4.1 Patterns

Can you predict the next number in the pattern 5, 9, 13, 17, ...? If you said 21, you are right. What if you were asked to find the 100th number? This would be more difficult. In this section, we will develop a linear equation to represent all values of the pattern.

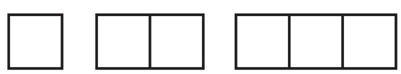
To find the 100th number of the pattern above, a general equation is needed.

The common difference is 4.

1st number: $5 = 1 + 4(1)$
2nd number: $9 = 1 + 4(2)$
3rd number: $13 = 1 + 4(3)$
4th number: $17 = 1 + 4(4)$

So the 100th number is $1 + 4(100) = 401$. A general equation is $t = 1 + 4n$, where n is the number of the term, and t is the term itself.

Example 1 Determine the perimeter of the 10th set of blocks.


...

► *Solution*: The perimeters are 4, 6, 8, ... with a common difference of 2.

Set 1: Perimeter $= 2 + 2(1) = 4$
Set 2: Perimeter $= 2 + 2(2) = 6$
Set 3: Perimeter $= 2 + 2(3) = 8$
Set 4: Perimeter $= 2 + 2(4) = 10$

let P = perimeter and n = number of squares (or length of rectangle)

The general equation is $P_n = 2 + 2n$

Therefore the 10th set of blocks has a perimeter of $P_{10} = 2 + 2(10) = 22$.

Note: It is possible for a sequence to have a common difference that is negative.
For example, the sequence: 13, 8, 3, − 2... has a common difference of 8 − 13 = − 5.

Example 2 Write an equation relating t to n.

n	0	1	2	3	4
t	4	6	8	10	12

► *Solution*: As n goes up by 1 unit, t goes up by 2 units.

Therefore $t = 2n + b$, with b an unknown value.

If $n = 0$, then $t = 2(0) + b$
$= b$

Therefore b must be 4, and $t = 2n + 4$.

Example 3 Find the 100th term of the pattern: 7, 4, 1, − 2, − 5, ...

► *Solution*: This pattern is decreasing by 3.

Therefore $t = -3n + b$, with b an unknown value.

If the first term is $n = 1$, with $t = 7$, then $7 = -3(1) + b \rightarrow b = 10$

Therefore $t = -3n + 10$.

The 100th term can be found by substituting 100 for n: $t = -3(100) + 10$
$= -300 + 10$
$= -290$

The 100th term is -290.

Example 4 Rent-A-Wreck rents a car for $30.00 per day, plus 20¢ per kilometre driven.

a) Write an equation relating cost to kilometres driven per day.

b) What is the cost if the car was driven 120 kilometres for one day.

c) If $57.40 was charged to a customer for a one day rental, how many kilometres were driven?

► *Solution*: **a)** $C = 0.20n + 30$

b) $C = 0.20(120) + 30$
$= 54$

A customer would be charged $54.00.

c) $54.70 = 0.20n + 30$
$0.20n = 57.40 - 30$
$0.20n = 27.40$
$n = 137$

The customer drove 137 kilometres.

4.1 Exercise Set

1. In the equation $A = 3p$, determine A when p is:

a) 3 ________ **b)** 5 ________

c) 11 ________ **d)** 17 ________

e) -6 ________ **f)** $x + 2$ ________

2. In the equation $S = 4n - 2$, determine S when n is:

a) 4 ________ **b)** 9 ________

c) 18 ________ **d)** 23 ________

e) 41 ________ **f)** $x - 1$ ________

3. Determine the common difference in each linear pattern.

a) 2, 5, 8, 11, … ________ **b)** 6, 10, 14, 18 … ________

c) $-4, -1, 2, 5, \dots$ ________ **d)** 15, 10, 5, 0, … ________

e) $-5, -11, -17, -23, \dots$ ________ **f)** $-8, -3, 2, 7, \dots$ ________

g) $\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2},\ 4\sqrt{2}, \dots$ ________ **h)** $\sqrt{2}+\sqrt{3},\ \sqrt{2},\ \sqrt{2}-\sqrt{3}, \dots$ ________

4. Determine the next three numbers in each linear pattern.

a) 2, 5, 8, 11, … _____ , _____ , _____ **b)** 6, 10, 14, 18, … _____ , _____ , _____

c) $-4, -1, 2, 5, \dots$ _____ , _____ , _____ **d)** 15, 10, 5, 0, … _____ , _____ , _____

e) $-5, -11, -17, -23, \dots$ _____ , _____ , _____ **f)** $-8, -3, 2, 7, \dots$ _____ , _____ , _____

g) 11, 8, 5, 2, … _____ , _____ , _____ **h)** $-4.1, -3.7, -3.3, \dots$ _____ , _____ , _____

5. Write an equation relating t to n. (It must work for every pair of values)

a)

n	1	2	3	4
t	2	5	8	11

b)

n	1	2	3	4
t	6	10	14	18

c)

n	1	2	3	4
t	− 4	− 1	2	5

d)

n	1	2	3	4
t	15	10	5	0

e)

n	1	2	3	4
t	− 5	− 11	− 17	− 23

f)

n	1	2	3	4
t	1	1.5	2	2.5

6. Determine the 50th term of the following linear pattern.

a) 2, 5, 8, 11, …

b) 6, 10, 14, 18, …

c) − 4, − 1, 2, 5, …

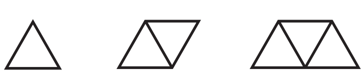
d) 15, 10, 5, 0, …

e) −5, − 11, − 17, −23, …

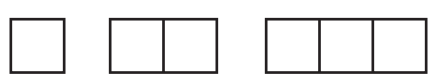
f) 0, $\frac{1}{2}$, 1, $\frac{3}{2}$, …

7. In the following patterns, determine the number of sides needed to produce 10 polygons.

a)

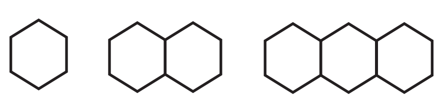


b)



c)

d)



8. The total cost of publishing a school yearbook is a fixed price, plus a cost for each yearbook printed.

C	300	500	700	900
t	0	100	200	300

a) Using the data provided, write an equation relating the total cost (C) to the number of year books printed (n).

b) Calculate the cost of printing 650 yearbooks.

