KEY

Financial investing is the process of setting money aside with the intention of receiving a greater amount in return in the future. The money made back is called the **interest**. There are two forms of interest: **simple** and **compound**.

Simple interest: calculated as a percent of an initial investment.

Compound interest: calculated from the initial investment and on any previously earned interest.

Simple Interest:

$$I = P \cdot r \cdot t \qquad A = P + I \quad \text{or} \quad A = P(1 + r \cdot t)$$

where,
- I = interest earned
- P = the principal amount, or the money on which interest is paid
- r = the percent charged for money borrowed, or the interest rate (given yearly)
- t = the time the money is invested (given in years)
- A = the final amount (future)

Ex 1) Find the future amount of an investment of $8000 (P) at simple interest for 5 (t) years at 6%.

$r = 0.06$

method 1

$I = Prt$
$= (8000)(0.06)(5)$
$= 2400$

$A = P + I$
$= 8000 + 2400 = 10400$

Method 2

$A = P(1 + rt)$
$= 8000(1 + (0.06)(5))$
$= 8000(1.3)$
$= \$10400$

Ex 2) Yazia borrowed $5200 (P) at 7.5% ($r = 0.075$) simple interest to build a swimming pool. If she paid $2340 (I) interest, find the term of the loan ($t = ?$) and monthly payments.

$I = Prt$

$2340 = (5200)(0.075)t$

$\frac{2340}{390} = \frac{390t}{390}$

$t = 6$ years

Monthly payments
Need to pay off
$A = 5200 + 2340 = 7540$
in 6 years

$\frac{7540}{6(12)} = \frac{7540}{72} = \$104.73$

total # of payments

from the $

Discount Loans:
Sometimes the interest on a loan is paid up front by deducting the amount of interest the lender gives you, called a discount loan.

t P r=0.08
Ex 3) Nate obtained a 2 year $6000 loan for university. The rate was 8% simple interest and the loan was a discounted loan.

a) Find the discount (interest owed)
b) Find the amount on Monday Nate received ($P - I$)
c) Find the actual interest rate

(a) $I = Prt$

$I = (6000)(0.08)(2)$
$= 960$

(b) $P - I$

$6000 - 960$
$= 5040$
↑
New principal

(c) $960 = (5040)r(2)$

$\frac{960}{10080} = \frac{10080r}{10080}$

$0.09523 = r$
x100
$r = 9.52\%$

Compound Interest
Interest can be compounded more that once a year, such as semi annually, quarterly, monthly or daily. or weekle
2 4 12 365 52

Compound Interest Formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where,
A = final amount
P = principal or starting amount
r = rate of yearly interest
n = number of times yearly interest is compounded per year
t = time, in years

Ex 4) To have savings for university, the parents of a child invest $25000 in a savings plan paying 6%, interest compounded quarterly. How much money will they have in 18 years? P r=0.06 n=4 t

$$A = 25000\left(1 + \frac{0.06}{4}\right)^{4 \cdot 18} = \$73\,028.95$$

P=? t r=0.042
Ex 5) How much would you have to invest into a 10 year bond paying 4.2% compounded weekly to make it worth $5000 at the end of its term?
n=52 A

$$5000 = P\left(1 + \frac{0.042}{52}\right)^{52 \cdot 10}$$

$$\frac{5000}{1.52170} = \frac{P(1.52170)}{1.52170} \rightarrow P = \$3285.80$$

Effective Interest Rates

The effective rate is the **simple interest rate** that is equivalent to what the **compound interest rate** would pay out over a given time.

Effective Interest Rate

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

where,
E = effective rate
n = number of periods per year the interest is calculated
r = interest rate per year

Ex 1) Find the effective interest rate when the started rate is 6% compounded quarterly.

$r = 0.06$
$n = 4$

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$
$$= \left(1 + \frac{0.06}{4}\right)^4 - 1 = 0.06136$$
$$= 6.14\%$$
effective rate

6% is called the nominal rate.

Ex 2) Which savings account is a better investment: (a) 5.25% compounded daily or (b) 5.3% compounded semi-annually?

