1.1 – Number Systems

KEY

Learning Target: to examine and understand all the components of the real number system

Toolkit:	**Main Ideas:**
• Placing numbers on number lines • Anything you remember about classifying Real Numbers	

Definitions:

Natural Numbers – $\{1, 2, 3, \ldots\}$ The counting numbers

Whole Numbers – $\{0, 1, 2, 3, \ldots\}$ zero and the counting numbers

Integers – $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$ neg. counting #'s, zero, pos. counting #'s

Rational Numbers – All numbers that can be written as a fraction $\frac{m}{n}$ ↙ integers $(n \neq 0)$

* Decimals: repeating ex. $\frac{1}{3} = 0.\overline{3}$

terminating ex $\frac{1}{4} = 0.25$

Irrational Numbers – All numbers that cannot be written as a fraction, a terminating decimal, or repeating decimal

Ex. $\pi, \sqrt{2}, \sqrt{7}, \ldots$

Real Numbers – All the rational numbers and irrational numbers combined

** If not "real", then a number is complex (imaginary)

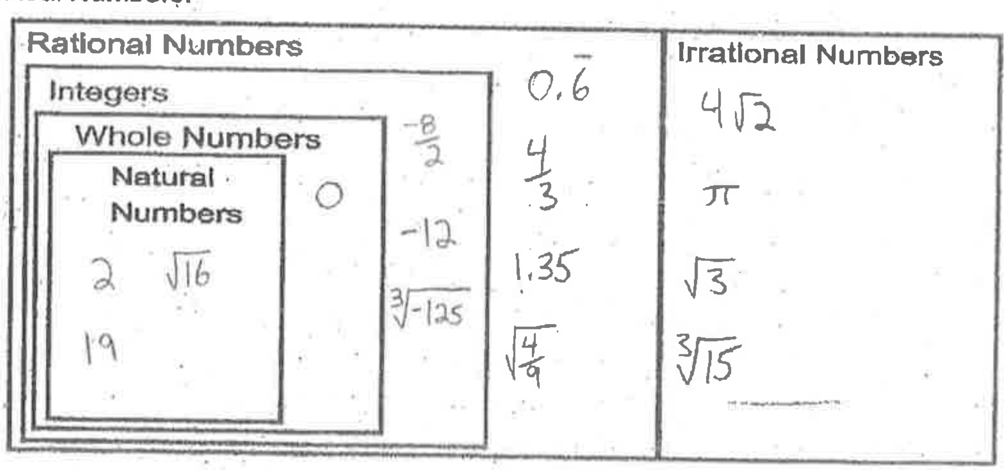
Classifying Real Numbers

Ex. 1) Where do these numbers belong in the diagram of Real Numbers?

$2 \quad 0.\overline{6} \quad 4\sqrt{2} \quad \frac{4}{3} \quad \frac{-8}{2} \quad -12 \quad \pi \quad 0 \quad \sqrt{16}$

$1.35 \quad \sqrt[3]{-125} \quad \sqrt{3} \quad \sqrt[3]{15} \quad 19 \quad \sqrt{\frac{4}{9}}$

Real Numbers:



True or False	Ex 2) State whether each statement is **true or false.** a) Every integer is a natural number ___False___ −3 not natural b) All whole numbers are integers ___True___ c) Every real number is a rational number ___False___ irrational $\sqrt{2}$ is real, but not rational
List of numbers	Ex 3) Consider the list of numbers: 0, −4, −1.3, $0.\bar{7}$, $\frac{3}{5}$, $\sqrt{17}$, 13, $-\sqrt{25}$, 3.232232223… $= -5$ **List all:** a) Natural Numbers 13 b) Whole Numbers 0, 13 c) Integers $0, -4, 13, -\sqrt{25}$ d) Rational numbers $0, -4, -1.3, 0.\bar{7}, \frac{3}{5}, 13, -\sqrt{25}$ e) Irrational Numbers $\sqrt{17}$, 3.232232223… f) Real numbers All of them
State the number System	Ex 4) State the number systems each of the following belong to: a) $\sqrt[3]{125}$ $= 5$ natural, whole, integer, rational, real b) $7.\overline{59}$ rational, real c) $\sqrt{27}$ $= 5.196152423\ldots$ Irrational, Real

1.2 – Greatest Common Factor and Least Common Multiple

Learning Target: to understand prime and composite numbers, and to find the greatest common factor (GCF) and least common multiple (LCM) of numbers.

Toolkit:

- Division
- Multiplication
- Writing repeated multiplication using powers
  Ex. $\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{2^5} \times \underbrace{3 \times 3 \times 3 \times 3}_{3^4}$

Main Ideas:

Definitions

Factor – A term which divides evenly into another term. *Ex. The factors of 24 are* 1, 2, 3, 4, 6, 8, 12, 24

Factoring – The decomposition of a number into the product of other numbers, which then multiplied together give the original value. *Ex. 12 can be factored into* $2 \times 2 \times 3$ *.