9. The cost of renting a car is $50, plus 20¢ per kilometer travelled.

a) Write an equation relating cost (C) to the number (n) of kilometers travelled.

b) Calculate the cost of travelling 480 kilometers.

10. The cost of sending the school's basketball team to a tournament is a fixed entrance fee, plus a cost per student. If 10 players cost $260, and 15 players cost $300

a) What is the cost per student?

b) What is the entrance fee to the tournament?

11. In 1994, 45% of students graduated from university in less than 5 years. In 2000, 41% of students graduated from university in less than 5 years.

a) Write an equation relating the year of graduation to the number of years spent in university.

b) If this linear trend continues, what percent of university students will graduate in less than 5 years at the end of 2012?

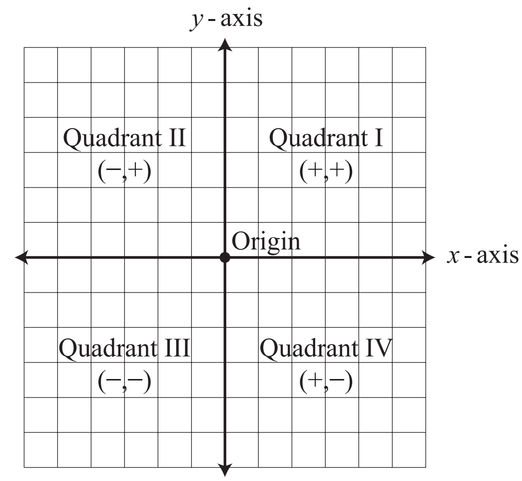
12. The sequence of numbers 0, 1, 1, 2, 3, 5, 8, 13, . . . is known as the Fibonacci sequence. Determine the next five numbers of the Fibonacci sequence.

4.2 Linear Systems

Many applications of Mathematics involve equations with two or more variables. This section will cover how to graph equations of two variables.

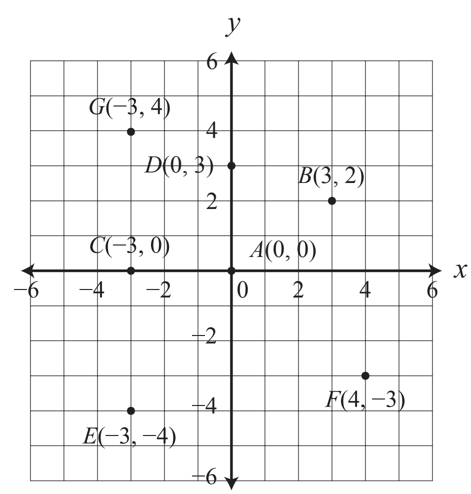
Coordinate System

Just as you can represent a real number by points on the real number line, you can represent an **ordered pair** by points in a plane called a rectangular coordinate system.



To each ordered pair (x, y) there is a unique point in the plane.

- The ordered pair ***A* (0, 0)** is located at the origin.
- The ordered pair ***B* (3, 2)** is located three units to the right, and two units up, from the origin.
- The ordered pair ***C* (− 3, 0)** is located three units to the left of the origin, on the x-axis.
- The ordered pair ***D* (0, 3)** is located three units up from the origin, on the y-axis.
- The ordered pair ***E* (− 3, − 4)** is located three units to the left, and four units down, from the origin.
- The ordered pair ***F* (4, − 3)** is located four units to the right, and three units down, from the origin.
- The ordered pair ***G* (− 3, 4)** is located three units to the left, and four units up, from the origin.



Ordered pairs **(4, − 3)** and **(− 3, 4)** plot different points. That is why they are called ordered pairs, because it makes a difference which number comes first.

Linear Equations

A linear equation means the equation of a straight line.

Slope Intercept Form

$y = mx + b$

$m = \text{slope} = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$, $b = y$-intercept

Graphing a Linear Equation

Method 1:

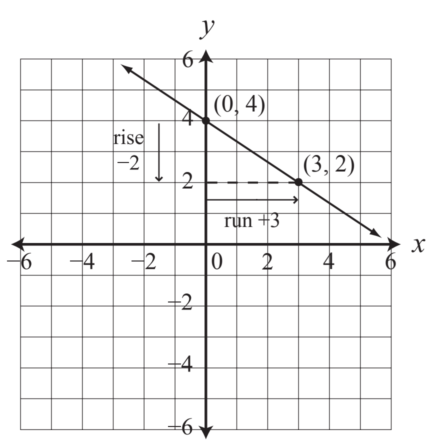
Step 1: Write the equation in form $y = mx + b$.
Step 2: Plot the y-intercept, or any known point.
Step 3: Travel up if slope is positive, or down if slope is negative, by the distance given by the rise. Then travel right a distance given by the run. Mark the new point. Draw a line connecting the new point to the other known point; extend it in both directions, and add arrows to show it goes forever.

Method 2:

Step 1: Write the equation in form $y = mx + b$.
Step 2: Let x = three values that are divisible by the 'run' or denominator of the slope.
Step 3: Solve for y.
Step 4: Plot three points from step 2 and draw a line through them.

Example 1 Graph $y = -\frac{2}{3}x + 4$.

► *Solution*: Method 1: Plot the y-intercept of 4, go down 2, and right 3 units. Mark the new point (3, 2). Draw a straight line through the y-intercept point (0, 4) and the new point (3, 2).