Note: the higher the effective rate, the better the investment

(a) $n = 365$
$r = 0.0525$

$$E = \left(1 + \frac{0.0525}{365}\right)^{365} - 1 = 0.0539 = 5.39\%$$
↑ more

(b) $n = 2$
$r = 0.053$

$$E = \left(1 + \frac{0.053}{2}\right)^2 - 1 = 0.0537 = 5.37\%$$

5.25% is better

Annuities

An **annuity** is a savings plan where the investor makes a regular fixed payment into a compound interest account and the interest does not change for the length of the investment. Instead of depositing one large sum at the beginning of the term, the principal amount is broken up into smaller payments.

Future Amount of an Annuity

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{n \cdot t} - 1\right]}{\frac{r}{n}}$$

where,

F = the future amount of the annuity
r = annual interest rate
t = term of the annuity in years
R = the regular periodic payment
n = the number of payments per year

R r=0.05

Ex 3) Find the future amount of an annuity with quarterly payments of \$600 at 5% compounded quarterly for 10 years.

n=4 t=10

$$F = \frac{600\left[\left(1+\frac{0.05}{4}\right)^{4 \cdot 10} - 1\right]}{\frac{0.05}{4}} = \$30\,893.73$$

os t

Ex 4) Suppose you want to save up to purchase a new car in 5 years. What amount must you save semi-monthly if the car will cost \$50000 and interest is calculated semi-monthly at 6%? n=24 F

r=0.06

What is R?

$$50000 = \frac{R\left[\left(1+\frac{0.06}{24}\right)^{24 \cdot 5} - 1\right]}{\frac{0.06}{24}}$$

$$\frac{50000}{139.74} = \frac{R(139.74)}{139.74}$$

$$R = 357.80$$

Stocks

A much quicker (yet riskier) way to have your money grow in value is in the stock market. If you buy stocks or shares in a company, you become a **shareholder**. As a shareholder, you are part owners of the company, and receive part of the company's profit in the form of dividends. The size of the dividend is based on the amount of the profit the company makes.

Stocks are bought and sold on a **stock exchange**. The price of stocks chance from day to day depending on the profitability of the company. Investors buy and sell stocks through a **stock broker.** The set of all stocks a person owns is called a portfolio.

P/E Ratio

The price/earnings (p/e) ratio is the most common measure of how expensive a stock is. It is the ratio of the current selling price of a stock, to the company's earnings per share.

P/E Ratio

$$\text{P/E Ratio} = \frac{\text{yesterday's closing price}}{\text{annual earning per share}}$$

The higher the p/e ratio, the riskier the investment, since a high p/e signifies high expectations.

Yield for a Stock

The yield is the income return on an investment. The yield is expressed as a annual percentage rate based on the investment cost, current market value or face value. The higher the risk, the higher the yield potential.

Yield for a Stock

$$\text{Yield for a Stock} = \frac{\text{annual dividend per share}}{\text{closing price of stock}}$$

Ex 1) The stock listing for Microsoft Corp is shown below.

Stock	High	Low	Div	YLD%	P/E	Vol	Close	Net CHG
MSFT	64.10	63.75	1.44	2.26	30.40	28.37 M	63.84	+0.22

a) What was the highest price of the stock in the past 52 weeks?
b) What was the lowest price of the stock in the past 52 weeks?
c) What was the amount of dividends paid per share in the last year?
d) If you have 200 shares, how much did you make in dividends last year?
e) How many shares were traded yesterday?
f) What was the closing price per share the day before yesterday?
g) If the annual earnings per share for Microsoft is $2.02, find the P/E ratio at yesterday's closing pr
h) What is Microsoft's current stock yield?

(a) 64.10

(b) 63.75

(c) 1.44

(d) 1.44x200 = $288

(e) 28.37 mil

(f) Closing price today 63.84, went up 0.22 from yesterday

so 63.84 - 0.22 = 63.62

(g) $\frac{63.84}{2.02} = 31.60$

(h) 1.44 ÷ 63.84 = 2.26%

Buying and Selling Stocks

Ex 2) Marie purchased 150 shares for $126.20 and later sold all of her shares for $140.30. If the commission rate was 1.6%, find Marie's profit/loss on the transaction.

