Chapter ~~4~~ 6 – Quadratic Equations Problem Solving

Name(s): KEY

Date: ____________ Per: ____

1) Solve: $-x^2 - 10x - 16 = 0$ by accurately graphing the corresponding function (from vertex form).

$$y = -x^2 - 10x - 16$$

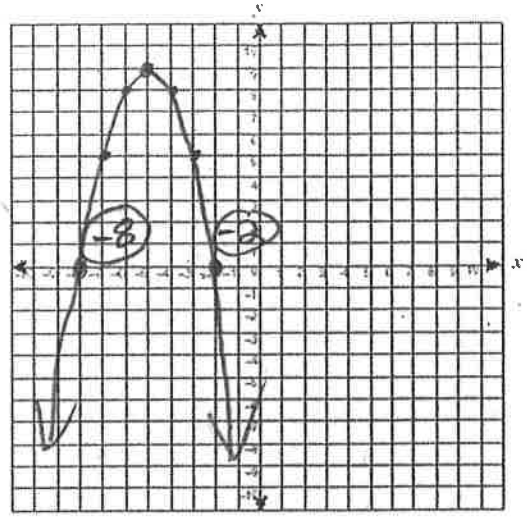
$$y = -(x^2 + 10x + 25 - 25) - 16$$

$b = 10$, $\frac{b}{2} = \frac{10}{2} = 5$ save, $5^2 = 25$ use

$$y = -(x^2 + 10x + 25) + 25 - 16$$

$$y = -(x+5)^2 + 9$$

vertex $(-5, 9)$, opens down, reg. count... find x-ints!



ANSWER to 1): $x = -8, -2$

2) Solve the equation $2x^2 - 5x - 3 = 0$ by

a) factoring.

$a = 2$, so decomp!

$\frac{-6 + 1 = -5}{-6 \times 1 = -6}$ $(2x-3)$

$$2x^2 + 1x - 6x - 3 = 0$$

$$x(2x+1) - 3(2x+1) = 0$$

$$(2x+1)(x-3) = 0$$

$$x = -\frac{1}{2}, 3$$

ANSWER: $x = -\frac{1}{2}, 3$

b) completing the square

$$2x^2 - 5x - 3 = 0$$

$$\frac{\cancel{2}(x^2 - \frac{5}{2}x)}{\cancel{2}} = \frac{3}{2}$$

$b = -\frac{5}{2}$

$\frac{b}{2} = -\frac{5}{4}$ save

$\left(-\frac{5}{4}\right)^2 = \frac{25}{16}$ use

$$\left(x^2 - \frac{5}{2}x + \frac{25}{16}\right) = \frac{3 \times 8}{2 \times 8} + \frac{25}{16}$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{24 + 25}{16}$$

$$\sqrt{\left(x - \frac{5}{4}\right)^2} = \pm\sqrt{\frac{49}{16}}$$

$$x - \frac{5}{4} = \frac{\pm 7}{4}$$

pos

$x - \frac{5}{4} = \frac{7}{4}$

$x = \frac{7}{4} + \frac{5}{4}$

$x = \frac{12}{4}$

$\boxed{x = 3}$

neg

$x - \frac{5}{4} = \frac{-7}{4}$

$x = \frac{-7}{4} + \frac{5}{4}$

$x = -\frac{2}{4}$

$\boxed{x = -\frac{1}{2}}$

ANSWER: $x = -\frac{1}{2}, 3$

c) using the quadratic formula

$$2x^2 - 5x - 3 = 0$$

$a = 2$, $b = -5$, $c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

pos

$x = \frac{5+7}{4}$

$x = \frac{12}{4}$

$\boxed{x = 3}$

neg

$x = \frac{5-7}{4}$

$x = \frac{-2}{4}$

$\boxed{x = -\frac{1}{2}}$

ANSWER: $x = -\frac{1}{2}, 3$

3) Solve $3x^2 - 4x - 8 = 0$ by completing the square. Leave the answer in exact form.

$$3x^2 - 4x - 8 = 0 \quad (+8 \;\; +8)$$

$$3x^2 - 4x = 8$$

$$\frac{\cancel{3}\left(x^2 - \frac{4}{3}x\right)}{\cancel{3}} = \frac{8}{3}$$

$b = \frac{4}{3}$

$\frac{b}{2} = \left(-\frac{2}{3}\right)$ save

$\left(-\frac{2}{3}\right)^2 = \left(\frac{4}{9}\right)$ use

$$\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) = \frac{8^{\times 3}}{3_{\times 3}} + \frac{4}{9}$$

$$\left(x - \frac{2}{3}\right)^2 = \frac{24}{9} + \frac{4}{9}$$

$$\sqrt{\left(x - \frac{2}{3}\right)^2} = \pm\sqrt{\frac{28}{9}}$$

$$x - \frac{2}{3} = \frac{\pm\sqrt{28}}{3} \quad \left(+\frac{2}{3} \;\; +\frac{2}{3}\right)$$

$$x = \frac{2}{3} \pm \frac{\sqrt{4}\sqrt{7}}{3}$$

$$x = \frac{2 \pm 2\sqrt{7}}{3}$$

ANSWER: $x = \frac{2 \pm 2\sqrt{7}}{3}$

4) Write the equations in standard form with roots:

a) -3 and 5

$x = -3$, $x = 5$
$+3 \;\; +3$, $-5 \;\; -5$

$(x+3) = 0$, $(x-5) = 0$

so... $(x+3)(x-5) = 0$

$x^2 - 5x + 3x - 15 = 0$

$x^2 - 2x - 15 = 0$

ANSWER: $x^2 - 2x - 15 = 0$

b) $\frac{3}{2}$ and $-\frac{1}{3}$

$x = \frac{3}{\cancel{2}}$ ($\times 2$), $x = \frac{-1}{\cancel{3}}$ ($\times 3$)

$2x = 3$, $3x = -1$
$-3 \;\; -3$, $+1 \;\; +1$

$(2x-3) = 0$, $(3x+1) = 0$

so... $(2x-3)(3x+1) = 0$

$6x^2 + 2x - 9x - 3 = 0$

$6x^2 - 7x - 3 = 0$

ANSWER: $6x^2 - 7x - 3 = 0$

5) Determine the value of the discriminant and the nature of the roots for $2x^2 - 7x + 3 = 0$.

