*) | |
|---|---|---|
| -3 | -7 | (-3,-7) |
| -2 | -4 | (-2,-4) |
| -1 | -1 | (-1,-1) |
| 0 | 2 | etc. |
| 1 | 5 | |
| 2 | 8 | |
| 3 | 11 | |

b) see graph

c) Yes, left side inc by 1, right side inc by 3
Graph is a straight line

d) $y = 3x + 2$

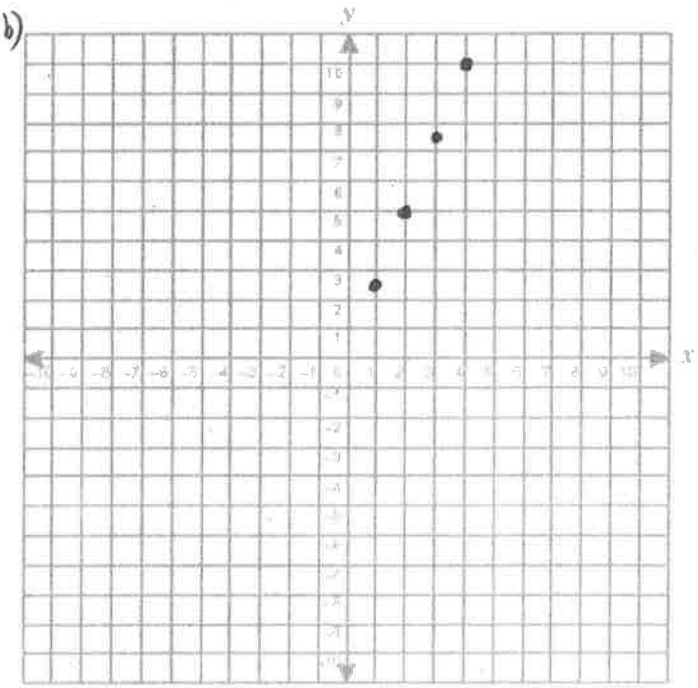


Ex3
It costs $2.50 to rent a movie online.
a) Complete the table
b) Graph the points, but don't draw a line.
c) What is the equation?

a)

| Movie (*x*) | Cost (*y*) |
|---|---|
| 0 | 0 |
| 1 | 2.50 |
| 2 | 5.00 |
| 3 | 7.50 |
| 4 | 10.00 |

b)



c) $y = 2.50x$

Does the cost depend on the number of movies, or does the number of movies depend on cost?

The cost depends on the number of movies

What is an independent variable, and what is a dependent variable?

The dependent variable depends on the independent variable
The first column (x value) is always the independent variable

**Why did we connect the dots with a line in Ex2, but not in Ex3 ?**

The data in between the points have meaning in Ex2, but don't in Ex 3.

What is the difference between discrete data and continuous data?

Discrete Data:
When data in between the points have no meaning. For discrete data, do not draw a line

Continuous Data:
When data between the points does have meaning. For continuous data, draw a line.

A math problem, like Ex4, that is not attached to a specific situation (it is just a numerical example), is automatically **continuous**.

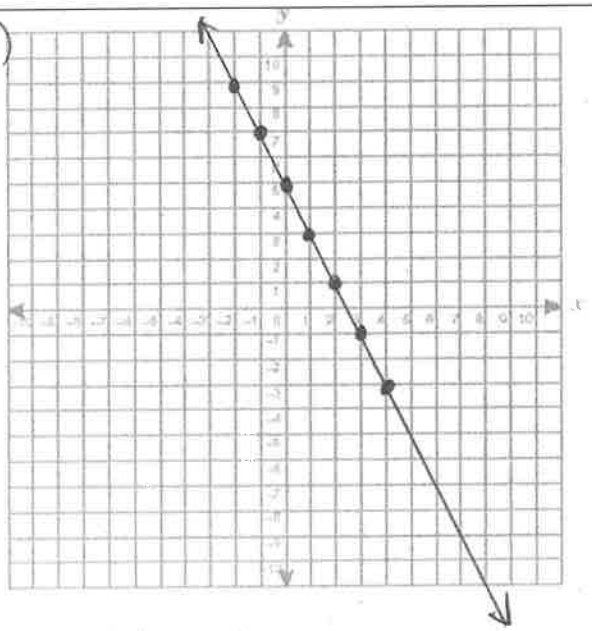
Ex4
A relation has the equation $y = -2x + 5$.
a) Create a table of values for values of $x$ from -2 to 4. Find $y$ for each using the equation.
b) Graph the relation.
c) Is the relation linear?