* *This is 12 written as the product of prime factors*

12 → 4, 3 → $2 \times 2 \times 3$

Prime Number – A whole number that has exactly two distinct factors: 1 and itself. *Ex.* 2, 3, 5, 7, 11, 13, ...

Composite Number – A whole number greater than 1 that has more than two distinct factors. *Ex.* 4, 6, 8, 9, 10, ...

** The whole numbers 0 and 1 are neither prime nor composite ** See pg. 61 for an explanation

Greatest Common Factor (GCF) – The largest number that divides each of the given numbers exactly.

Ex. The GCF of 24 and 36 is 12

Common FACTORS of 24 and 36: 1, 2, 3, 4, 6, 12

Least Common Multiple (LCM) – The smallest common non-zero multiple of two or more whole numbers, or the smallest number that is divisible by all the numbers.

Ex. The LCM of 24 and 36 is 72.

multiples of 24: 24, 48, 72, 96, ...

multiples of 36: 36, 72, 108, ...

Product of Primes

Ex 1) Completely factor each of the numbers

a) 28

$\boxed{28 = 2 \times 2 \times 7}$

$\boxed{28 = 2^2 \times 7}$

b) 96

$\boxed{96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3}$

$\boxed{96 = 2^5 \times 3}$

c) 325

$\boxed{325 = 5 \times 5 \times 13}$

$\boxed{325 = 5^2 \times 13}$

d) 4560

$\boxed{4560 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 19}$

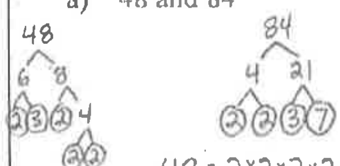
$\boxed{4560 = 2^4 \times 3 \times 5 \times 19}$

Finding the GCF

Finding the Greatest Common Factor:
1) Write each number as the product of prime factors
2) List each COMMON factor the LEAST number of times it appears in any one number
3) Multiply these factors together to get the GCF

Ex. 2) Find the GCF

a) 48 and 84



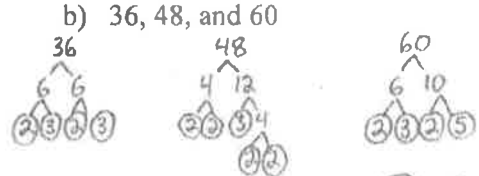
$48 = 2\times2\times2\times2\times3 = 2^4\times3$

$84 = 2\times2\times3\times7 = 2^2\times3\times7$

*common primes are 2 and 3

$GCF = 2^2\times3 = 4\times3 = \boxed{12}$

b) 36, 48, and 60



$36 = 2\times2\times3\times3 = 2^2\times3^2$

$48 = 2\times2\times2\times2\times3 = 2^4\times3$

$60 = 2\times2\times3\times5 = 2^2\times3\times5$

* common primes are 2 and 3

$GCF = 2^2\times3 = 4\times3 = \boxed{12}$

Finding the LCM

Finding the Least Common Multiple:
1) Write each number as the product of prime factors
2) Select the primes that occur the **greatest number of times** in any one factor
3) Multiply these primes together to get the LCM

Ex. 3) Find the LCM

a) 60 and 72

$60 = 2\times2\times3\times5 = 2^2\times3\times5$

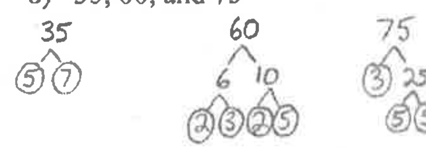
$72 = 2\times2\times2\times3\times3 = 2^3\times3^2$

*unique primes are 2, 3, and 5

** highest power of primes anywhere: $2^3, 3^2, 5^1$

$\therefore L.C.M. = 2^3\times3^2\times5 = 8\times9\times5 = \boxed{360}$

b) 35, 60, and 75



$35 = 5\times7$

$60 = 2\times2\times3\times5 = 2^2\times3\times5$

$75 = 3\times5\times5 = 3\times5^2$

* unique primes are 2, 3, 5, 7

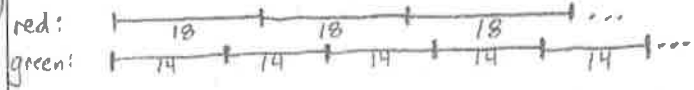
** highest power of primes: $2^2, 3, 5^2, 7$

$\therefore L.C.M. = 2^2\times3\times5^2\times7 = 4\times3\times25\times7 = \boxed{2100}$

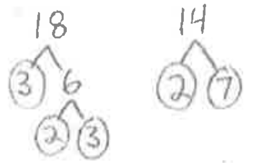
Word Problem

like #12, 13 in 2.2

You have red bungee cords that are 18cm long and green bungee cords that are 14 cm long. What is the shortest length of connected bungees you can make with each colour so that they make the same length?