Method 2: Pick three points divisible by 3:

x	y
0	
3	
−3	

Solve for three missing y values:

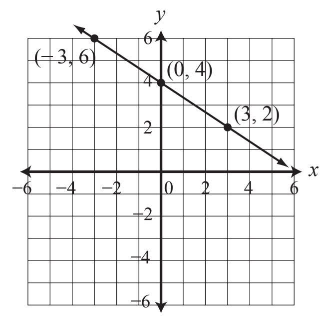
$y = -\frac{2}{3}x + 4$ $y = -\frac{2}{3}x + 4$ $y = -\frac{2}{3}x + 4$

$y = -\frac{2}{3}(0) + 4$ $y = -\frac{2}{3}(3) + 4$ $y = -\frac{2}{3}(-3) + 4$

$y = 4$ $y = 2$ $y = 6$

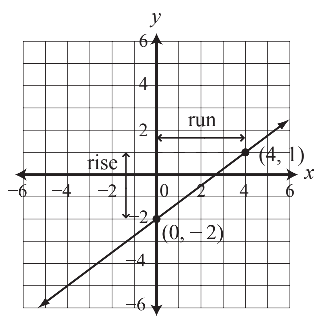
Plot the three points (0, 4), (3, 2) and (− 3, 6) and draw a line through the points.

x	y
0	4
3	2
−3	6



Example 2 Graph $y = \frac{3}{4}x - 2$.

► *Solution*: Method 1: Plot the y-intercept of − 2, go up 3 units then right 4 units. Mark the new point (4, 1). Draw a straight line through the y-intercept point (0, − 2), and the new point (4, 1).



Method 2: Pick three points divisible by 3:

x	y
−4	
0	
−4	

Solve for three missing y values:

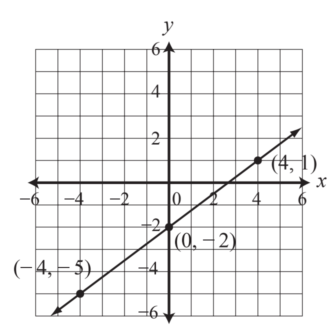
$y = \frac{3}{4}x - 2$ $y = \frac{3}{4}x - 2$ $y = \frac{3}{4}x - 2$

$y = \frac{3}{4}(-4) - 2$ $y = \frac{3}{4}(0) - 2$ $y = \frac{3}{4}(4) - 2$

$y = -5$ $y = -2$ $y = 1$

Plot the three points (− 4, − 5), (0, − 2) and (4, 1) and draw a line through the points.

x	y
−4	−5
0	−2
−4	1



4.2 Exercise Set

1. Fill in the blanks using $<$, or $>$.

a) If a point (x, y) is in quadrant I, then x ___ 0, and y ___ 0.

b) If a point (x, y) is in quadrant II, then x ___ 0, and y ___ 0.

c) If a point (x, y) is in quadrant III, then x ___ 0, and y ___ 0.

d) If a point (x, y) is in quadrant IV, then x ___ 0, and y ___ 0.

2. Fill in the blanks.

a) If $xy > 0$, then the point (x, y) is either in quadrant _______ or quadrant _______ .

b) If $xy < 0$, then the point (x, y) is either in quadrant _______ or quadrant _______ .

3. Without plotting, determine which quadrant the points are found in.

a) $(4, -2)$ ________ **b)** $(6, 3)$ ________

c) $(-1, 3)$ ________ **d)** $\left(-2, -\frac{3}{2}\right)$ ________

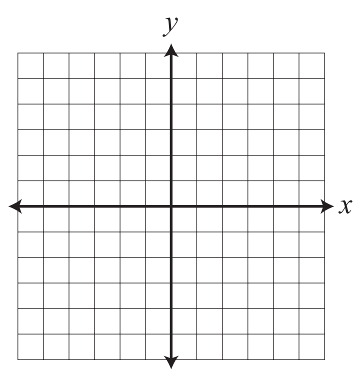
4. Plot the points on the grid provided.

A $(-3, 1)$ B $(-4, -2)$

C $(-5, 0)$ D $(0, 2)$

E $(3, -5)$ F $(4, 3)$

G $(4, 0)$ H $(0, -4)$



5. Find the coordinates of each point.

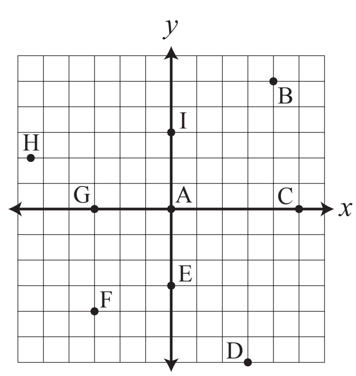
A (,) B (,)

C (,) D (,)

E (,) F (,)

G (,) H (,)

I (,)



