Key

5.1A

**– Different Forms of Linear Relations**

**Learning Target:** to graph linear relations from standard form and slope-intercept form

**Toolkit:**
- graphing lines using a point and the slope
- x and y intercepts

**Slope-Intercept Form**

$$y = mx + b$$

Slope

y-intercept (y when $x = 0$)

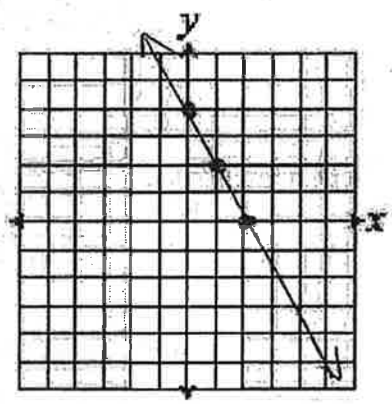
**Graph using Slope-Intercept form**

Step 1) Identify the y-intercept $(0, b)$ and graph this point.
Step 2) Graph two more points using the slope, counting from the y-intercept.
Step 3) Draw a line connecting the points to obtain the graph.

**Ex 1)** Graph the following: $y = -2x + 4$

y-int : $(0, 4)$

Slope $= \frac{-2}{1}$ ← down 2 ← right 1




**Standard Form**

positive or 0 ⟶ $Ax + By = C$

We want A, B and C to be **integers**, and **A≥ 0**. If the equation is not in the form we want, we can multiply the entire equation by an integer to change it to the proper form.

**Ex 2)** Change the following to proper standard notation:

a) $(-2x - 3y = 6)^{\times -1} \Rightarrow 2x + 3y = -6$

negative A value

multiply each term by -1

b) $\left(\frac{2}{3}x - 2y = -1\right)^{\times 3} \Rightarrow 2x - 6y = -3$

fraction

multiple each term by the denominator

**Rewriting Standard form in Slope-intercept form**

We can take a linear equation from standard form and write it in slope-intercept form. We do this because it is easier to graph a linear equation from slope-intercept form than standard form.

**Ex 3)** Change $2x - 3y = 12$ into slope-intercept form and identify the slope and y-intercept:

**Step 1:** move the *x* term to the right side of the equals sign.
**Step 2:** Divide ever term by the y coefficient.
**Step 3:** Simplify.

$$2x - 3y = 12$$
$$-2x \qquad -2x$$
$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{12}{-3}$$
$$y = \frac{2}{3}x - 4$$

Note: From standard form – slope: $m = \frac{-A}{B}$, y-int: $b = \frac{C}{B}$.

**Ex 4)** Change $y = \frac{2}{3}x - 1$ to standard form:

**Step 1:** remove fractions by multiplying by the LCM of the denominator.
**Step 2:** Isolate the constant by moving the X term to the left hand side of the =.
**Step 3:** make A positive by multiplying every term by -1 (if necessary).

$$\left(y = \frac{2}{3}x - 1\right) \times 3$$
$$3y = 2x - 3$$
$$-2x \qquad -2x$$
$$(-2x + 3y = -3) \times -1$$
$$2x - 3y = 3$$

**Graph from Standard Form**

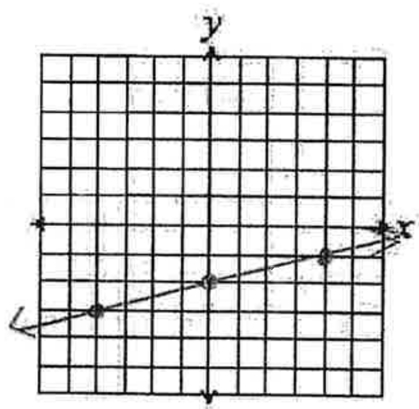
Step 1) Write the equation in slope-intercept form by solving for y.
Step 2) Identify the y-intercept $(0, b)$ and graph this point.
Step 3) Graph two more points using the slope, counting from the y-intercept.
Step 4) Draw a line connecting the points to obtain the graph.

