Name: NOTES KEY
Date: _____

CHAPTER 5 NOTES – Polynomials

Calendar of Chapter: See the 'Homework' link on the webpage

What You'll Learn:

5.1 – Using correct vocabulary to describe polynomials
5.2 – Performing addition and subtraction of polynomials using like terms
5.3 – Performing multiplication on monomials and polynomials using exponent laws
5.P (Power) – Performing operations on polynomials involving exponents
5.4 – Performing division on monomials and polynomials using exponent laws

What is the difference between an expression and an equation?

What is a term?

5.1 – Defining Polynomials

Focus: Classifying and ordering polynomials

Warmup:
How many terms does each expression have, and what is each term?
a) $y + 7$ b) $x^2 - 2x + 3$
c) -1 d) $2t - 9 + t^3 - t^2$
e) $2x^2y^5z^7$

a) $y+7$ (b) x^2-2x+3 (c) -1 (d) $2t-9+t^3-t^2$

2 terms: $y, 7$ — 3 terms: $x^2, -2x, 3$ — 1 term: -1 — 4 terms: $2t, -9, t^3, -t^2$

(e) $2x^2y^5z^7$
1 term

What is a monomial?
What is a binomial?
What is a trinomial?
What is a polynomial?
**HINT: Look at the prefix of each word!*

A one term expression
A two term expression
A three term expression
An expression with two or more terms (poly= plural)

Ex1 – Classify each expression as one **or more** of the 4 terms above:
a) $x - 2y$
b) $-17x^2y^2z^2$
c) $4x$
d) $2x^2 - 5 + 16xy$
e) $x + y - z + 5$

a) $x-2y$ — binomial, polynomial
(b) $-17x^2y^2z^2$ — monomial
(c) $4x$ — monomial
d) $2x^2-5+16xy$ — trinomial, polynomial
(e) $x+y-z+5$ — polynomial

What is the 'degree' of a monomial?

the sum of the exponents on the variables

Ex2 – State the degree
a) $2x$
b) $-3x^2y^3z$
c) -7

a) $2x^1$ — degree 1
(b) $-3x^2y^3z^1$ — degree 6
(c) -7 — degree 0

What is the **leading term** of a polynomial?

the term with the highest degree

What is the degree of a polynomial?

the degree of the leading term

Ex3 – State the degree of each polynomial
a) $x^2 + 3x + 7$
b) $3y - 2y^3 + 2y^2$
c) $4x^6 + 2x^3y^4$
d) $25x^3y + 36x^2y^2$

a) $x^2 + 3x^1 + 7$ (b) $3y - 2y^3 + 2y^2$ c) $4x^6 + 2x^3y^4$

(2) 1 0 1 (3) 2 6 (7)

degree 2 degree 3 degree 7

(d) $25x^3y^1 + 36x^2y^2$

4 4

degree 4

Ex4 – State the degree for '*x*' of each polynomial
a) $x^2 + 3x + 7$
b) $3y - 2y^3 + 2y^2$
c) $4x^6 + 2x^3y^4$
d) $25x^3y + 36x^2y^2$

a) $x^2 + 3x^1 + 7$ (b) $3y - 2y^3 + 2y^2$ (c) $4x^6 + 2x^3y^4$

(2) 1 0 0 0 0 6 3

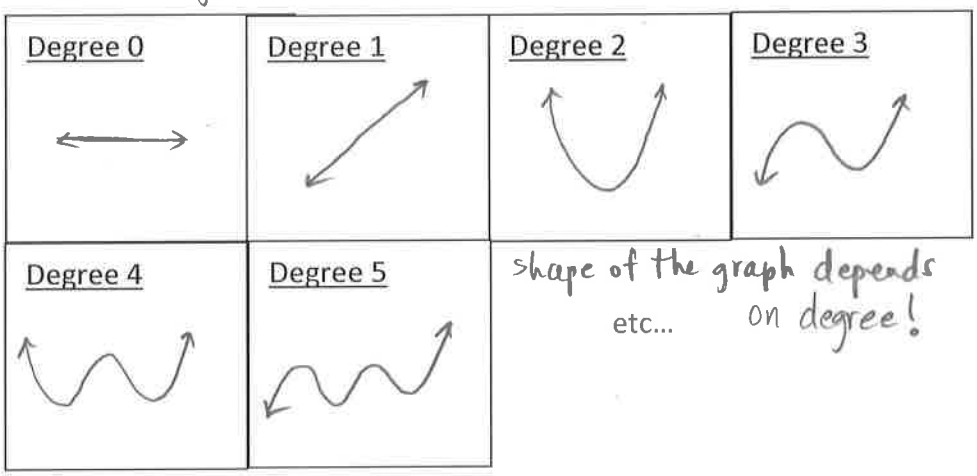
degree 2 degree 0 degree 6

(d) $25x^3y + 36x^2y^2$

3 2

degree 3

Why is the leading term for a polynomial important in math?



shape of the graph depends on degree!

etc...