Purchase price = 150 x 126.20 = 18930.00
commission = 189300 x 0.016 (+) 302.88
Total cost = 19232.88

selling price = 150 x 140.30 = 21045.00
commission = 21045.00 x 0.016 (−) 336.72
Total cost = 20708.28

profit
20708.20
− 19232.88
= 1475.40

Ex 3) An investor bought a stock at $24 per share. Her quarterly dividend was 42 cents per share. She sold the stock after 3 years at $29.50 per share. What was her total gain per share, excluding commission?

Div 0.42 x 4 x 3 = 5.04

Share increase 29.50 − 24.00 = 5.50

Gain 504 + 5.50 = 10.54 per share.

Bonds

Bonds represent a borrower's promise to make certain payments of interest and repayment of the principal. Bonds can be bought and sold through banks, bond exchanges, and other financial institutions.

current yeild

$$\frac{\text{annual interest}}{\text{market price}}$$

Ex 4)

interest

Bond Exchange Quotations per $100 Par Value

	Bond			Current Yield	Vol	High	Low	Close	Net Change
A	6.75	S	2230	6.7	22	120.4	102.2	100.2	+0.10
B	7.00	S	2025	8.9	38	78	78.5	78.3	+0.60
C	5.20	S	2028	5.5	8	95	94	94.5	−0.20
D	6.50	S	2023	7.3	40	89	88.5	88.5	−0.75
E	8.80	S	2020	-----	250	96	95.5	96	+0.25

10 × 100

a) Find the market price for 5 of $1000 bond A at the day's closing price.
b) How much bond interest will be paid semi-monthly to an investor who owns eight $1000 bond B?
50 × 100
c) Find the discount on a $5000 bond C at the day's low price.
d) Find the current yield for Bond E.

(a) $100.2 \times 10 \times 5 = \$5010$

10 shares of 100

(b) $\frac{0.07}{2} \times 1000 = \$35$

0.08 × 100

(c) $94 \times 50 = 4700$

$5000 - 4700 = 300$

(d) $\frac{8.80}{96} = 0.092 = 9.2\%$

Ex 5) Ten years before maturity a $5000, 8% bond is quoted at 96.5. Find the rate of the current yield.

Annual interest $= 5000 \times 0.08 = 400$

market price $= 50 \times 96.5 = 4825$

current yeild $= \frac{400}{4825} = 0.0829 = 8.29\%$

Ex 6) Adam buys a bond with a face value of $1000 issued 18 months ago. The maturity date is 5 years from the issued date at a simple interest rate of 6%. If he paid $950 for the bond and kept it until the maturity date, what is his profit? What percent return does he get per year?

6% simple interest

$$A = P(1 + rt)$$

$$= 1000(1 + (0.06)(5))$$

$$= 1300$$

profit = Final amount − P

$= 1300 - 960$

$= \$350$

Return $= \frac{\text{profit}}{}$

When some people buy a car, furniture, tv, etc they may need to take out a loan and make payments. This is called **installment buying**.

Loan Terminology

Down Payment: a percentage of the item's cost pay upfront.
Fixed Installment Loan: a loan where equal, usually monthly payments are made.
Amount Financed: the amount that has been borrowed (price of the item – down payment)
Installment Price: the total price of the item, including interest (sum of all payments + down payment)
Finance Charge: the interest charged for not paying immediately (installment price – cost of the item)

Ex 1) Rachel bought a new car with a retail price of \$35000. Her down payment was \$3000 and she had to pay \$838.14 per month for 48 months. Calculate her total cost for the car, including interest.

Amount financed: 35000 − 3000 = 32000 actual cost

installment payments: 838.14 × 48 = \$40038.72

Total cost: 3000 + 40038.72 = 43038.72 cost + interest

Ex 2) New appliances for a house cost \$15000. The full cost was financed over three years at 9% simple interest per year.

a) Find the finance charge
b) Find the installment price
c) Find the monthly payment

3 years → monthly payments
3 × 12 = 36 payments

(a) $I = Prt$
$= (15000)(0.09)(3) = \$4050$

(b) 15000 + 4050 = 19050

(c) 19050 ÷ 36 = \$529.17

Credit Cards

Credit cards charge interest on any unpaid balance each month. The rate is much higher than bank rates, so paying off credit cards regularly is encouraged.

