

1.1 – Factoring Trinomials of the form $x^2 + bx + c$, where $a=1$

Name:

Date:

KEY

Goal: to use models and algebraic strategies to multiply binomials and to factor trinomials.

Toolkit:

- Factoring

Main Ideas:

Descending order: the terms are written in order from the term with the greatest exponent to the term with the least exponent.

Ascending order: the terms are written in order from the term with the least exponent to the term with the greatest exponent.

Steps for Factoring a Trinomial in the form: $x^2 + bx + c$, where $a=1$

With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 1: If needed, re-order the terms in descending powers of the variable (*biggest to smallest*)

Step 2: Find two numbers that multiply to equal the c term and add to equal the b term (add to the middle, multiply to the end)

Step 3: Factor into two binomials using the numbers from step 2, with the variable from the question placed first in each Bracket

*** we will refer to this method as "the simple way" ***

Multiplying two binomials

Ex 1) Expand and Simplify: $(x-1)(x-7)$ use FOIL

$$= x^2 - 7x - 1x + 7$$

$$= x^2 - 8x + 7$$

Remember: expanding and factoring are opposite operations...they UNDO each other!

Factoring a trinomial in the form $x^2 + bx + c$

Ex 2) Factor the trinomial: $x^2 - 8x + 7$...we should end up with $(x-1)(x-7)$!

$$\begin{array}{r} -7 \times -1 = 7 \\ -7 + -1 = -8 \end{array}$$

$$= (x-7)(x-1)$$

Notice that a (the number in front of the x^2) will always end up being 1 in these questions!

Ex 3) Factor: $a^2 - 2a - 8$

$$\begin{array}{r} -4 \times 2 = -8 \\ -4 + 2 = -2 \end{array}$$

$$= (a-4)(a+2)$$

Factoring a trinomial written in ascending order

Ex 4) Factor: $-30 + 7m + m^2 \rightarrow m^2 + 7m - 30$

reorder

$$\begin{array}{r} -3 \times 10 = -30 \\ -3 + 10 = +7 \end{array}$$

$$= (m-3)(m+10)$$

GCF of -5!

now, a = 1
"simple way"

Ex 5) Factor: $\frac{-5h^2}{-5} - \frac{20h}{-5} + \frac{60}{-5}$

Always check to see if there is a GCF you can factor out first! IF there is a negative number in front of the x^2 , factor out the negative as well.

$$= -5(h^2 + 4h - 12)$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ -2 \times 6 = -12 \\ -2 + 6 = 4 \end{array}$$

$$= -5(h-2)(h+6)$$

Difference of Squares

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

Ex 6) Factor: $x^2 - 16$

$$\begin{array}{l} \sqrt{x^2} = x \\ \sqrt{16} = 4 \end{array}$$

$$= (x+4)(x-4)$$

Ex 7) Factor: $x^4 - 9$

$$\begin{array}{l} \sqrt{x^4} = x^2 \\ \sqrt{9} = 3 \end{array}$$

$$= (x^2+3)(x^2-3)$$

Ex 8) Factor: $\frac{50x^2}{2} - \frac{2y^2}{2}$

GCF of 2!

$$= 2(25x^2 - y^2)$$

$$\begin{array}{l} \sqrt{25x^2} = 5x \\ \sqrt{y^2} = y \end{array}$$

$$= 2(5x+y)(5x-y)$$

1.2 – Factoring $ax^2 + bx + c$

Learning Target: to factor trinomials of the form $ax^2 + bx + c$, where the a value $\neq 1$

Toolkit:

- Multiplying binomials
- Factoring by grouping

When $a \neq 1$ in a trinomial of the form $ax^2 + bx + c$, and it can't be factored out, then another process is needed.

This process is called DECOMPOSITION, and uses factoring by grouping.