How do you order the terms in a polynomial?

from highest degree term to lowest degree term

Ex5 – Arrange in descending powers of '*x*'
a) $3x - x^2 + 2x^3 - 6$
b) $2x^2y^5 - 3x + 2x^3y^2$

a) $3x - x^2 + 2x^3 - 6$ (b) $2x^2y^5 - 3x + 2x^3y^2$

1 2 3 0 2 1 3

$2x^3 - x^2 + 3x - 6$ $2x^3y^2 + 2x^2y^5 - 3x$

What are **like terms**?

terms that have the same exponent(s) raised to the same exponent(s)

Ex6 - Simplify
a) $3x^2 - x - 4x^2 + 7x$
b) $-xy + 2 - 5xy + x - 8$

a) $3x^2 - x - 4x^2 + 7x$ (b) $-xy + 2 - 5xy + x - 8$

$-x^2 + 6x$ $-6xy + x - 6$

5.2 – Adding and Subtracting Polynomials

Focus: To add and subtract polynomials using like terms.

Warmup:

Draw trays, then simplify:
$x^2 - 2x + 5x - 3x^2$

$x^2 - 2x + 5x - 3x^2$

$-2x^2 + 3x$

How do you add/subtract like terms?

Add or subtract the coefficients and leave the variable(s) the same.

Ex1 – Add
Suppose an x^2 was a square, an x was a strip, and a constant was a dot. Draw a picture for the situation below:
$(2x^2 + 3x + 4) + (x^2 + 2x + 3)$

$(2x^2+3x+4) + (x^2+2x+3)$

x^2 = 

x = 

1 = 

Now, add the like shapes together, and write your answer:

$3x^2 + 5x + 7$

Now, do Ex1 without shapes:

$(2x^2 + 3x + 4) + (x^2 + 2x + 3)$

$2x^2 + 3x + 4 + x^2 + 2x + 3$

$3x^2 + 5x + 7$

What steps are involved in adding polynomials?

1. See if there is a number to distribute into each bracket. If it's only a 1 for each, rewrite without brackets
2. Identify like terms
3. Combine like terms.

Ex2 – Simplify

a) $(3y^2 - 8y + 3) + (2y^2 + 8y - 9)$

$3y^2 - 8y + 3 + 2y^2 + 8y - 9$

$5y^2 - 6$

b) $(5x^3 + 7x - 9) + (-8x + 11 + 4x^3)$

$5x^3 + 7x - 9 - 8x + 11 + 4x^3$

$9x^3 - x + 2$

Subtracting Polynomials

ex3 - Simplify

$(4x^2 - 5x + 7) - (3x^2 + 2x - 5)$

$(4x^2 - 5x + 7) - 1(3x^2 + 2x - 5)$

$4x^2 - 5x + 7 - 3x^2 - 2x + 5$

$x^2 - 7x + 12$

What is the extra step necessary when subtracting polynomials?

The second bracket has a -1 out front which must be distributed

ex4 - Simplify

a) $(3y^2 - y + 5) - (4y^2 - y + 1)$

a) $(3y^2 - y + 5) - 1(4y^2 - y + 1)$

$3y^2 - 1y + 5 - 4y^2 + 1y - 1$

$-y^2 + 4$

b) $(-x^2y + 3xy) - (4xy^2 + xy - 2x^2y)$

(b) $(-x^2y + 3xy) - 1(4xy^2 + xy - 2x^2y)$

$-x^2y + 3xy - 4xy^2 - xy + 2x^2y$

$x^2y + 2xy - 4xy^2$

5.3 – Multiplying Polynomials

Focus: How to multiply monomials so only one monomial results.
How to multiply a monomial to a polynomial.

Warmup:
Simplify in power form:
a) $3^2 \times 3^5$
b) $(-7)^4(-7)$
c) $y^6 \times y^3$

a) 3^{2+5} 3^7 (b) $(-7)^{4+1}$ $(-7)^5$ (c) y^{6+3} y^9

Think Back:
What is the exponent law for multiplying powers with the same base?

add exponents

Ex1 – Multiply by writing in expanded form:
$(6x^2)(-3x^4)$

$(6x^2)(-3x^4)$
$6x x(-3)xxxx$
$6(-3)xxxxxx$
$-18x^6$

$(6x^2)(-3x^4)$
$-18x^6$

Now, do the example again without using expanded form:

What are the steps to multiplying monomials?