a)

| x | y |
|---|---|
| -2 | 9 |
| -1 | 7 |
| 0 | 5 |
| 1 | 3 |
| 2 | 1 |
| 3 | -1 |
| 4 | -3 |

$y = -2x + 5$
$y = -2(-2) + 5$
$y = 4 + 5$
$y = 9$

b)



c) Yes, the relation is linear as the graph is a straight line

If you know a relation is linear, how many points do you need to plot the line?

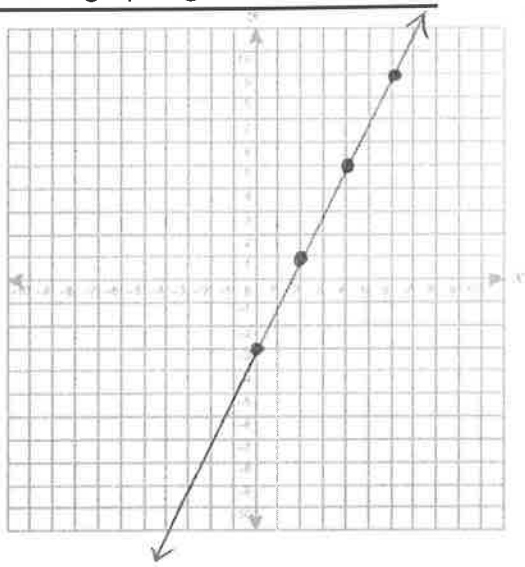
Minimum 2 points, but 3 points is better to confirm your line is correct

## 4.2B – Linear Systems: Slope, *y*-intercept, & the *y = mx + b* Equation

Focus: Learn and use the concepts of slope and *y*-intercept when graphing linear relations.

**Warmup:**
Graph ***y = 2x – 3***
using the table of values

| x | y |
|---|---|
| 0 | -3 |
| 2 | 1 |
| 4 | 5 |
| 6 | 9 |



Using the equation
***y = mx + b***
what is the ***m*** value and what is the ***b*** value of the equation in the **warmup**?

$y = 2x - 3$
$y = 2(0) - 3$
$y = -3$

$m = 2$

$b = -3$

What is a **b** value?

The 'b value' is the y-intercept, which is where the line crosses the y axis.

What is an **m** value?

The 'm value' is the slope (steepness) of the line, defined by $\frac{\text{rise}}{\text{run}}$. For example above, slope = $2 = \frac{2 \leftarrow \text{up } 2}{1 \leftarrow \text{right } 1}$

What is the difference between a positive & negative slope?

- a positive slope rises as it goes to the right
- a negative slope drops as it goes to the right

Describe the steps to graph a line using *y*-intercept and slope.

① Get the equation into $y = mx + b$ form (get 'y' by itself!)
② Identify and plot the y-intercept on the grid.
③ Identify the slope, make it a fraction (if not already). Put any negative on top. If top is positive, count UP from the y-intercept. If it's negative, count down. Then count right by the value of the denominator. Plot new point, then continue.