Steps for factoring $ax^2 + bx + c$, $a \neq 1$ by DECOMPOSITION

- Step 1:** As with any factoring question, check to see if you can factor out a GCF
- Step 2:** If needed, re-order the terms in **descending** powers of the variable
- Step 3:** Find two numbers that multiply to equal ac and add to equal b
(multiply to the product (\times) of the first and last, and add to the middle!)
- Step 4:** Rewrite the expression but split or *decompose* the middle (b) term, using the two numbers from step 3
- Step 5:** Now the expression has FOUR terms, so we can factor by grouping the first two terms and the last two terms
- Step 6:** When factoring by grouping, the two resulting binomials need to be identical!
These matching binomials are now the **COMMON FACTOR**, and can be factored out...and what is left become the components of the second bracket.

Ex. 1) Factor:

Factor by grouping
* remove
GCF from both pairs

So... binomials
match; factor out
($x+2$) from both
terms

$$4x^2 + 11x + 6$$

$$= 4x^2 + 8x + 3x + 6$$

$$= 4x(x+2) + 3(x+2)$$

$$= (x+2)(4x+3)$$

GCF? no! Reorder? no!

$a=4$, $a \neq 1$, so... decomp!

$$\begin{array}{r} 8 \times 3 = 24 \quad (4 \times 6) \\ 8 + 3 = 11 \end{array}$$

so... split $11x$ into $8x+3x$ (or $3x+8x$)

Ex 2) $-7m - 10 + 6m^2$

$$\begin{aligned}
 &= 6m^2 - 7m - 10 \\
 &= \underbrace{6m^2 - 12m}_{6m(m-2)} + \underbrace{5m - 10}_{5(m-2)} \\
 &= 6m(m-2) + 5(m-2) \\
 &= (m-2)(6m+5)
 \end{aligned}$$

GCF? no
reorder? yes!

$a=6$, so decomp!

$$\frac{-12}{6} \times \frac{5}{(6)(-10)} = \frac{-60}{(6)(-10)} \quad (a \times c)$$

$$\frac{-12}{6} + \frac{5}{1} = -7 \quad (b)$$

so... split $-7m$ into $-12m + 5m$!

Ex 3) $8p^2 - 18p - 5$

$$\begin{aligned}
 &= 8p^2 + 2p - 20p - 5 \\
 &= 2p(4p+1) - 5(4p+1) \\
 &= (4p+1)(2p-5)
 \end{aligned}$$

GCF? no
reorder? no

$a=8$, so decomp!

$$\frac{2}{8} \times \frac{-20}{(8)(-5)} = \frac{-40}{(8)(-5)} \quad (a \times c)$$

$$\frac{2}{8} + \frac{-20}{1} = -18 \quad (b)$$

so... split $-18p$ into $2p$ and $-20p$!

Ex 4) $\frac{24}{2}x^2 - \frac{10}{2}x - \frac{4}{2}$

$$\begin{aligned}
 &= 2(12x^2 - 5x - 2) \\
 &= 2(\underbrace{12x^2 + 3x}_{3x(4x+1)} - \underbrace{8x - 2}_{2(4x+1)}) \\
 &= 2[3x(4x+1) - 2(4x+1)] \\
 &= 2(4x+1)(3x-2)
 \end{aligned}$$

GCF? yes!
reorder? no

$a=12$, so decomp!

$$\frac{3}{12} \times \frac{-8}{(12)(-2)} = \frac{-24}{(12)(-2)}$$

$$\frac{3}{12} + \frac{-8}{1} = -5$$

so... split $-5x$ into $3x - 8x$!

Ex 5) $\frac{-16x^3}{-4x} + \frac{20x^2}{-4x} + \frac{24x}{-4x}$

$$\begin{aligned}
 &= -4x(4x^2 - 5x - 6) \\
 &= -4x(\underbrace{4x^2 - 8x}_{4x(x-2)} + \underbrace{3x - 6}_{3(x-2)}) \\
 &= -4x[4x(x-2) + 3(x-2)] \\
 &= -4x(x-2)(4x+3)
 \end{aligned}$$

GCF? yes!
reorder? no

$a=4$, so decomp!

$$\frac{-8}{4} \times \frac{3}{(4)(-6)} = \frac{-24}{(4)(-6)}$$

$$\frac{-8}{4} + \frac{3}{1} = -5$$

so... split $-5x$ into $-8x + 3x$!

square brackets
here... just to
help keep work
organized