**Ex 4)** Graph the following: $x - 4y = 8$

$$x - 4y = 8$$
$$-x \qquad -x$$
$$\frac{-4y}{-4} = \frac{-x}{-4} + \frac{8}{-4}$$
$$y = \frac{1}{4}x - 2$$

slope: $\frac{1}{4}$ ← up 1 ← right 4

yint → (0, -2)



5.1B

**[redacted] – Different Forms of Linear Relations**

**Learning Target:** to write an equation of a line from a graph or given information.

**Toolkit:**
- slope – $m$
- standard form of a line
- slope-intercept form of a line

Writing an equation of a line using $y = mx+b$

By substituting given values for a slope and a point on a line into $y = mx + b$ we can find the equation of a line.

Ex 1) Write the equation of a line (in slope-intercept form) that passes through the point $(3, -2)$ and has a slope $m = -2$.

x y

(1) Plug in x, y and m
(2) Solve for b
(3) Rewrite $y=mx+b$ using m and b

$y = -2x + 4$

$y = mx + b$

$-2 = (-2)(3) + b$

$-2 = -6 + b \Rightarrow b = 4$

$+6 \quad +6$

Ex 2) Write the equation of a line (in standard form) that passes through $(6, -2)$ and has a slope $m = \frac{1}{3}$.

x y

*hint: write using slope-intercept form, then change to standard form*

$y = mx + b$

$-2 = (\frac{1}{3})(6) + b$

$-2 = \frac{6}{3} + b$

$-2 = 2 + b$

$-2 \quad -2$

$b = -4$

$\rightarrow y = \frac{1}{3}x - 4$

change to standard form

positive A value, no fractions

$(y = \frac{1}{3}x - 4) \times 3$

$3y = x - 12$

$-x \quad -x$

$(-x + 3y = -12) \times -1 \Rightarrow \boxed{x - 3y = 12}$

Point-slope form

$$y - y_1 = m(x - x_1)$$

The point given is $(x_1, y_1)$ and the slope of the line is $m$.
We can start with this formula and change it into slope-intercept form or standard form.

Ex 3) Using the point-slope form, write the equation of the line that passes through $(1, -2)$ with a slope of $m = -3$. Write the line using all three forms.

$x_1, y_1$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = (-3)(x - 1)$$

point slope form $\rightarrow$ $y + 2 = -3(x - 1)$

Standard Form

$y + 2 = -3(x - 1)$

$y + 2 = -3x + 3$

$+3x \qquad +3x$

$3x + y + 2 = 3$

$\qquad -2 \quad -2$

Standard form $\rightarrow$ $3x + y = 1$

Slope-intercept form

$y + 2 = -3(x - 1)$

$y + 2 = -3x + 3$

$-2 \qquad -2$

$y = -3x + 1$

Ex 4) Given the graph, write the equation in standard form, slope-intercept form, and point slope form:

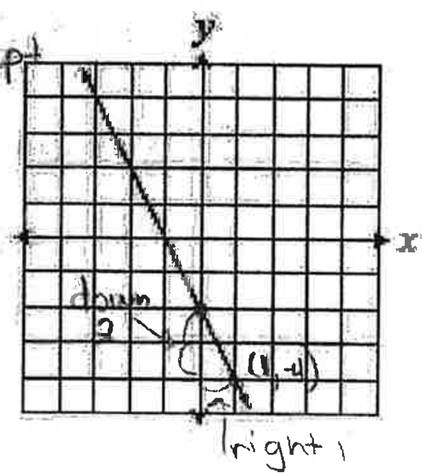
① Write in slope intercept form

y intercept $\rightarrow b = -2$

Slope $\rightarrow m = \frac{-2}{1} = -2$

$y = mx + b$

slope intercept form $\rightarrow$ $y = -2x - 2$




$y = -2x - 2$

$+2x \qquad +2x$

Standard form $\rightarrow$ $2x + y = -2$

point-slope form

$m = -2, \ (x_1, y_1) = (1, -4)$

$y - y_1 = m(x - x_1)$

$y - (-4) = -2(x - 1)$

$y + 4 = -2(x - 1)$

# 5.2A

**~~[illegible]~~ – Special Cases of Linear Equations**

**Learning Target:** to understand equations of horizontal and vertical lines, and how to find an equation of a line using two points.