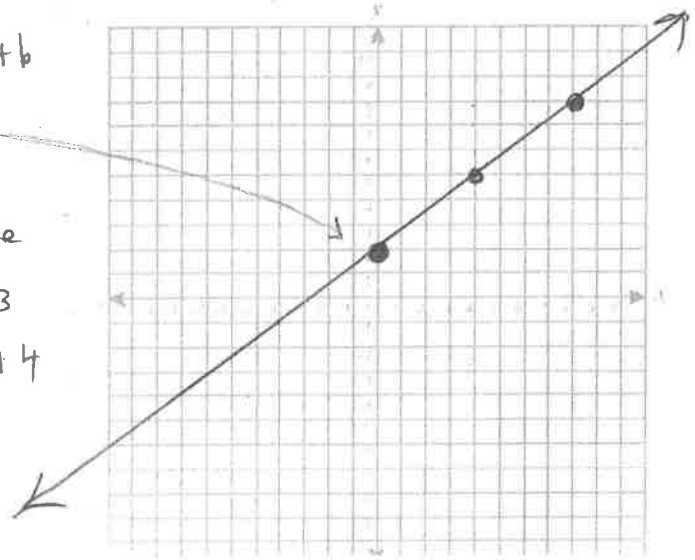
Ex1 - Graph
$y = \frac{3}{4}x + 2$

① already in $y = mx + b$

② $b = 2$ = y-int

③ $m = \frac{3}{4}$ = slope

$\frac{\text{rise}}{\text{run}} = \frac{3 \leftarrow \text{up } 3}{4 \leftarrow \text{right } 4}$



Ex2 – Graph

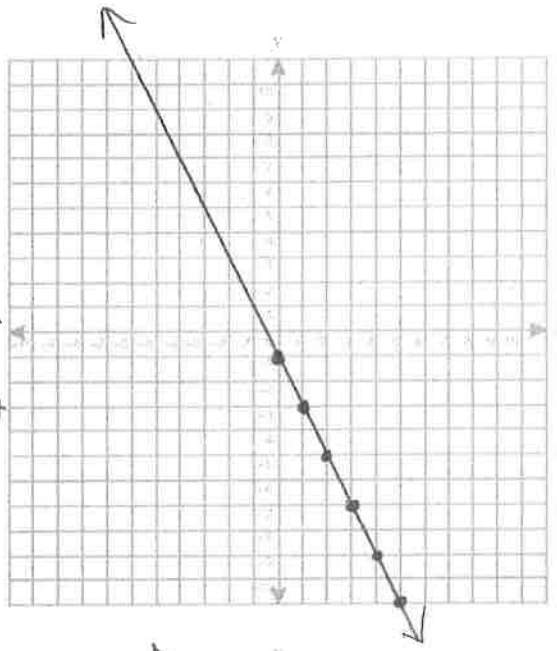
***y = -1 – 2x***

① $y = -1 - 2x$

becomes $y = -2x - 1$

② $b = -1$ (y-int)

③ $m = \text{slope} = -2 = \frac{-2}{1}$ ← down 2, → right 1



Ex3 – Graph

$x + y = 7$

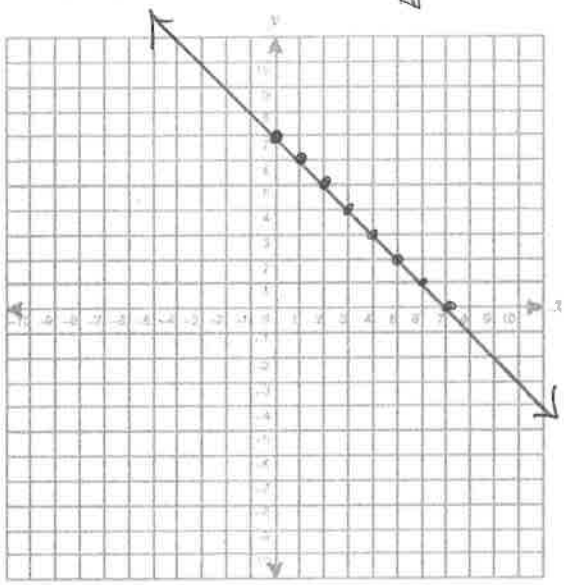
① $x + y = 7$

$-x \quad -x$

$y = -x + 7$

$y = -1x + 7$

② $b = 7$ ③ $m = \frac{-1}{1}$ ← down, → right



What is true of any point on a line?

Any point on a line will satisfy the equation of the line.

ex) the point $(9, -2)$ is on the line. It should satisfy the equation:

$x + y = 7$; $9 + (-2) = 7$

✓ yes.