6. Determine whether the given ordered pair is a solution of the equation.

a) $(2, 7);\ y = 3x + 1$ y / n

b) $(-2, -1);\ y = -2x + 3$ y / n

c) $(6, 5);\ y = \frac{2}{3}x + 1$ y / n

d) $(-8, 1);\ y = -\frac{3}{4}x - 5$ y / n

e) $(3, 0);\ 2x - 3y = 6$ y / n

f) $(3, 0);\ 2x + 3y = 6$ y / n

7. Determine the missing ordered pair values for the given equations.

a) $y = -\frac{3}{4}x + 2$

x	y
0	
	0
4	

b) $y = 3x - 6$

x	y
0	
	0
	6

c) $y = -\frac{3}{4}x - \frac{5}{2}$

x	y
0	
	0
−6	

d) $y = \frac{8}{3}x - 8$

x	y
0	
	0
	4

e) $y = -2x$

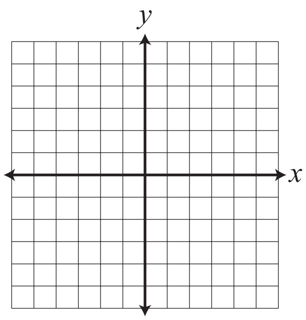
x	y
0	
	0
−3	

f) $y = -2$

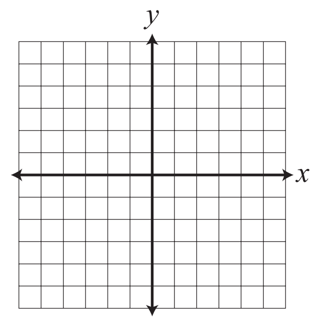
x	y
−3	
0	
4	

8. Graph the equation and identify the y-intercept

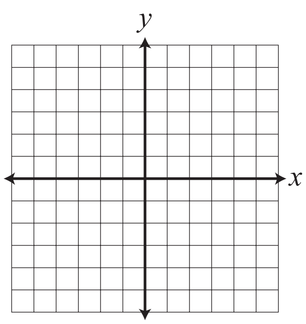
a) $y = x + 1$



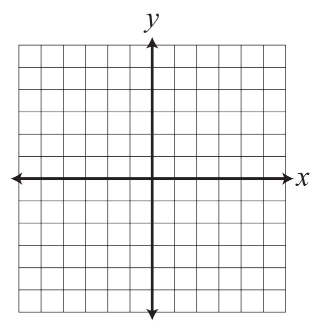
b) $y = 3x - 2$



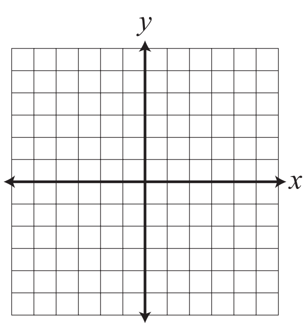
c) $y = -\,2x + 1$



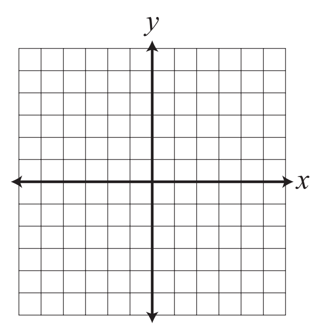
d) $y = -\frac{5}{3}x + 2$



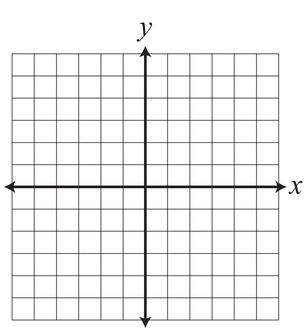
e) $y = \frac{1}{3}x + 2$



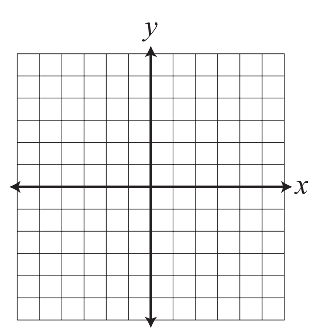
f) $y = \frac{1}{2}x - 1$



g) $y = \frac{3}{2}x + 1$

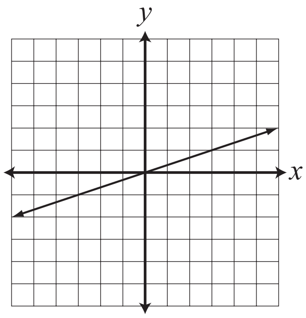


h) $y = -\frac{1}{2}(x - 4)$

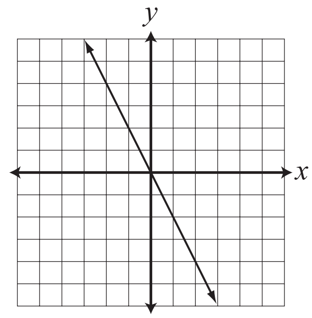


9. Write an equation for the graph.

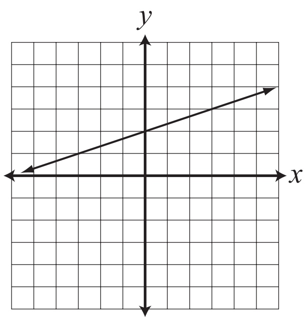
a)



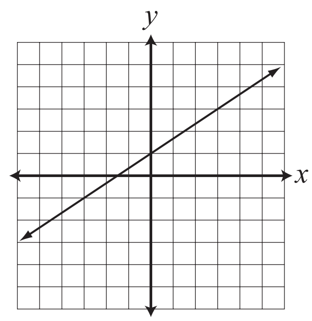
b)



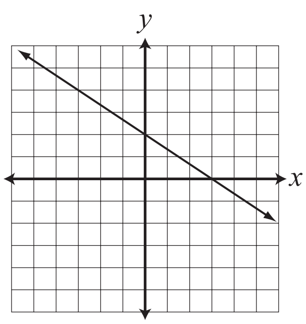
c)



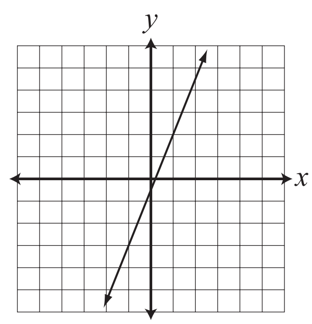
d)



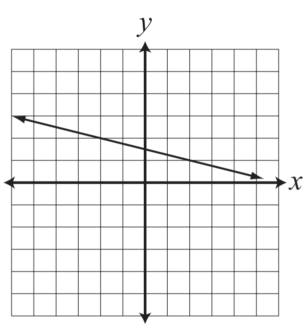
e)