① Multiply coefficients
② add exponents on variables

Ex2 - Multiply
a) $(3a)(4b^3)$
b) $-x^2(2xy)$
c) $-2y^3(4y^2)$

a) $(3a)(4b^3)$ $12ab^3$ (b) $-x^2(2xy)$ $-2x^3y$ (c) $-2y^3(4y^2)$ $-8y^5$

Ex3 – Simplify
a) $(2x^2y)(-3xy)$
b) $(x^3)(-3xy^3)(4y)$
c) $(-2a^3bc^2)(-5ac^4y^2)$

a) $(2x^2y)(-3xy)$ $-6x^3y^2$ (b) $(x^3)(-3xy^3)(4y)$ $-12x^4y^4$

(c) $(-2a^3bc^2)(-5ac^4y^2)$
$= 10a^4bc^6y^2$

Multiplying Monomials with Polynomials:

Warmup:
Expand
-2(3x – 7)
**Think back to last chapter.*

$-2(3x-7)$

$-6x+14$

Ex4 – Expand
a) 2x(x – 6)
b) –y(y² – 7)

a) $2x(x-6)$

$2x^2-12x$

(b) $-y(y^2-7)$

$-y^3+7y$

Ex5 – Expand & Simplify

m(m + 2) - 3m(m – 5)

$m(m+2)-3m(m-5)$

$m^2+2m-3m^2+15m$

$-2m^2+17m$

Ex6 – Expand & Simplify
a) 4x(x² – x + 7)
b) y(y + 3) – y(y – 2)
c) 3(x² + 2x – 5) – x(x + 1)

a) $4x(x^2-x+7)$

$4x^3-4x^2+28x$

b) $y(y+3)-y(y-2)$

y^2+3y-y^2+2y

$5y$

c) $3(x^2+2x-5)-x(x+1)$

$3x^2+6x-15-x^2-x$

$2x^2+5x-15$

5.P – Powers of Monomials

Focus: How to work with monomials that have an exponent.

Warmup:
Simplify in power form:
a) $(3^4)^5$
b) $(9^2)^3$
c) $(x^6)^2$

a) $(3^4)^5 = 3^{4\times5} = 3^{20}$

b) $(9^2)^3 = 9^{2\times3} = 9^6$

c) $(x^6)^2 = x^{6\times2} = x^{12}$

Think Back:
What is the exponent law when you have a power raised to an exponent?

multiply exponents

Ex1 – Simplify
a) $(x^2)^3$
b) $(y)^4$

a) $(x^2)^3 = x^{2\times3} = x^6$

b) $(y)^4 = y^{1\times4} = y^4$

Ex2 – Simplify
$(a^4b^7)^3$

Ex2) $(a^4b^7)^3 = a^{4\times3}b^{7\times3} = a^{12}b^{21}$

Ex3 – Simplify using expanded form:
$(2x^2y^3)^2$

$(2x^2y^3)^2$

expanded: $(2x^2y^3)(2x^2y^3)$
$= 2xxyyy\,2xxyyy$
$= 4x^4y^6$

Now, simplify using exponent laws:

$(2x^2y^3)^2$

$= 2^2x^{2\times2}y^{3\times2}$
$= 4x^4y^6$ SAME!!

Ex4 - Simplify
a) $(-x^2y)^2$
b) $(-2a^2bc^3)^3$

a) $(-1x^2y^1)^2$
$= (-1)^2x^{2\times2}y^{1\times2}$
$= 1x^4y^2 = x^4y^2$

b) $(-2a^2b^1c^3)^3$
$(-2)^3a^{2\times3}b^{1\times3}c^{3\times3}$
$-8a^6b^3c^9$

Ex5 - Simplify
$-(m^2n^5)^0$

$-1(m^2n^5)^0$
$-1(m^{2\times0}n^{5\times0})$
$-1m^0n^0$
$-1(1)(1)$
$= -1$

OR:
$-1(m^2n^5)^0$
$-1(1)$
$= -1$

Ex6 - Simplify
a) $(2x^2y^3)^4(-xy^2)^3$
b) $(ab^3c)^2(2a^2bc^2)^3$

**Remember, if you want to confirm an answer, simplify it in expanded form!*
Do Ex6 (b) in expanded form:

a) $(2x^2y^3)^4(-1x^1y^2)^3$

$2^4x^{2\times4}y^{3\times4}(-1)^3x^{1\times3}y^{2\times3}$

$16x^8y^{12}(-1)x^3y^6$

$= -16x^{11}y^{18}$

b) $(a^1b^3c^1)^2(2a^2b^1c^2)^3$

$a^2b^6c^2\,2^3a^6b^3c^6$

$8a^8b^9c^8$

$(ab^3c)^2(2a^2bc^2)^3$

$(ab^3c)(ab^3c)(2a^2bc^2)(2a^2bc^2)(2a^2bc^2)$

$abbbcabbbc2aabcc2aabcc2aabcc$

$2(2)(2)aaaaaaaabbbbbbbbbccccccccc$

$= 8a^8b^9c^8$

Ex7 - Simplify

$\left(\frac{2x}{3y^2}\right)^2$

$\left(\frac{2x}{3y^2}\right)^2$

expanded form:

$\left(\frac{2x}{3y^2}\right)\left(\frac{2x}{3y^2}\right)$

$= \frac{2(2)xx}{3(3)yyyy}$

$= \frac{4x^2}{9y^4}$

OR:

$\left(\frac{2x}{3y^2}\right)^2$

$= \frac{2^2x^2}{3^2y^{2\times2}}$

$= \frac{4x^2}{9y^4}$

5.4 – Dividing Polynomials

Focus: How to divide monomials so only one monomial results, and how to divide a polynomial by a monomial.

Warmup:
Simplify in power form:
a) $4^5 \div 4^3$

b) $\frac{2^7}{2^6}$

c) $y^3 \div y$

Think Back:
What is the exponent law for dividing powers with the same base?

a) $4^5 \div 4^3$
4^{5-3}
4^2

b) $\frac{2^7}{2^6}$
$= 2^{7-6}$
$= 2^1$
$= 2$

c) $y^3 \div y^1$
$= y^{3-1}$
$= y^2$

subtract exponents

Ex1 – Divide using expanded form:

$$\frac{8x^5}{2x^2}$$

$$\frac{8x^5}{2x^2} = \frac{8xxxxx}{2xx} = 4x^3$$

Now, divide using exponent laws:

$$\frac{8x^5}{2x^2} = \frac{8}{2}x^{5-2} = 4x^3$$

What are the steps for dividing monomials?

1. Divide coefficients
2. subtract exponents on variables

Ex2 - Simplify
a) $16y \div 4y$
b) $-12abc \div -2bc$
c) $18x^5y^2 \div -6x^2y$

a) $\frac{16y}{4y} = 4$

b) $\frac{-12abc}{-2bc} = 6a$

c) $\frac{18x^5y^2}{-6x^2y^1}$
$= -3x^3y$

Ex3 – Divide
a) $\frac{21x^5yz^2}{-7xyz}$

b) $\frac{10m^8n^5}{5m^3n^2}$

a) $\frac{21x^5y^1z^2}{-7x^1y^1z^1}$
$= -3x^4z$

b) $\frac{10m^8n^5}{5m^3n^2}$
$= 2m^5n^3$

Ex4 – Simplify

a) $\frac{2x^2y}{6x^{-2}y^4}$

b) $\frac{8a^2b^4c^3}{a^4bc^4}$

How do negative exponents work again?

Dividing a polynomial by a monomial:

What is another way to write: $\frac{3+1}{5}$?

Ex5 - Divide

$\frac{5xyz+10xy}{5xy}$

What are the steps involved with dividing a polynomial by a monomial?

Ex6 - Simplify

a) $\frac{18x^2y^2z^2-12x^3yz^2}{6xyz}$

b) $\frac{-10a^5b^4+5a^6b^8}{-5ab^4}$

c) $\frac{27y^3-9y+18y^2}{9y}$

a) $\frac{2x^2y^1}{6x^{-2}y^4}$

$= \frac{2}{6}x^{2-(-2)}y^{1-4}$

$= \frac{1}{3}x^4y^{-3}$

$= \frac{1x^4}{3y^3} = \frac{x^4}{3y^3}$

b) $\frac{8a^2b^4c^3}{a^4b^1c^4}$

$= 8a^{-2}b^3c^{-1}$

$= \frac{8b^3}{a^2c}$

A negative exponents means that variable is 'trapped' in the denominator

ex $x^{-2} = \frac{1}{x^2}$

$\frac{3+1}{5} = \frac{3}{5} + \frac{1}{5}$

$\frac{5xyz+10xy}{5xy} = \frac{5xyz}{5xy} + \frac{10xy}{5xy} = z+2$

Divide each term on the top by the term on the bottom

a) $\frac{18x^2y^2z^2-12x^3yz^2}{6xyz}$ ① ②

$= 3xyz - 2x^2z$

b) $\frac{-10a^5b^4+5a^6b^8}{-5ab^4}$ ① ②

$= 2a^4 - a^5b^4$

c) $\frac{27y^3-9y+18y^2}{9y}$ ① ② ③

$3y^2 - 1 + 2y$