Ex4 – Graph

***3x – 2y = 6***

① $-3x - 2y = 6$

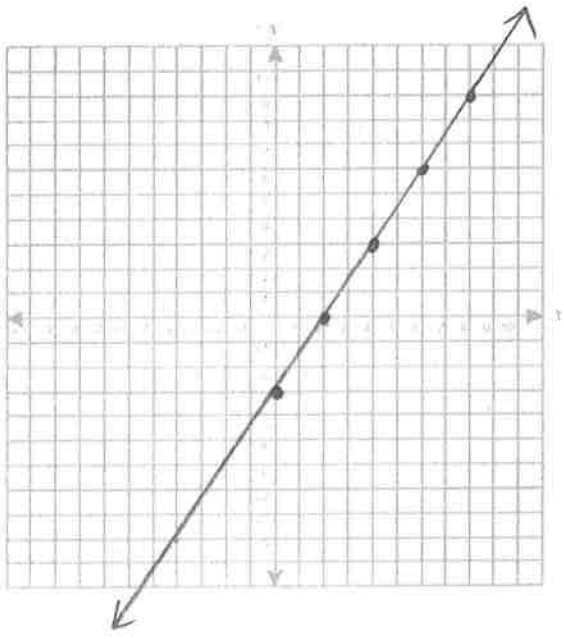
$-2y - 3x = 6$

$-3x \quad -3x$

$\frac{-2y}{-2} = \frac{-3x + 6}{-2}$

$y = \frac{3}{2}x - 3$

② $b = -3$ ③ $m = \frac{3}{2}$ ← up 3, → right 2



## 4.3 – Graphing Equations in the Form $Ax + By = C$

Focus: Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

**Warmup:**

Suppose you have a cedar plank that is 10 ft long.

a) How many different ways could you cut it into two pieces?

b) In words, how are the lengths of the two pieces related?

c) If $x$ = the length of the first piece, and $y$ = the length of the second piece, write an equation for the relation.

d) Make a table of values using only whole numbers.

e) Graph the equation. Should you join the points?

f) Is the relation linear?

(d)

| Piece #1 ($x$) | Piece #2 ($y$) |
|---|---|
| 1 | 9 |
| 2 | 8 |
| 3 | 7 |
| 4 | 6 |
| 5 | 5 |
| 6 | 4 |
| 7 | 3 |

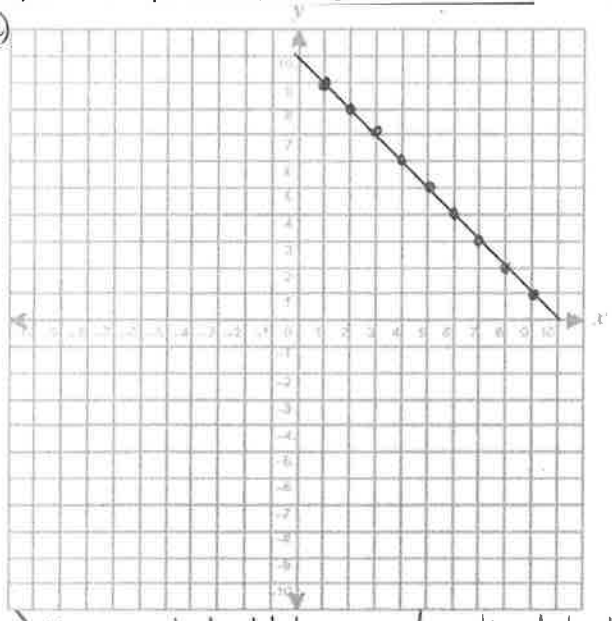
etc

a) infinite

b) they always sum to 10

c) $x + y = 10$

(e)



e) Yes, points should be joined as the data is continuous (you can theoretically cut the plank wherever you wish)

f) Yes, the graph makes a straight line

Ex1

Graph the equation $3x - 2y = 6$:

$3x - 2y = 6$

$3(0) - 2y = 6$

$-2y = 6$

$\frac{-2y}{-2} = \frac{6}{-2}$

$y = -3$

$3(1) - 2y = 6$

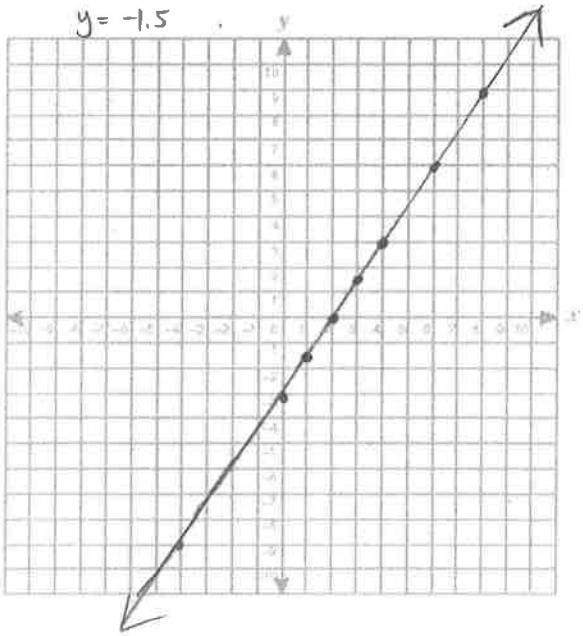
$\frac{3}{-2} - 2y = \frac{6}{-2}$

$\frac{-2y}{-2} = \frac{3}{-2}$

$y = -1.5$

OR do $y = mx + b$ principles

| $x$ | $y$ |
|---|---|
| 0 | -3 |
| 1 | -1.5 |
| 2 | 0 |
| 3 | 1.5 |
| 4 | 3 |



What is another name for a 'slanted' line?

oblique

diagonal

Sometimes, only one variable appears in an equation, for example, $x = 2$ or $y = -5$

Ex2
Graph $x = 2$

Hint: The only requirement for a point on the graph is that the $x$ value must be 2. So $y$ can be anything, as long as $x$ is 2.

What kind of line is produced?

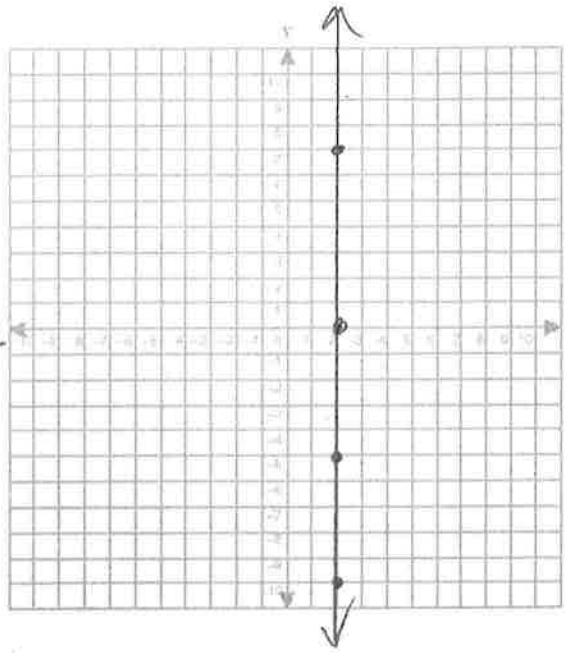
An equation is basically a set of rules. For example, $x+y=10$, the rule is that $x$ and $y$ must always add to 10

$x=2$
Rule is that $x$ must be 2
No rule for $y$.

| x | y |
|---|---|
| 2 | -10 |
| 2 | -5 |
| 2 | 0 |
| 2 | 7 |

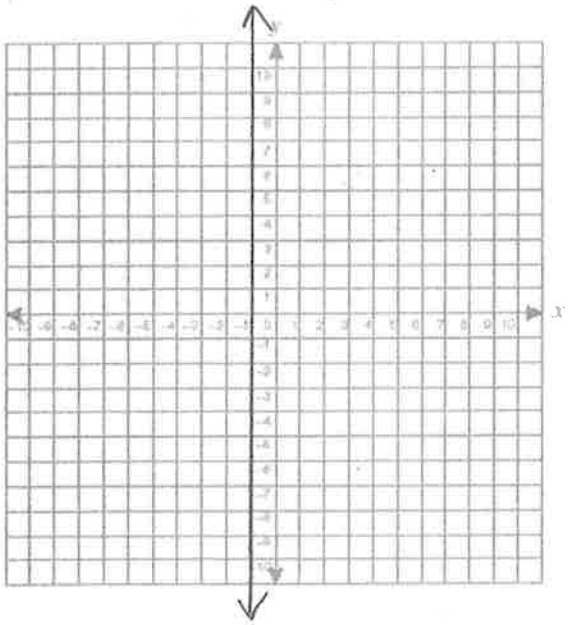
(can be anything...)