LCM question!



$18 = 2\times3\times3 = 2\times3^2$

$14 = 2\times7$

* unique primes are 2, 3, 7

** highest power of primes: $2, 3^2, 7$

$\therefore LCM = 2\times3^2\times7 = 2\times9\times7 = \boxed{126}$

The shortest length you could make would be 126cm

1.3 – Squares and Square Roots

Learning Target: to understand perfect squares and perfect cubes, and square roots and cube roots

Toolkit:
- Writing a number as the product of prime factors
- The opposite operation of squaring is the square root.
  Ex. $5^2 = 25$ *and* $\sqrt{25} = 5$
- The opposite operation of cubing is the cube root:
  Ex. $2^3 = 2 \times 2 \times 2 = 8$ *and* $\sqrt[3]{8} = 2$

Main Ideas:

Definitions

Perfect Squares – Numbers with square roots that are rational (can be written as the product of two equal factors)

ex $\sqrt{25} = \sqrt{5 \times 5} = \underline{\underline{5}}$, $\boxed{\sqrt{x^2} = x}$

List of perfect square whole numbers:
0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484....

Perfect Cubes – Numbers with cube roots that are rational (can be written as the product of three equal factors)

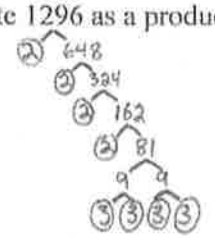
ex $\sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$, $\boxed{\sqrt[3]{x^3} = x}$

List of perfect cube whole numbers:
0, 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000....

Finding Square Roots Without a Calculator

Ex 1) Determine the square root of 1296 without a calculator.

Step 1: Write 1296 as a product of its prime factors



$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Step 2: Re-order the prime factors into TWO identical groups. (If you can't do this, your number is NOT a perfect square).

$$1296 = (2 \times 2 \times 3 \times 3)(2 \times 2 \times 3 \times 3)$$

Step 3: Multiply out each group again to see what number it represents

$$= (\ 36\ )(\ 36\ )$$

Since 1296 can be written as the product (×) of TWO equal factors: 36 × 36, we can determine that the square root of 1296 is 36.

We write $\sqrt{1296} =$ 36.

Terminology: radical, radicand, index:

index

radical

$\sqrt[2]{1296}$

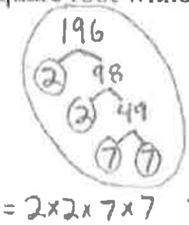
radicand
1296

*an empty index means a 2 (square root)

Finding Cube Roots Without a Calculator

Ex 2) Find the square root without a calculator

a) $\sqrt{196}$



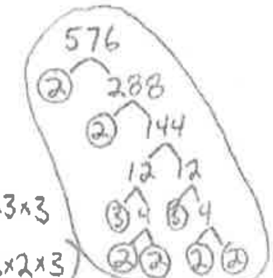
196 = 2×2×7×7

now make 2 identical groups → 196 = (2×7)(2×7)

196 = (14)(14)

so... $\sqrt{196} = \boxed{14}$

$\sqrt{576}$



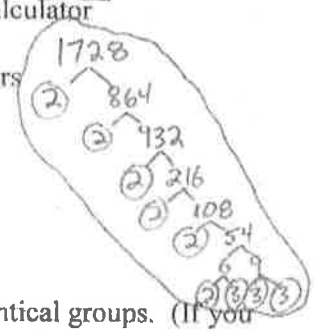
576 = 2×2×2×2×2×2×3×3

576 = (2×2×2×3)(2×2×2×3)

576 = (24)(24)

So... $\sqrt{576} = \boxed{24}$

Ex 3) Determine the cube root of 1728 without a calculator



Step 1: Write 1728 as the product of its prime factors

1728 = 2×2×2×2×2×2×3×3×3

Step 2: Re-order the prime factors into THREE identical groups. (If you can't, your number is NOT a perfect cube)

1728 = (2×2×3)(2×2×3)(2×2×3)

Step 3: Multiply out each group again to see what it represents

1728 = (12)(12)(12)

Since 1728 can be written as the product (×) of THREE equal factors: 12 × 12 × 12, we can determine that the cube root of 1728 is 12.

index

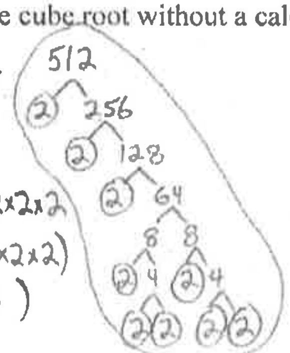
We write $\sqrt[3]{1728} = 12$

Radical, radicand, index?