**Toolkit:**

- slope – $m$
- standard form of a line
- slope-intercept form of a line
- point-slope form of a line

**Horizontal Lines**

From the last chapter, we know that the **slope** of a horizontal line is 0.
Let's plug that into the slope-intercept formula and simplify:

$$y = mx + b$$

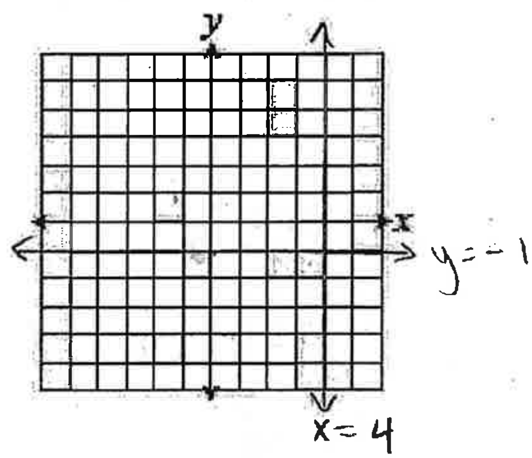
$$y = (0)x + b \Rightarrow y = b$$

So, the equation of a horizontal line with y-intercept k is $y = k$.

**Vertical Lines**

From the last chapter, we know that the **slope** of a vertical line is undefined.
Because of this, we write the equation of a vertical like with x-intercept k as $x = k$.

**Ex 1)** Graph the following:
$y = -1$ and $x = 4$




**Ex 2)** Write the equation of the **horizontal line** that passes through the point $(1, 3)$.

We only need the y-value (x, y)

$\Rightarrow y = 3$

**Ex 3)** Write the equation of the **vertical line** that passes through the point $(-2, -4)$.

We only need the x-value (x, y)

$\Rightarrow x = -2$

| | |
|---|---|
| Writing a line through 2 points | In 5.1, we were given a point and the slope and we could write the equation of the line. Now what if we are given two points on the line? From the two points given, we can calculate the Slope. |
| Ex 4) | Write the equation of the line that passes through A(1, −2) and B(2, −4) in slope-intercept form. |

$x_1\ y_1 \quad x_2\ y_2$

1) Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-2)}{2-1} = \frac{-4+2}{1} = \boxed{-2}$$

2) Use the slope and one point to find b.

$$y = mx + b$$
$$-2 = (-2)(1) + b$$
$$-2 = -2 + b$$
$$+2 \quad +2$$
$$b = 0$$

$$\Rightarrow y = -2x$$

| | |
|---|---|
| Ex 5) | Write the equation of the line that passes through A(3, −1) and B(−3, 3) in standard form. |

$x_1\ y_1 \quad x_2\ y_2$

1) Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-1)}{-3-3} = \frac{3+1}{-3-3} = \frac{4}{-6}$$

$$m = \frac{-2}{3}$$

2) Use the slope and one point to find b.

$$y = mx + b$$
$$-1 = \frac{-2}{\cancel{3}}(\cancel{3}) + b$$
$$-1 = -2 + b$$
$$+2 \quad +2$$
$$b = 1$$

$$\Rightarrow y = \frac{-2}{3}x + 1$$

3) Change to Standard form

$$\left(y = \frac{-2}{3}x + 1\right) \times 3 \leftarrow \text{remove fraction}$$

$$3y = -2x + 3$$
$$+2x \quad +2x$$

$$2x + 3y = 3 \leftarrow \text{put in the right order}$$

5.2B

**[illegible] – Special Cases of Linear Equations**

**Learning Target:** to compare equations of line and determine if they are parallel, perpendicular or neither.

**Toolkit:**
- slope – $m$
- standard form of a line
- slope-intercept form of a line
- point-slope form of a line

**Parallel and Perpendicular lines**

From last chapter we learned that **parallel lines** have the same slope but different y-intercept.
**Perpendicular lines** have slopes that are negative reciprocals
We can use this information to determine if equations of lines are parallel, perpendicular or neither.
Note: two or more equations put together is called a **system of equations.**