a vertical line that's 2 to the right of the origin



Ex3
Graph $x = -1$

a vertical line that's 1 to the left of the origin

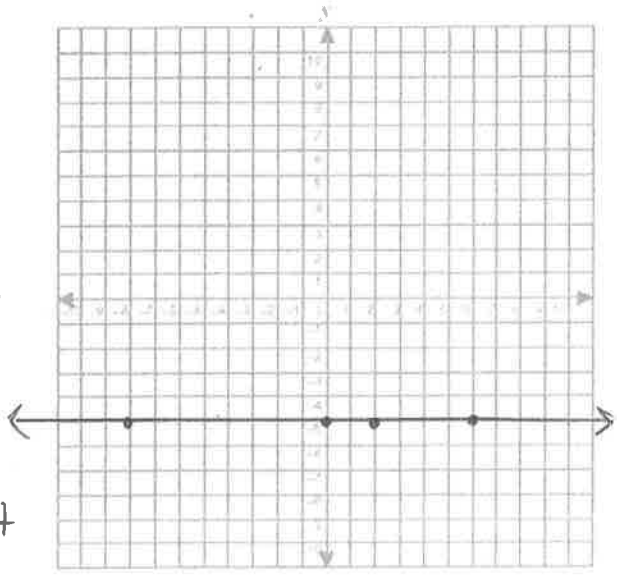


Ex4
Graph $y = -5$
The only requirement is that for each point, the $y$ value must be -5.

What kind of line is produced?

| x | y |
|---|---|
| -8 | -5 |
| 0 | -5 |
| 2 | -5 |
| 6 | -5 |

horizontal line five down from the origin



What kind of equation produces a:
a) oblique line?
b) vertical line?
c) horizontal line?

a) an equation with x and y in it
b) an equation with just 'x' in it
c) an equation with just 'y' in it.

## 4.4 & 6.6 – Matching Equations to Graphs & Using Graphs to Estimate Values

Focus: Use interpolation and extrapolation to estimate values on a graph.

**4.4 Warmup:**
Discuss and find a method to match each equation with a line:

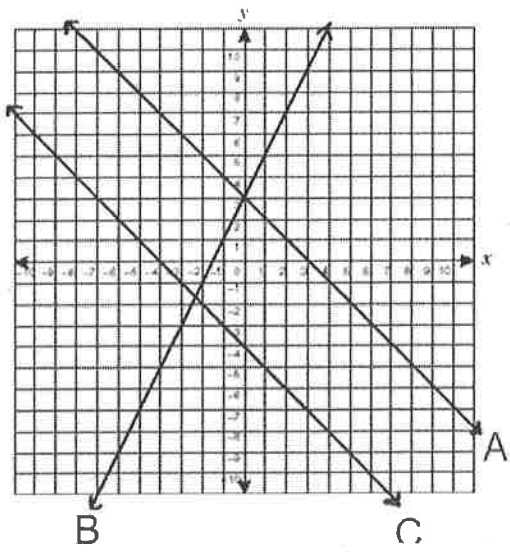
Describe how you did the matching.

1) $y = 2x + 3$ = B (pos slope)

2) $y = -x - 4$ = C

3) $y = -x + 3$ = A (neg slope)

-use y-intercepts
- then use slopes




**6.6 – *Estimating Values***

Ex1 - **Chocolate Bars**

a) Estimate the total revenue when 250 choc. bars are sold?