1728 3

Ex 4) Find the cube root without a calculator

a) $\sqrt[3]{512}$



512 = 2×2×2×2×2×2×2×2×2

3 identical groups: 512 = (2×2×2)(2×2×2)(2×2×2)

512 = (8)(8)(8)

so... $\sqrt[3]{512} = \boxed{8}$

b) $\sqrt[3]{2744}$

2744 = 2×2×2×7×7×7

2744 = (2×7)(2×7)(2×7)

2744 = (14)(14)(14)

So... $\sqrt[3]{2744} = \boxed{14}$

*** you may use your calculator to help factor a number into a product of primes...but try not to ;-)

1.4 – Rational and Irrational Numbers

Learning Target: understanding rational and irrational numbers, and approximating irrational numbers

Toolkit:	**Main Ideas:**
• Finding a square root • Finding a cube root • Multiplication • Estimating • Radicals $\sqrt[n]{x}$ radical; index = n; radicand = x	

Rational Number: A number that can be represented as a fraction (a decimal that terminates OR repeats)

Ex. $\frac{3}{4} = 0.75$, $\frac{1}{3} = 0.33333 = 0.\overline{3}$

Irrational Number: A non-repeating OR non-terminating decimal value. *When writing an irrational number as a decimal, it is just an approximation.* Ex. $\pi = 3.14159\ldots$, $\sqrt{3} = 1.73205\ldots$

Perfect squares to memorize: $\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{49}$, $\sqrt{64}$, $\sqrt{81}$, $\sqrt{100}$, $\sqrt{121}$, $\sqrt{144}$

=2 =3 =4 =5 =6 =7 =8 =9 =10 =11 =12

Perfect cubes to memorize: $\sqrt[3]{8}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, $\sqrt[3]{125}$, $\sqrt[3]{216}$

=2 =3 =4 =5 =6

Ex 1) Evaluate the following radicals; identify the radicand and index for each

a) $\sqrt{16} = 4$
$(\sqrt{4 \times 4})$

Radicand: 16
Index: 2

b) $\sqrt[3]{64} = 4$
$(= \sqrt[3]{4 \times 4 \times 4})$

Radicand: 64
Index: 3

*** If no index is written, it is a* 2.

Estimating Square Roots

Ex 2) Estimate the value of $\sqrt{20}$ to one decimal place.

Step 1: Find the two perfect squares that your radicand is between

$\sqrt{16}$, $\sqrt{20}$, $\sqrt{25}$

$=4$ $\boxed{=4.?}$ $=5$

Step 2: Find which of the two perfect squares is closest to your radicand; this will determine the decimal point of your root estimation.

20 is 4 away from 16
20 is 5 away from 25
} so slightly closer to $\sqrt{16}$, so decimal slightly closer to 4.

Now evaluate $\sqrt{20}$ on your calculator…how close was the estimate? $\boxed{\text{estimate } 4.4}$

$\sqrt{20} = 4.4721$ … the estimate was close!

☺

Estimating Cube roots

Ex 3) Estimate the value of $\sqrt[3]{16}$ to one decimal place

Step 1: Find the two perfect CUBES that your radicand is between

$\sqrt[3]{8}$, $\sqrt[3]{16}$, $\sqrt[3]{27}$

$=2$ $\boxed{=2.?}$ $=3$

Step 2: Find which of the two perfect cubes is closest to your radicand; this will determine the decimal point of your root estimation.

16 is 8 away from 8 } so closer to $\sqrt[3]{8}$,
16 is 11 away from 27 } so decimal closer to 2.

estimate $\boxed{2.3 \text{ or } 2.4}$

Now evaluate $\sqrt[3]{16}$ on your calculator…how close was the estimate?

$\sqrt[3]{16} = 2.5198\ldots$ estimate was close!

Ex 4) Using a calculator, approximate the irrational numbers to 2 decimal places

a) $\sqrt{4.5}$ $=2.12$ b) $\sqrt{45}$ $=6.71$ c) $\sqrt{450}$ $=21.21$ d) $\sqrt{0.45}$ $=0.67$

Number lines

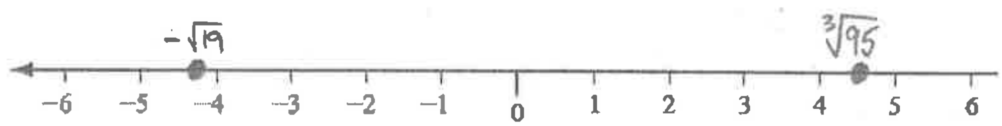
Ex 5) Estimate the position of $-\sqrt{19}$ and $\sqrt[3]{95}$ on the number line

$-\sqrt{16}$, $-\sqrt{19}$, $-\sqrt{25}$

$=-4$ $\boxed{=-4.2}$ $=-5$

$\sqrt[3]{64}$, $\sqrt[3]{95}$, $\sqrt[3]{125}$

$=4$ $\boxed{=4.5}$ $=5$



Ex 6) Without using the root function of your calculator, approximate the number to one decimal place

a) $-\sqrt{62}$

$-\sqrt{49}$, $-\sqrt{62}$, $-\sqrt{64}$

$=-7$ $\boxed{=-7.9}$ $=-8$

since 62 is only 2 away from 64

b) $\sqrt[3]{35}$

$\sqrt[3]{27}$, $\sqrt[3]{35}$, $\sqrt[3]{64}$

$=3$ $\boxed{=3.2}$ $=4$

Since 35 is 8 from 27, and 29 from 64, much closer to 27!