Ex 1) In the following systems of equations, determine if the lines are parallel, perpendicular or neither: change to slope - intercept form

a) $2x + 6y = 5$ and $3x - y = 2$

$2x+6y=5$
$-2x \quad -2x$
$\frac{6y}{6} = \frac{-2x}{6} + \frac{5}{6}$
$y = -\frac{1}{3}x + \frac{5}{6}$

$3x - y = 2$
$-3x \quad -3x$
$\frac{-y}{-1} = \frac{-3x}{-1} + \frac{2}{-1}$
$y = 3x - 2$

$m_1 = -\frac{1}{3}$
$m_2 = 3$
↑ perpendicular

b) $-3x + 2y = -2$ and $2x - 3y = -6$

$-3x + 2y = -2$
$+3x \quad +3x$
$\frac{2y}{2} = \frac{3x}{2} - \frac{2}{2}$
$y = \frac{3}{2}x - 1$

$2x - 3y = -6$
$-2x \quad -2x$
$\frac{-3y}{-3} = \frac{-2x}{-3} - \frac{6}{-3}$
$y = \frac{2}{3}x + 2$

$m_1 = \frac{3}{2}$
$m_2 = \frac{2}{3}$
↑ Neither!

c) $x - 2y = -6$ and $-x + 2y = -2$

$x - 2y = -6$
$-x \quad -x$
$\frac{-2y}{-2} = \frac{-x}{-2} - \frac{6}{-2}$
$y = \frac{1}{2}x + 3$

$-x + 2y = -2$
$+x \quad +x$
$\frac{2y}{2} = \frac{x}{2} - \frac{2}{2}$
$y = \frac{1}{2}x - 1$

$m_1 = \frac{1}{2}$
$m_2 = \frac{1}{2}$
↑ parallel

5.3

**– Equations of Parallel and Perpendicular Lines**

**Learning Target:**

**Toolkit:**
- slope – $m$
- parallel lines
- perpendicular lines
- Equations of a line

Warm up

Find the slopes of the lines parallel and perpendicular to the following:

a) $y = 2x + 1$ (m)

parallel : $m = 2$
perpendicular
$m = \frac{-1}{2}$

b) $y = 2$

horizontal line, $m = 0$
parallel : $m = 0$
perpend : $m =$ undefined

c) $x - 2y = 4$

$-x \quad -x$

$\frac{-2y}{-2} = \frac{-x}{-2} + \frac{4}{-2}$

$y = \frac{1}{2}x + 4$

par : $m = \frac{1}{2}$, per : $m = -2$

Finding lines parallel and perpendicular to the lines given

When asked to find a line that crosses through the point given and is parallel or perpendicular to the line given:

**Step 1)** Determine the slope of the line given.

**Step 2)** A line parallel: keep the slope the same
A line perpendicular: take the negative reciprocal.

**Step 3)** Use $y = mx + b$ and plug in the appropriate slope and $(x, y)$ coordinates given.

**Step 4)** Solve for b.

**Step 5)** Write the equation of the new line using $m$ and $b$ in $y = mx + b$.

**Step 6)** If necessary, rewrite in the form asked.

Ex 1) Find the equation of the line, in standard form, that passes through the point $(-3, 7)$ and is parallel to the line $2x - 3y = 6$.

① $2x - 3y = 6$
$-2x \quad -2x$
$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$
$y = \frac{2}{3}x - 2$
$m = \frac{2}{3}$

② parallel, same slope

③ $y = mx + b$
$7 = \frac{2}{3}(-3) + b$

④ $7 = -2 + b$
$+2 \quad +2$
$b = 9$

⑤ $y = \frac{2}{3}x + 9$

⑥ $\left(y = \frac{2}{3}x + 9\right) \times 3$ · no fractions, · $A > 0$

$3y = 2x + 27$
$-2x \quad -2x$
$(-2x + 3y = 27) \times -1 \Rightarrow 2x - 3y = -27$

Ex 2) Find the equation of the line, in standard form, that passes through the point $(6, -4)$ and is perpendicular to $2x - y = 3$.