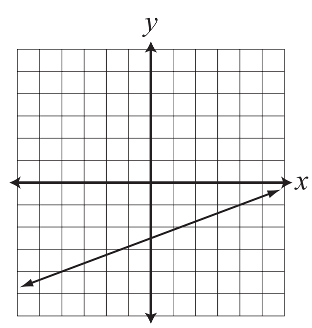
f)



g)



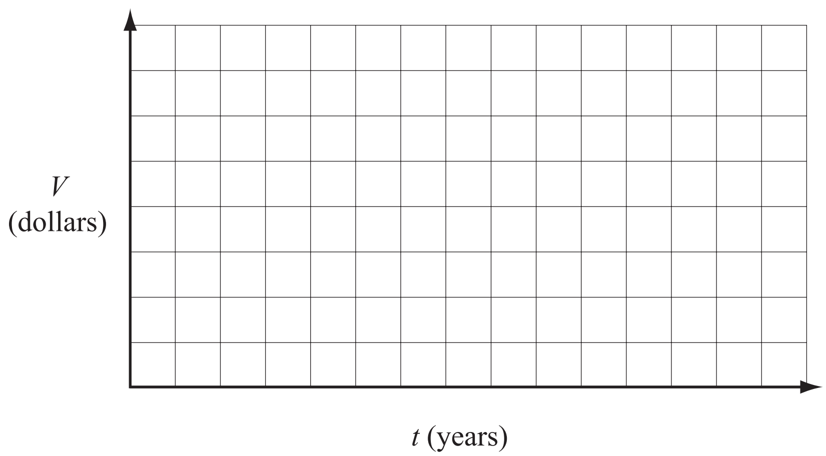
h)



10. The value (V) in dollars of a stereo bought is given by the equation $V = -100t + 600$ where t is the number of years since first buying the stereo.

a) Find the value of the stereo after zero, two and four years.

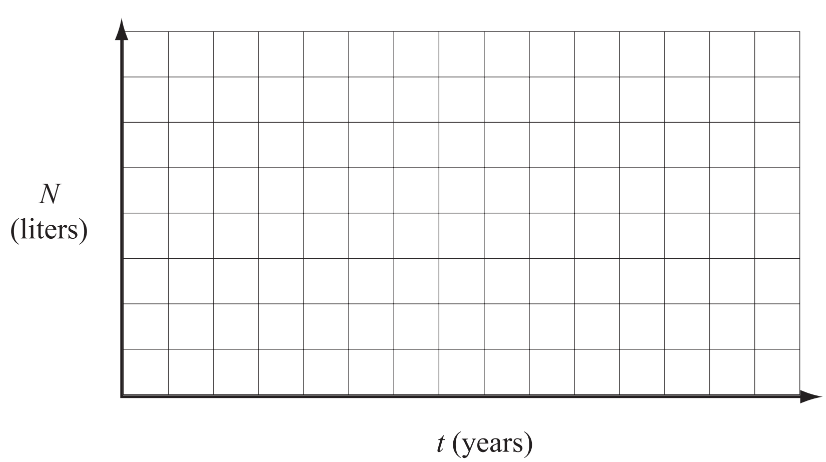
b) Graph the equation and then use the graph to estimate the value of the stereo after $3\frac{1}{2}$ years.



11. The number of liters (N) of soft drinks consumed each year by the average Canadian teenager is approximated by the equation $N = 0.4t + 20$, where t is the age.

a) Find the number of liters of soft drinks consumed at age 13, 14 and 15.

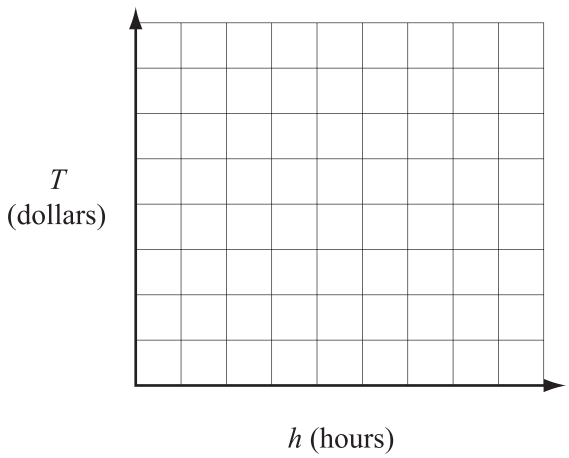
b) Graph the equation and use the graph to estimate the amount of soft drinks consumed by a 19 year old.



12. The cost (T) in dollars of dues and fees at a health spa is $T = 80h + 100$, where h is the number of hours spent with a personal trainer.

a) Find the cost for a member who uses two, four or eight hours with a personal trainer.

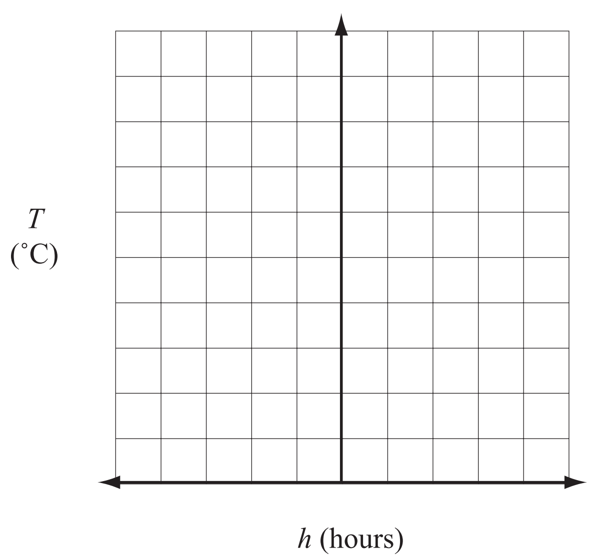
b) Graph the equation and then use the graph to estimate the cost of dues and fees for six hours of a personal trainer.