1.5A – Exponential Notation and Fractional Exponent

Learning Target: To relate rational exponents and radicals

Toolkit:
- Taking Square and Cube Roots
- Converting Decimals to Fractions
- Order of Operations

What is an exponent?

An exponent tells us how many time we are multiplying the base by itself

Example: $2^4 = 2 \times 2 \times 2 \times 2$

What about negative signs with the base?

If the negative sign is inside the brackets with the base, we need to include it in our multiplications. Otherwise, leave it in the front.

Example: $(-5)^2$

$= (-5) \times (-5)$

$= 25$

-5^2

$= -5 \times 5$

$= -25$

Exponent Laws

Exponents of 0 and 1
$a^1 = a$ for any number a.
$a^0 = 1$ for any non-zero number a.

Ex. 1) Simplify

a) $5^0 = 1$

b) $5^1 = 5$

Product Rule

$$a^m \times a^n = a^{m+n}$$

Ex. 2) Simplify

a) $4^3 \times 4^4$

$= 4^{3+4}$

$= 4^7$

b) $x^6 \times x^{10}$

$= x^{6+10}$

$= x^{16}$

Quotient Rule

$$\frac{x^m}{x^n} = x^{m-n}$$

Ex. 3) Simplify

a) $\frac{(-4)^5}{(-4)^2}$

$(-4)^{5-2}$

$= (-4)^3$

b) $x^6 \div x$

x^{6-1}

x^5

Power Rule
$(x^m)^n = x^{m \times n}$

Ex 4) Simplify

a) $(3^5)^4$

$3^{5\times4} = 3^{20}$

b) $(x^8)^2$

$x^{8\cdot2} = x^{16}$

A Product to a Power

$$(ab)^n = a^n \times b^n$$

Ex. 5) Simplify

a) $(3x)^3$

$3^3 x^3$

$\boxed{27x^3}$

b) $(2xy)^4$

$2^4 x^4 y^4$

$\boxed{16x^4y^4}$

A Fraction to a Power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Ex. 6) Simplify

a) $\left(\frac{4}{5}\right)^3$

$\frac{4^3}{5^3}$

$= \boxed{\frac{64}{125}}$

b) $\left(\frac{x}{y}\right)^4$

$\frac{x^4}{y^4}$

Fractional Exponents: Rewriting powers in Radical and Exponent form

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number,

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

Ex. 1) Write each power in radical form and evaluate without using a calculator

a) $1000^{\frac{1}{3}}$

1000 is a perfect cube $\rightarrow$ $\sqrt[3]{1000}$

$= 10$

Change to a fraction

b) $25^{0.5}$

$= 25^{\frac{1}{2}}$

$= \sqrt{25}$

$= 5$

c) $(-8)^{\frac{1}{3}}$

$\sqrt[3]{-8}$

(-2)

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$\frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}}$

$\frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{\sqrt[4]{2\cdot2\cdot2\cdot2}}{\sqrt[4]{3\cdot3\cdot3\cdot3}}$

$= \frac{2}{3}$

* Flower Power *

$x^{\frac{m}{n}}$ ← power/flower on top

↖ root on the bottom

$(\sqrt[n]{x})^m$ ← power

↑ root

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

Ex. 2) Write the following in radical form:

a) $26^{\frac{2}{5}}$ ← power ← root

$(\sqrt[5]{26})^2$

b) $25^{\frac{3}{2}} = (\sqrt[2]{25})^3 = (\sqrt{25})^3$

Ex. 3) Write the following in exponent form:

a) $(\sqrt{6})^5$ root, ← power

$6^{\frac{5}{2}}$

if no number written in the root, there is a 2!

b) $(\sqrt[4]{19})^3$ root, → power

$19^{\frac{3}{4}}$

* order of operations*
evaluate inside the bracket first

Ex. 4) Evaluate:

a) $100^{\frac{3}{2}}$

$= (\sqrt{100})^3$

$= 10^3$

$\boxed{1000}$

b) $(-27)^{\frac{4}{3}}$

$= (\sqrt[3]{(-27)})^4$

$= (-3)^4$

$(-3) \times (-3) \times (-3) \times (-3)$

$\boxed{= 81}$

c) $32^{\frac{4}{5}}$

$= (\sqrt[5]{32})^4$

$= (\sqrt[5]{\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{\text{five 2's}}})^4$

$= 2^4$

$\boxed{= 16}$

1.5B – Negative Exponents

Learning Target: To relate rational exponents and radicals

Toolkit:
- Taking Square and Cube Roots
- Converting Decimals to Fractions
- Order of Operations

What is a reciprocal?