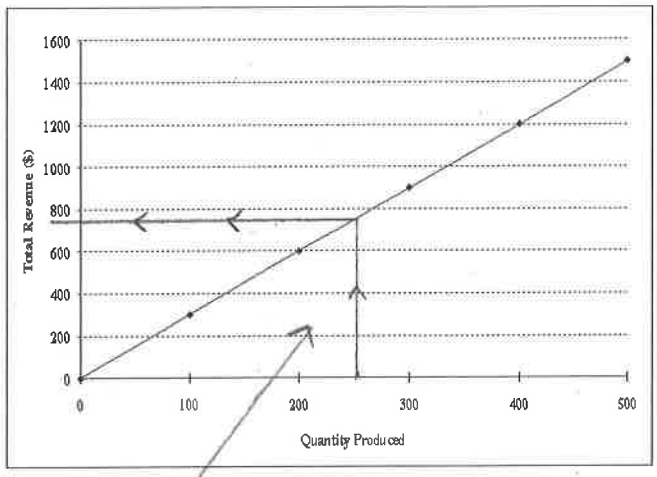
b) Estimate how many choc. bars are sold to give a total revenue of $1000?

c) Estimate the total revenue if 800 choc. bars are sold.

a) $750

b) 333

c) $2400




a & b was interpolating

Interpolation is the estimation of a value that is between data points

See graph above

In Ex1, you did some **inter**polating. Can you take a guess as to what **interpolation** is?

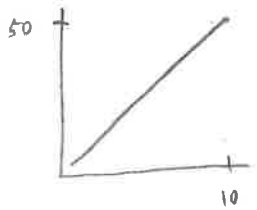
Can you draw a visual of interpolation?

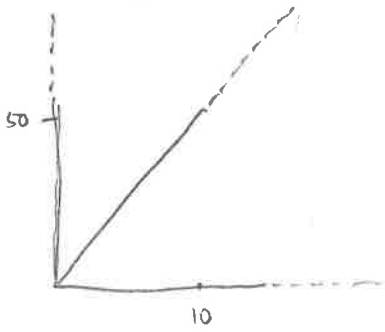
c was extrapolating

Extrapolation is the estimation (or prediction) of a value that lies beyond the data

In Ex1, you did some **extra**polating. Can you take a guess as to what **extrapolation** is?

Can you draw a visual of extrapolation?



With a partner, think of an actual situation In life where you use:
a) Interpolation:

b) Extrapolation

Ex1 – **A Taxi Ride**
a) What is the cost of a 4km ride?
b) What is the cost of an 8km ride?
c) What is the cost of a 20km ride?
d) Can you make an equation for this?
e) Does your equation support your estimates for parts **b** and **c**?

a) Answers will vary

b) Answers will vary

a) about $12.80

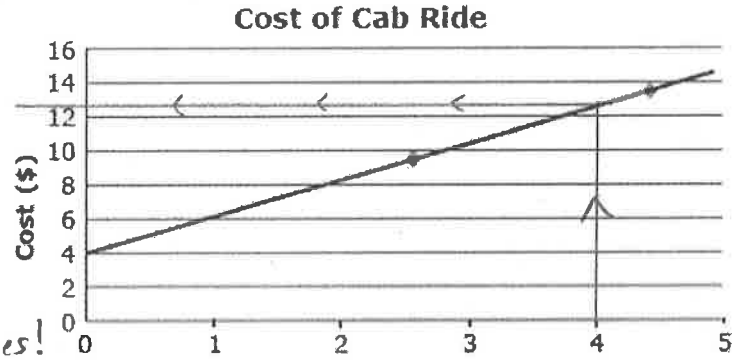
b) ~12.80 + 8.80
= $21.60

c) 12.80 + 4(8.80)
= $48.00

d) C = 2.20d + 4

e) C = 2.20(8) + 4
= $21.60 ✓ yes!

C = 2.20(20) + 4
= $48 ✓ yes!




Ex2 – Print Job

**Watch 'ACT ONE'**
*What questions come to mind?*
How long will it take to print all of the sheets

*What information is necessary to solve the problem?*
How many sheets are there?
How long does it take to print one sheet?

**Watch 'ACT TWO'**
*With a partner, solve the problem to the best of your ability.*

**Watch 'ACT THREE'**
*How did you go about solving the problem?*
*Who's guess was closest?*
*What elements from this chapter did we use? Why?*