13. The temperature in degrees celsius (T) in Victoria, BC on July 1, 2008 could be approximated by $T = -1.2h + 20$, where h is the number of hours since 5:00 pm.

a) Find the temperature at 3:00 pm, 7:00 pm and 9:00 pm.

b) Graph the equation and use the graph to estimate the temperature at 1:00 pm.



4.3 Graphing Equations in the Form Ax + By = C

In the previous section, we graphed equations in the y-intercept form $y = mx + b$, where m was the slope of the linear equation and b was the y-intercept. In this section, we will graph linear equations in **standard form** $Ax + By = C$.

Graphing a Linear Equation in Standard Form ($Ax + By = C$)

Step 1: To find the y-intercept, set $x = 0$.
To find the x-intercept, set $y = 0$.

Step 2: Then pick any value for x, and solve for y to get a third point.

Step 3: Plot three points from steps 1 and 2, and draw a straight line through them.

Note: The y-intercept is where the graph crosses the y-axis. (When x = 0)

Example 1 Graph $3x + 2y = 6$.

► *Solution*: Three values picked:

x	y
0	
	0
−2	

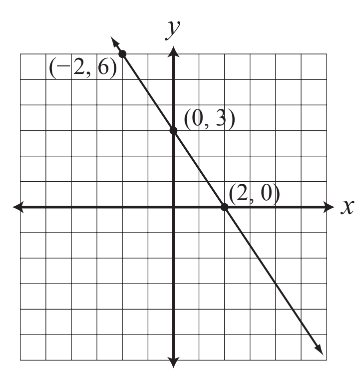
Solving for the three missing values:

$$3x + 2y = 6 \qquad 3x + 2y = 6 \qquad 3x + 2y = 6$$
$$3(0) + 2y = 6 \qquad 3x + 2(0) = 6 \qquad 3(-2) + 2y = 6$$
$$y = 3 \qquad x = 2 \qquad 2y = 12$$
$$y = 6$$

Therefore ordered pairs are:

x	y
0	3
2	0
−2	6

Plotting the three points (0, 3), (2, 0), and (− 2, 6), and drawing a straight line through the points, extending past the last points in each direction:



Horizontal or Vertical Equations

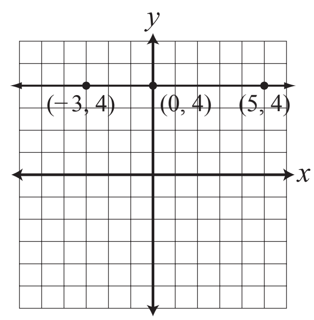
Consider the equation $y = 4$. This equation can also be written as $0 \cdot x + y = 4$. No matter what number is selected for x, y is 4.

Therefore a table of values for this equation could have any value in the x column, and only 4 in the y column.

For example:

x	y
-3	4
0	4
5	4

Plotting the points (− 3, 4), (0, 4), and (5, 4) on the graph and connecting the points:



1. The line is horizontal.
2. The line is parallel to the x-axis.
3. The y-intercept is 4.
4. Any point $(x, 4)$ is a solution to the equation.

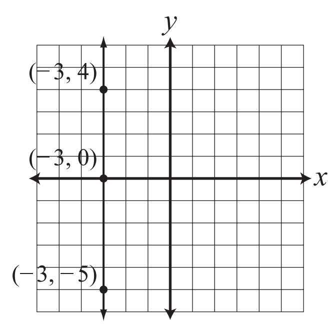
Now consider the equation $x = -3$. This equation can be written as $x + 0 \cdot y = -3$. No matter what number is selected for y, x is − 3.

Therefore a table of values for this equation could have only − 3 in the x column, and any value in the y column.

For example:

x	y
-3	4
-3	0
-3	-5

Plotting the points (− 3, 4), (− 3, 0), and (− 3, − 5) on the graph and connecting the points:



1. The line is vertical.
2. The line is parallel to the y-axis.
3. The x-intercept is − 3.
4. Any point $(-3, y)$ is a solution to the equation.

Summary of Horizontal and Vertical Lines

The graph $y = a$ is a **horizontal line** with y-intercept (0, a).

The graph $x = a$ is a **vertical line** with x-intercept (a, 0).