*(hint: **nature of roots** means HOW MANY roots are there?)*

$a = 2$, $b = -7$, $c = 3$

discriminant $\rightarrow b^2 - 4ac$

$= (-7)^2 - 4(2)(3)$

$= 49 - 24$

$= +25$... which is > 0!

$\therefore$ two possible roots

DISCRIMINANT: $= +25$

NATURE OF ROOTS: two roots

6) Solve by factoring. $4n^2 - 11n + 6 = 0$

$a=4,$

$-8 + -3 = -11$

$-8 \times -3 = \underset{(4)(6)}{24}$

so decomp

$4n^2 - 8n - 3n + 6 = 0$

$4n(n-2) - 3(n-2) = 0$

$(n-2)(4n-3) = 0$

so... $n = 2, \quad n = \frac{3}{4}$

ANSWER: $n = 2, \quad n = \frac{3}{4}$

7) Solve by the **quadratic formula**

$2x^2 + 4x - 5 = 0$

$a = 2, \quad b = 4, \quad c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16 + 40}}{4}$$

$$x = \frac{-4 \pm \sqrt{56}}{4}$$

$$x = \frac{-4 \pm \sqrt{4} \cdot \sqrt{14}}{4}$$

$$x = \frac{-4 \pm 2\sqrt{14}}{4}$$

$$x = \frac{\overset{\div 2}{2}(-2 \pm \sqrt{14})}{4 \div 2}$$

$$x = \frac{-2 \pm \sqrt{14}}{2}$$

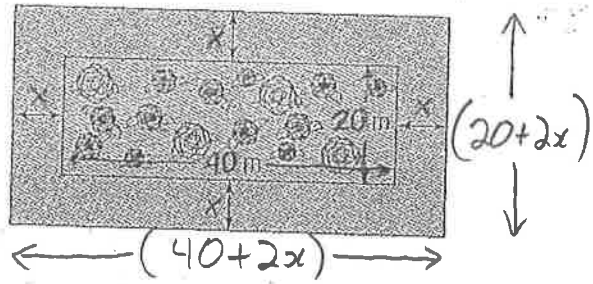
ANSWER:

$x = \frac{-2 \pm \sqrt{14}}{2}$

* turn over

8) Sebastian plans to build a uniform walkway around a rectangular flower bed that is 20m by 40m. There is enough material to make a walkway that has a total area of 700m^2. What is the width of the walkway?

Area of Flower bed $= (20)(40) = 800 m^2$



$A_{total} = A_{Flower\ bed} + A_{walkway}$

$= (800) + (700) = 1500 m^2$

total length $= (40+2x)$ total width $= (20+2x)$

so... $l \cdot w =$ area

$(40+2x)(20+2x) = 1500$

$800 + 80x + 40x + 4x^2 = 1500$ (-800, -800)

$4x^2 + 120x = 700$

$4(x^2 + 30x) = 700$

$(x^2 + 30x) = \frac{700}{4}$

$(x^2 + 30x + 225) = 175 + 225$

$b = 30$, $\frac{b}{2} = 15$ save, $15^2 = 225$ use

$\sqrt{(x+15)^2} = \pm\sqrt{400}$

$(x+15) = \pm 20$

pos: $x + 15 = 20$ — $* \boxed{x = 5}$

neg: $x + 15 = -20$ — $\boxed{x = -35}$ REJECT

SENTENCE ANSWER: The width of the walkway is 5m

9) Lola popped a baseball straight up with an initial upward velocity of 48ft/s. The height, *h*, in feet, of the ball above the ground is modeled by the function $h(t) = 3 + 48t - 16t^2$, where *t* is time, in seconds. How long was the ball in the air if the catcher catches the ball 3ft above the ground?

height!

height, h, = 3

so... $3 + 48t - 16t^2 = h(t)$

$-16t^2 + 48t + 3 = 3$

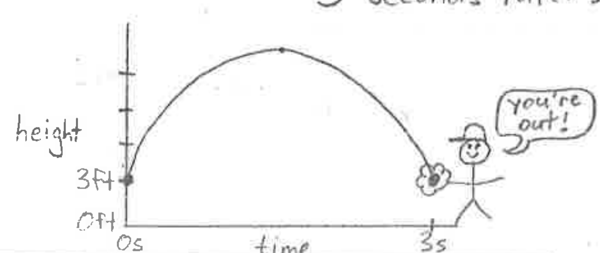
$-16t^2 + 48t = 0$

$-16t(t-3) = 0$

$t = 0$, $t = 3$

$t = 0$... the ball was 3ft. in the air when Lola hit it.

$t = 3$... The ball was 3ft. high when the catcher caught it ... 3 seconds later!



SENTENCE ANSWER: The ball was in the air for 3 seconds