Ex 3) On March 1, James had an unpaid balance of \$1350.50 on his credit card. He made purchases of \$1200.60 over Spring Break and made a payment of \$500 on the balance. The monthly interest on the unpaid balance was 7%. Find the finance charges, and the new balance on April 1.

New balance: 1350.00 + 1200.60 − 500 = \$2050.60

Interest: \$2050.60 × 0.07 = \$143.54 ← Finance charge

New Balance: \$2050.60 + 143.54 = \$2194.14

8.5 – Canadian Mortgages

A mortgage is a particular type of loan exclusively used for real estate. The bank or other financial institutions give these loans. Over a period of many years, the borrower repays the loan, plus interest until they eventually own the property free and clear.

Because these loans are usually very large, banks use the property as collateral. If the borrower can no longer make the required payments, the bank can evict the home's tenants and sell the house for them to clear the debt.

Calculating Canadian Mortgages

Mortgages are installment loans. To find the monthly payment for a mortgage:

1. Find the down payment amount.
2. Subtract the down payment from the total cost.
3. Use the yearly formula to get the month payment

$$E = \frac{L \cdot r(1+r)^n}{(1+r)^n - 1}$$

L $r = 0.05$ $n = 30$

Ex 1) Find the yearly installment payment on \$100000 at 5% interest compounded yearly over 30 years.

$$E = \frac{(100000)(0.05)(1+0.05)^{30}}{(1+0.05)^{30} - 1} = \frac{21609.91188}{3.321942} = \$6505.20$$

Canadian Monthly Mortgage Payment Formula

$$E = \frac{L \cdot i(1+i)^{12n}}{(1+i)^{12n} - 1} \quad \text{with } i = \left(1 + \frac{r}{2}\right)^{\frac{1}{6}} - 1$$

E = payment, L = loan amount, r = annual interest rate, n = time in years
i = effective interest rate for monthly payment

Note: The annual interest rate needs to be converted to a effective interest rate for monthly payments before calculating the payment. Round i to at least 6 decimal places for accuracy.

L

Ex 2) What is the monthly Canadian mortgage payment on a \$500000 mortgage at 5% compounded monthly over 25 years? $n = 25$

$r = 0.05$

$$i = \left(1 + \frac{0.05}{2}\right)^{\frac{1}{6}} - 1$$

$= 0.0041239$

$i = 0.412392\%$

$$E = \frac{(500000)(0.0041239)(1+0.0041239)^{12 \cdot 25}}{[(1+0.00412392)^{12 \cdot 25} - 1]}$$

$= \$2908.03$

monthly mortgage payment

Ex 3) A house sells for \$650000 and a 15% down payment is made. A mortgage was secured at 6% annual for 20 years.

a) find the down payment
b) find the amount of the mortgage
c) find the monthly payment
d) find the total cost of the mortgage
e) find the total interest paid
f) find the total cost of the house.

(a) 650000×0.15
$= \$97,500$
down payment

(b) $650000 - 97500$
$= \$552500$
Mortgage

(c) convert 6% annual to a monthly rate:

$$i = \left(1+\frac{r}{2}\right)^{\frac{1}{6}} - 1 = \left(1+\frac{0.06}{2}\right)^{\frac{1}{6}} - 1 = 0.00493862$$

$i =$ 0.00493862

$$E = \frac{L \cdot i(1+i)^{12n}}{(1+i)^{12n} - 1} = \frac{552500 \times 0.00493862 \times (1.00493862)^{240}}{(1.00493862)^{240} - 1}$$

$$= \frac{8900.751389}{2.262036} = \boxed{\$3934.84}$$

monthly payment

d) Total cost of mortgage = monthly payment × number of payments $(12 \times 20 = 240)$
$= \$3934.84 \times 240$
$\boxed{= \$944361.69}$

e) Total interest paid = mortgage payments − mortgage
$= 944361.69 - 552500 = \boxed{\$391861.69}$

f) Total cost of house = down payment + total cost of mortgage
$= 97500 + 944361.69$
$\boxed{= \$1041861.69}$