4.3 Exercise Set

1. Determine whether the given ordered pair is a solution to the following equations:

a) $(2, 3)$; $3x - 5y = -9$ y / n

b) $(0, 4)$; $\frac{1}{3}x + y = 4$ y / n

c) $(1, -1)$; $3y = 5 - 2x$ y / n

d) $(6, 8)$; $\frac{1}{3}x - \frac{1}{4}y = 4$ y / n

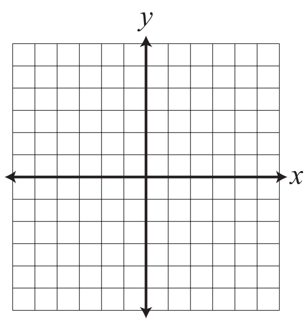
e) $(4, 2)$; $x = 4$ y / n

f) $(-1, 3)$; $y = -1$ y / n

2. Graph the equations using the table provided.

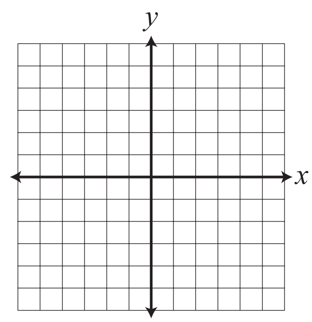
a) $2x + 3y = 6$

x	y
0	
	0
−3	



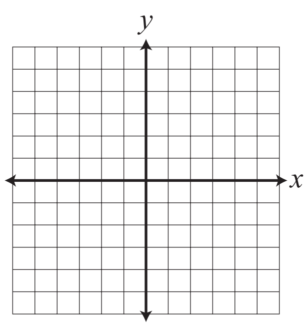
b) $2x + y = -4$

x	y
0	
	0
	4



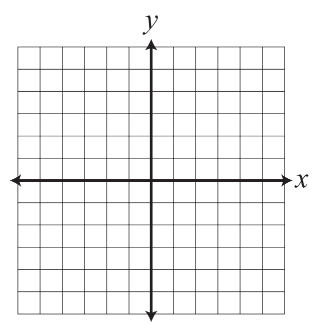
c) $2x - \frac{1}{2}y = 2$

x	y



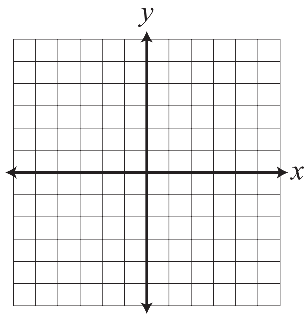
d) $3x + 2y = 5$

x	y



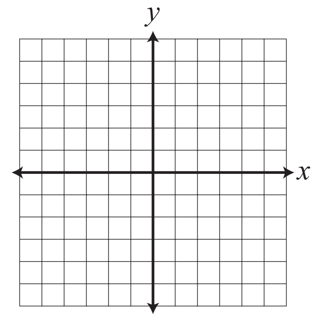
2. **e)** $\frac{2}{3}x - 0.4y = 2$

x	y



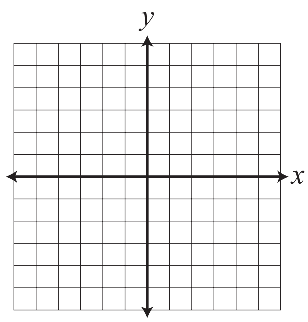
f) $2x + y = -1$

x	y



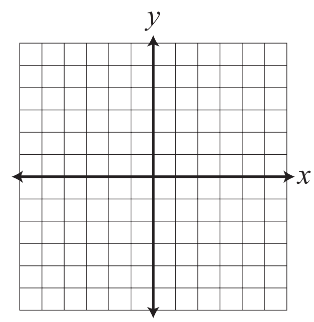
g) $\frac{3}{4}x + y = 1$

x	y



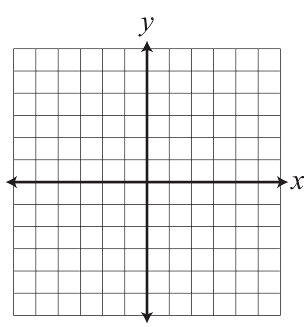
h) $\frac{2}{3}x - y = 2$

x	y



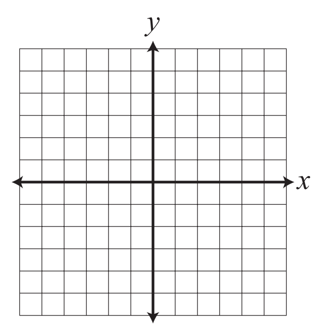
i) $0.4x - \frac{2}{3}y = 2$

x	y



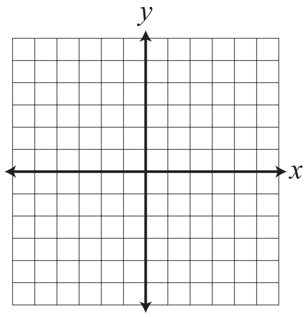
j) $\frac{1}{3}x + \frac{2}{3}y = 2$

x	y

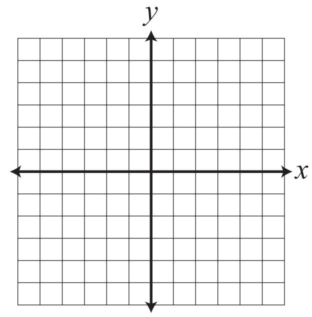


3. Graph

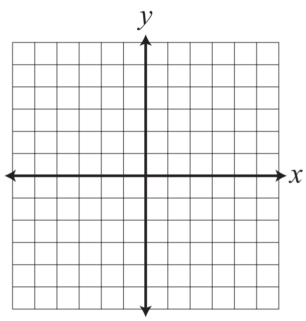
a) $x = 3$



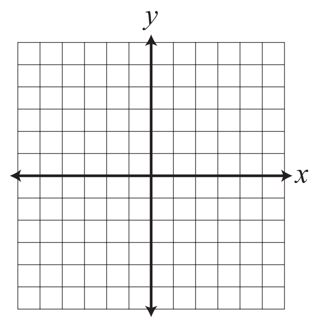
b) $y = 3$



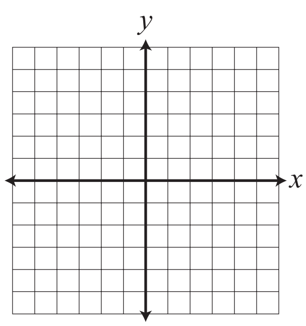
c) $x = -4$



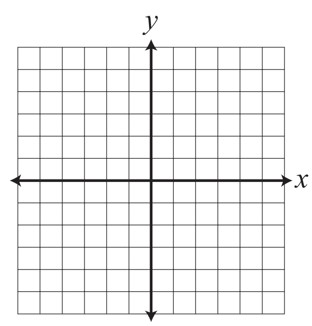
d) $y = -4$



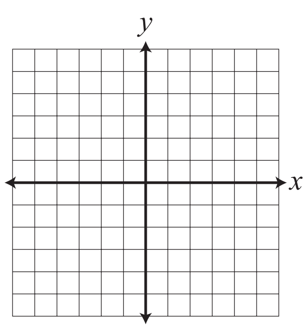
e) $x = 3\frac{1}{2}$



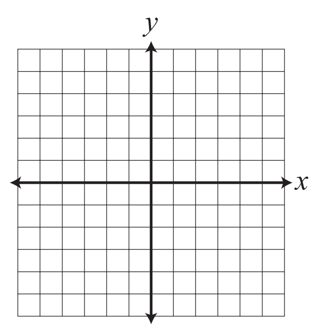
f) $y = -3\frac{1}{2}$



g) $x = 0$



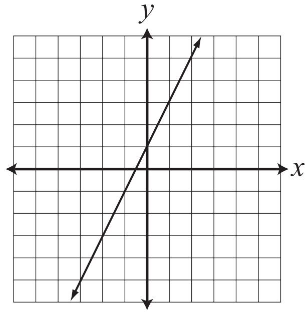
h) $y = 0$



4. Use the graphs to complete the tables.

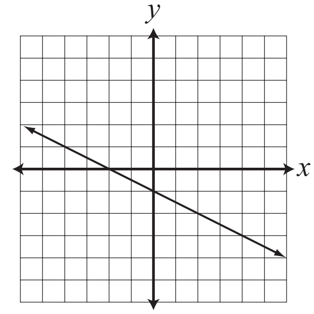