Two numbers with a product of 1 are reciprocals.

Ex. 1) Since $4 \times \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Ex. 2) Since $\frac{2}{3} \times \frac{3}{2} = 1$, the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals.

Powers with Negative Exponents

When x is an non-zero number and n is a rational number:

$$x^{-n} = \frac{1}{x^n} \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

Evaluate a power with a negative exponent

Evaluate each power:

Ex. 3)

a) 3^{-2}

$\frac{1}{3^2}$

$= \frac{1}{9}$

b) $(-5)^{-3}$

$\frac{1}{(-5)^3}$

$\frac{1}{-125} = \boxed{\frac{-1}{125}}$

c) $(-\frac{3}{4})^{-3}$

$\left(-\frac{4}{3}\right)^3$

$\frac{(-4)^3}{3^3} = \boxed{\frac{-64}{27}}$

d) $(\frac{10}{3})^{-2}$

$\left(\frac{3}{10}\right)^2$

$\frac{3^2}{10^2} = \boxed{\frac{9}{100}}$

Evaluate a power with a negative rational exponent

To evaluate a power with a negative *rational* (fraction) exponent:

Ex. 4) Evaluate $8^{-\frac{2}{3}}$

$= \frac{1}{8^{\frac{2}{3}}}$ *write with a positive exponent*

$= \frac{1}{(\sqrt[3]{8})^2}$ *re-write into radical form, then work from inside out*

$= \frac{1}{(2)^2}$ *evaluate (write answer with NO exponents)*

$= \frac{1}{4}$

Ex. 5) Evaluate:

change to positive exponent first!

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{3/2}$

$= \frac{16^{3/2}}{9^{3/2}}$

$= \frac{(\sqrt{16})^3}{(\sqrt{9})^3}$

$= \frac{4^3}{3^3}$

$= \boxed{\frac{64}{27}}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}}$

$= \left(\frac{36}{25}\right)^{1/2}$

$= \frac{36^{1/2}}{25^{1/2}}$

$= \frac{\sqrt{36}}{\sqrt{25}}$

$= \boxed{\frac{6}{5}}$

c) $16^{-\frac{5}{4}}$

$\frac{1}{16^{5/4}}$

$= \frac{1}{(\sqrt[4]{16})^5}$

$* \sqrt[4]{16}$

$= \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2}$

$= 2 \, *$

$\frac{1}{2^5}$

$= \boxed{\frac{1}{32}}$

negative on the outside

d) $-25^{-1.5}$

(hint: change 1.5 to a fraction in lowest terms)

$= -25^{-3/2}$

$= -\frac{1}{25^{3/2}}$

$= \frac{-1}{(\sqrt{25})^3}$

$= \frac{-1}{5^3}$

$= \boxed{\frac{-1}{125}}$

1.5C – Simplifying with Exponent Laws

Learning Target: To apply all of the exponent laws to simplify expressions

Toolkit:

- Exponent Laws
- Fractional and negative exponents
- Order of Operations with Fractions

Exponent Laws

For any integers m and n:		
Exponent of 1	$a^1 = a$	$3^1 = 3$
Exponent of 0	$a^0 = 1, \quad a \neq 0$	$(-5)^0 = 1$
Product Rule	$a^m \times a^n = a^{m+n}, \quad a \neq 0$	$2^3 \times 2^4 = 2^{3+4} = 2^7$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$	$\frac{3^5}{3^3} = 3^{5-3} = 3^2$
Power Rules	$(a^m)^n = a^{m \times n}$ $(ab)^n = a^n \times b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$(2^3)^4 = 2^{3\times4} = 2^{12}$ $(2x)^3 = 2^3 \times x^3$ $\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$
Negative Exponents	$a^{-n} = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$	$2^{-3} = \frac{1}{2^3}$ $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$ $\frac{2^{-3}}{3^{-4}} = \frac{3^4}{2^3}$
Rational Exponents	$\sqrt[n]{a} = a^{\frac{1}{n}}$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[3]{5} = 5^{\frac{1}{3}}$ $\sqrt[4]{5^3} = 5^{\frac{3}{4}}$

NOTE: write all powers with **POSITIVE EXPONENTS**

Ex. 1) Simplify by writing as a single power:

a) $6^2 \cdot 6^{-6}$

$= 6^{2+(-6)}$

$= 6^{-4}$

$= \boxed{\frac{1}{6^4}}$

b) $x^{-4} \cdot x^7$

x^{-4+7}

$\boxed{x^3}$

c) $m^7 \div m^{-2}$

$= m^{7-(-2)}$

$= m^{7+2}$

$= \boxed{m^9}$

d) $\frac{0.4^3}{0.4^4}$

$0.4^{3-4} = 0.4^{-1}$

$= \boxed{\frac{1}{0.4}}$

e) $(n^2)^{-4}$ $= n^{2\cdot(-4)}$

$= n^{-8}$

$= \frac{1}{n^8}$

Ex. 2) Simplify by writing as a single power.