$2x - y = 3$
$-2x \quad -2x$
$\frac{-y}{-1} = \frac{-2x}{-1} \frac{+3}{-1}$
$y = 2x - 3$
$m = 2$
perpendicular
$\Rightarrow m = \frac{-1}{2}$

$y = mx + b$
$-4 = \frac{-1}{2}(6) + b$
$-4 = -3 + b$
$+3 \quad +3$
$-1 = b$
$y = \frac{-1}{2}x - 1$

Change to Standard form

$(y = -\frac{1}{2}x - 1) \times 2$
$2y = -x - 2$
$+x \quad +x$
$\boxed{x + 2y = -2}$

No fractions, $A \geq 0$

Ex 3) Find the equation of the line, in standard form, that passes through the point $(-1, 3)$ and is perpendicular to the line $y = 6$.

$y = 6$ is a horizontal line, so $m = 0$

perpendicular
$\Rightarrow m =$ undefined

Vertical line passing through $(-1, 3)$ $\Rightarrow \boxed{x = -1}$

Ex 4) Find the equation of a line, in slope-intercept form, parallel to $2x + 5y = -10$ that has the same y-intercept as $4x - 3y = -9$.

parallel to
$2x + 5y = -10$
$-2x \quad -2x$
$\frac{5y}{5} = \frac{-2x}{5} \frac{-10}{5}$
$y = \frac{-2}{5}x - 2$
$m = \frac{-2}{5}$
Same slope

y intercept of $4x - 3y = -9$
change to $y = mx + b$
y int
$4x - 3y = -9$
$-4x \quad -4x$
$\frac{-3y}{-3} = \frac{-4x}{-3} \frac{-9}{-3}$
$y = \frac{4}{3}x + 3$ ← y int

so $y = \frac{-2}{5}x + 3$

5.4

## – Linear Applications and Modelling

**Learning Target:** to use graphs as a tool to help interpret data and reveal trends.

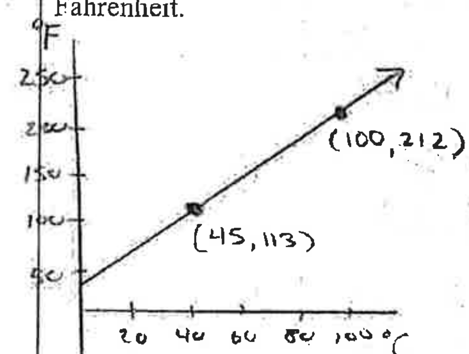
**Toolkit:**

- slope – $m = \frac{y_2 - y_1}{x_2 - x_1}$
- different forms of equations of a line
- plotting points $(x, y)$
-

Graphs are used to visually display data and reveal trends in the information given. We can use the graphs to answer questions regarding the information.

From the last chapter, we set up a linear equation from a word problem in the following way: dependant variable = rate of change × independent variable + initial value

Ex 1) Water boils at 212°F, or 100°C and when water is 45°C, it is 113°F. Graph the linear relation between °C and °F, and find the formula that converts Celsius to Fahrenheit.




Rate of change = slope = $\frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{212 - 113}{100 - 45} = \frac{99}{55} = \frac{9}{5}$

$\Rightarrow y = mx + b$

$212 = \frac{9}{5}(100) + b$

$212 = 180 + b$

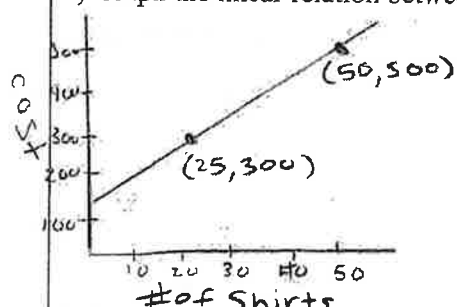
$-180 \quad -180$

$b = 32$

$\boxed{F = \frac{9}{5}C + 32}$

Ex 2) It costs a company \$300 to make 25 shirts and \$500 to make 50 shirts.

a) Graph the linear relation between the cost, C, and the number of shirts, N.




b) Find the rate of change and fixed amount for making shirts.

rate of change = slope = $\frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{500 - 300}{50 - 25} = \frac{200}{25} = \$8$ per shirt

c) Write the cost equation. $C = \text{rate of charge} \cdot N + i$

$500 = 8 \cdot (50) + i$

$500 = 400 + i$ $i = 100$ $\Rightarrow C = 8N + 100$

$-400$ $-400$

d) How much would it cost to make 32 shirts?