a) $\left[\left(-\frac{4}{7}\right)^2\right]^{-3} \div \left[\left(-\frac{4}{7}\right)^4\right]^{-5}$

$= \left(-\frac{4}{7}\right)^{-6} \div \left(-\frac{4}{7}\right)^{-20}$

$= \left(\frac{-4}{7}\right)^{-6-(-20)}$

$= \left(\frac{-4}{7}\right)^{-6+20}$

$= \boxed{\left(\frac{-4}{7}\right)^{14}}$

b) $\frac{(2.3^{-3})^{-5}}{2.3^5}$

$= \frac{2.3^{15}}{2.3^5}$

$= 2.3^{15-5}$

$= \boxed{2.3^{10}}$

c) $\frac{8^{\frac{5}{4}} \cdot 8^{-\frac{1}{4}}}{8^{\frac{3}{4}}}$

$= \frac{8^{\frac{5}{4}+(-\frac{1}{4})}}{8^{3/4}}$

$= \frac{8^{\frac{4}{4}}}{8^{3/4}}$

$8^{\frac{4}{4}-\frac{3}{4}} = \boxed{8^{\frac{1}{4}}}$

Ex. 3) Simplify **(write all powers with POSITIVE exponents).**

a) $(x^4y^{-2})(x^2y^3)$

$= x^{4+2}y^{-2+3}$

$\boxed{= x^6y}$

b) $(27x^6y^9)^{\frac{1}{3}}$

$= 27^{\frac{1}{3}}x^{6\cdot\frac{1}{3}}y^{9\cdot\frac{1}{3}}$

$= 27^{\frac{1}{3}}x^2y^3$

$= \sqrt[3]{27}\,x^2y^3$

$= \boxed{3x^2y^3}$

simplify inside first

c) $\left(\frac{6a^4b^{-3}}{14a^{-2}b^2}\right)^{-2}$

$= \left(\frac{2a^{4-(-2)}b^{-3-2}}{7}\right)^{-2}$

$= \left(\frac{2a^6b^{-5}}{7}\right)^{-2}$

$= \left(\frac{2a^6}{7b^5}\right)^{-2} = \left(\frac{7b^5}{2a^6}\right)^2 = \frac{7^2b^{10}}{2^2a^{12}}$

multiply every exponent by 2

$= \boxed{\frac{49b^{10}}{4a^{12}}}$

d) $\left(\frac{50m^2n^4}{2m^4n^2}\right)^{\frac{1}{2}}$ Simplify inside

$= (25m^{2-4}n^{4-2})^{\frac{1}{2}}$

$= (25m^{-2}n^2)^{\frac{1}{2}}$

$= \left(\frac{25n^2}{m^2}\right)^{\frac{1}{2}} = \frac{25^{\frac{1}{2}}n^1}{m^1}$

$= \frac{\sqrt{25}\,n}{m} = \boxed{\frac{5n}{m}}$

1.6 – Simplifying Radicals

Learning Targets: To express entire radicals as mixed radicals and mixed radicals as entire radicals

Toolkit:
- Understanding Radicals
- Identifying factors of a number

Definitions

1×1 2×2 3×3 4×4 . . .

Perfect Squares – 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 …

1×1×1 2×2×2 3×3×3 . . .

Perfect Cubes – 1, 8, 27, 64, 125, 216 …

What is an entire radical?

A radical sign with a number under it

ex $\sqrt{28}$ $\sqrt[3]{64}$

What is a mixed radical?

A number written as a product of a number and a radical.

ex $3\sqrt{5}$, $4\sqrt[3]{10}$

Equivalent Forms:

Ex. 1)

a) $\sqrt{16 \times 9}$ is equivalent to $\sqrt{16} \times \sqrt{9}$ because:

$= \sqrt{144}$ $= 4 \times 3$

$= 12$ = $= 12$

b) $\sqrt[3]{8 \times 27}$ is equivalent to $\sqrt[3]{8} \times \sqrt[3]{27}$ because:

$\sqrt[3]{216}$ 2×3

$= 6$ = 6

Multiplication Property of Radicals

$$\boxed{\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}, \text{ where n is a natural number and } a \text{ and } b \text{ are real numbers}}$$

We can use this property to simplify square roots and cube roots that are *not* perfect cubes, but have *factors* that are perfect squares or perfect cubes

Simplifying Square Roots	Factors: 1, 2, 3, 4, 6, 8, 12, 24 We can simplify $\sqrt{24}$ because 24 has a perfect square factor of 4. (hint: look at the list of perfect squares!) - Rewrite $\sqrt{24}$ as a product of two factors, with the first one being the perfect square: $\sqrt{24}$ $= \sqrt{4} \times \sqrt{6}$ * Now simplify $\sqrt{4} = 2$ $= 2\sqrt{6}$
Simplifying Cube Roots	We can also simplify $\sqrt[3]{24}$ because 24 has a perfect cube factor of 8. Factors: 1, 2, 3, 4, 6, 8, 12, 24 (hint: look at the list of perfect cubes!) - Rewrite $\sqrt[3]{24}$ as a product of two factors, with the first one being the perfect cube: $\sqrt[3]{24}$ $= \sqrt[3]{8} \cdot \sqrt[3]{3}$ * Now simplify $\sqrt[3]{8} = 2$ $= 2\sqrt[3]{3}$
Tip: If there is MORE than one perfect square or perfect cube factor, choose the LARGEST one!	Ex. 2) Simplify each Radical: (remember your list of perfect squares and perfect cubes!) a) $\sqrt{80}$ perfect square factors: 4, 16; $\sqrt{80}$; $= \sqrt{16} \cdot \sqrt{5}$; $= 4\sqrt{5}$ b) $\sqrt{32}$ perfect square factors: 4, 16; $\sqrt{32}$; $= \sqrt{16} \cdot \sqrt{2}$; $= 4\sqrt{2}$ c) $\sqrt[3]{162}$ perfect cube factors: 27; $\sqrt[3]{162}$; $\sqrt[3]{27} \cdot \sqrt[3]{6}$; $= 3\sqrt[3]{6}$ d) $\sqrt[3]{108}$ perfect cube factors: 27; $\sqrt[3]{108}$; $\sqrt[3]{27} \cdot \sqrt[3]{4}$; $3\sqrt[3]{4}$
How do simplify with radicals if the index is not 2 or 3?	Ex. 3) Simplify $\sqrt[4]{162}$ - Rewrite radical with the prime factorization of 162 - Since $\sqrt[4]{162}$ is a **fourth** root, look for a factor that appears 4 times $\sqrt[4]{162}$ $\sqrt[4]{81 \cdot 2}$ $\sqrt[4]{9 \cdot 9 \cdot 2}$ $\sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 2}$ ← prime factorization ... 3 is written 4 times! $= \sqrt[4]{3^4} \cdot \sqrt[4]{2}$ $= 3\sqrt[4]{2}$

Word Problem

Ex. 4) A cube has a volume of $128cm^3$. Write the edge length of the cube in simplest radical form.

cube has the same length, width and height

$V = l \cdot w \cdot h$

$V = e \cdot e \cdot e$

$V = e^3$

$128 = e^3$ ← cube root both sides

$e = \sqrt[3]{128}$ ← perfect cube factor of 128: 8, 64

$e = \sqrt[3]{64} \cdot \sqrt[3]{2}$

$= 4\sqrt[3]{2}$ cm

The cube has an edge length of $4\sqrt[3]{2}$ cm

How do you write a mixed radical as an entire radical?

Ex. 5) Write the mixed radical $4\sqrt{3}$ as an entire radical:

$4\sqrt{3}$
$= 4 \cdot \sqrt{3}$
$= \sqrt{16} \cdot \sqrt{3}$
$= \sqrt{16 \cdot 3}$
$= \sqrt{48}$

- Use the Multiplication Property of Radicals (re-write 4 as a radical.....think .....$4 = \sqrt{?}$.....$\sqrt{16}$!)
- Combine these under the same radical sign and multiply

(***NOTICE...these are the *opposite* steps to writing an entire radical as a mixed radical)

Ex. 6) Write each as an entire radical:

a) $5\sqrt{2}$	b) $3\sqrt{3}$	c) $3\sqrt[3]{2}$	d) $2\sqrt[3]{6}$
$5 = \sqrt{25}$	$3 = \sqrt{9}$	$3 = \sqrt[3]{27}$	$2 = \sqrt[3]{8}$
$= \sqrt{25} \cdot \sqrt{2}$	$= \sqrt{9} \cdot \sqrt{3}$	$= \sqrt[3]{27} \cdot \sqrt[3]{2}$	$= \sqrt[3]{8} \cdot \sqrt[3]{6}$
$= \sqrt{25 \cdot 2}$	$= \sqrt{9 \cdot 3}$	$= \sqrt[3]{27 \cdot 2}$	$= \sqrt[3]{8 \cdot 6}$
$= \boxed{\sqrt{50}}$	$= \boxed{\sqrt{27}}$	$= \boxed{\sqrt[3]{54}}$	$= \boxed{\sqrt[3]{48}}$

What do you do if the index is higher than 3?

Ex. 7) Write $3\sqrt[5]{2}$ as an entire radical:

$3 = \sqrt[5]{\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{5 threes, since the index is 5.}}} = \sqrt[5]{243}$

Rewrite 3 as $\sqrt[5]{\quad}$.....

Now using the Multiplication property of radicals...

$\sqrt[5]{243} \cdot \sqrt[5]{2}$

$= \sqrt[5]{243 \cdot 2} = \sqrt[5]{486}$