$N = 32$

$C = 8(32) + 100$

$C = 256 + 100$

$C = \$356$

It would cost \$356

e) How many shirts can you make for \$676? $C = 676$

$676 = 8N + 100$
$-100$ $-100$ $\Rightarrow \frac{576}{8} = \frac{8N}{8} \Rightarrow N = \boxed{72 \text{ shirts}}$

Ex 3) A printer costs \$960 new and is expected to be worth \$140 after 6 years. What will it be worth after 4 years?

$(\underset{(t)}{\text{years}}, \underset{(C)}{\text{cost}})$

*Create an equation for the cost depending on the year.

$(0, 960)$ and $(6, 140)$

Rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{140 - 960}{5 - 0} = \frac{-820}{5} = -\$16$ per year

$C = \text{rate of change} \cdot t + i$

$960 = -164(0) + i$

$960 = i$

$\Rightarrow C = -164t + 960$

when $t = 4 \Rightarrow C = -164(4) + 960$

$C = -656 + 960$

$C = \$304$

After 4 years, the printer is worth \$304.00

5.5

~~4.4~~ – **Function Notation**

**Learning Target:** to understand function notation and relate it to linear equations and (x, y) coordinates.

**Toolkit:**

- Equation of a line
- Substituting numerical values into an equation
- (x, y) coordinates

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**Warm up**

Given $y = -2x + 5$ and $x = 3$, what is the value for y ?

$y = -2(3) + 5$

$y = -6 + 5 \Rightarrow y = -1$

**Function Notation**

The notation $f(x)$ is called **function notation**. It is just another way of writing $y$ as a function.

For example, we can write $y = -2x + 5$ as $f(x) = -2x+5$.

Therefore, $f(x) = y$.

We read it as "f of x", because the equation is a function using the variable x.

Using the equation above, when we substitute in $x = 3$, we write it as follows:

using $f(x) = -2x+5$ ← anywhere you see x, plug in 3

$f(3) = -2(3)+5$

$f(3) = -1$

**Ex 1)** Given $f(x) = 3x - 4$, determine the coordinates of one point on the line for $f(-1)$.

$f(-1) = 3(-1) - 4$

$f(-1) = -3 - 4$

$f(-1) = -7 \Rightarrow (-1, -7)$

same as y

**Ex 2)** Given $f(x) = -2x + 1$, find the following:

a) $f(-3)$

$f(-3) = -2(-3)+1$

$= -6+1$

$f(-3) = -5$

b) $f(t)$ $= -2(t)+1$

$f(t) = -2t+1$

c) $f(2x)$

$f(2x) = -2(2x)+1$

$f(2x) = -4x+1$

d) $f(x-1)$

$f(x-1) = -2(x-1)+1$

$= -2x+2+1$

$f(x-1) = -2x+3$

Ex 3) Given $f(x) = 5x - 7$, find the coordinates of the point when $f(x) = -12$

$$-12 = 5x - 7$$ ← solve for x

same as $y$

so $y = -12$

$$\begin{aligned} +7 \quad &\quad +7 \\ -5 &= 5x \\ \frac{-5}{5} &= \frac{5x}{5} \\ x &= -1 \end{aligned}$$

$(-1, -12)$

Ex 4) Determine the slope-intercept function $f(x) = mx + b$ if $f(2) = -5$ and $f(-6) = -9$.

$f(2) = -5 \Rightarrow$ the point $(2, -5)$

$f(-6) = -9 \Rightarrow$ the point $(-6, -9)$

so, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-9-(-5)}{-6-2} = \frac{-9+5}{-8} = \frac{-4}{-8} = \frac{1}{2}$

$$\begin{aligned} y &= mx + b \\ -5 &= \tfrac{1}{2}(2) + b \\ -5 &= 1 + b \\ -1 \quad &\quad -1 \\ b &= -6 \end{aligned}$$

$\Rightarrow \boxed{y = \frac{1}{2}x - 6}$