a)

x	y
−2	
−1	
0	
1	
2	



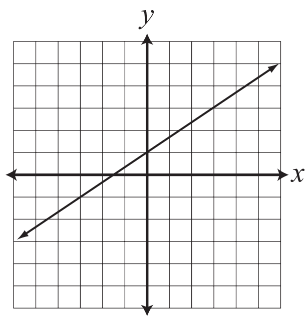
b)

x	y
−6	
−2	
0	
4	
6	



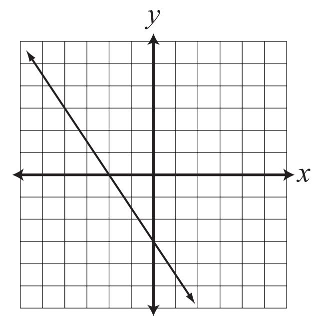
c)

x	y
	−3
	−1
	1
	3
	5



d)

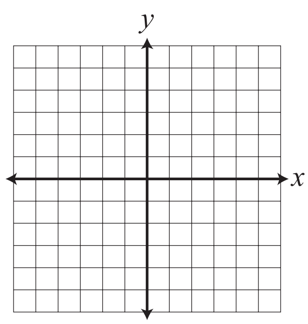
x	y
	6
−4	
	0
0	
	−6



5. Sketch a graph of a linear equation parallel to the given equation, that passes through the point indicated.

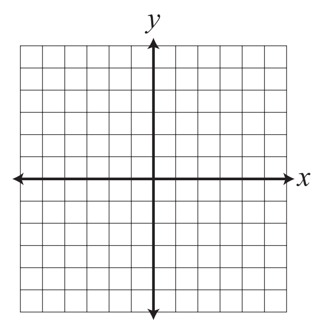
a) $y = 3x + 4,\ (0, 0)$

x	y



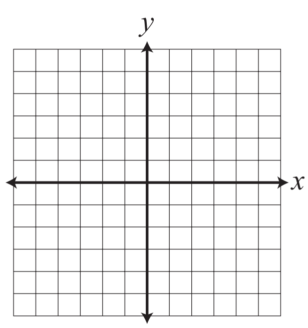
b) $y = -\frac{1}{3}x - 4,\ (0, 2)$

x	y



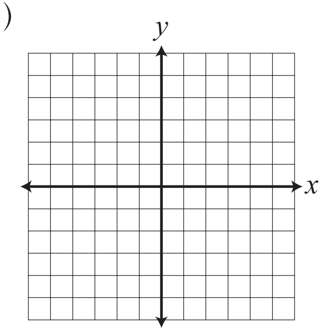
c) $2x - 3y = 0,\ (1, -2)$

x	y



d) $x + 2y = 6,\ (-1, -2)$

x	y

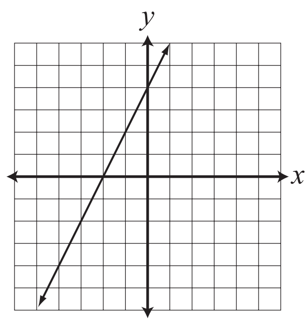


4.4 Matching Equations of Graphs

Equations can be matched to a particular graph by testing points on the graph in the equation. When testing points, use at least two points or, to be absolutely sure, three points.

Example 1 Which one of the equations match the graph?

a) $y = -2x + 4$ **b)** $y = 4x + 4$ **c)** $y = 2x + 4$



► *Solution*: Choose any two points on the graph. Using a point with x or y equal to zero makes the calculations easier, so try (0, 4), (− 2, 0). Substitute both these points into each equation to see if they make both equations true.

a) (0, 4): $y = -2x + 4$, $4 = -2(0) + 4$, $4 = 4$ (− 2, 0): $y = -2x + 4$, $0 = -2(-2) + 4$, $0 \neq 8$

b) (0, 4): $y = 4x + 4$, $4 = 4(0) + 4$, $4 = 4$ (− 2, 0): $y = 4x + 4$, $0 = 4(-2) + 4$, $0 \neq -4$

c) (0, 4): $y = 2x + 4$, $4 = 2(0) + 4$, $4 = 4$ (− 2, 0): $y = 2x + 4$, $0 = 2(-2) + 4$, $0 = 0$

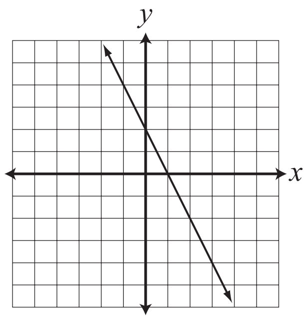
Equation **c)** has two true results. To verify this, test a third point on the graph: (− 1, 2).

$$y = 2x + 4$$
$$2 = 2(-1) + 4$$
$$2 = 2$$

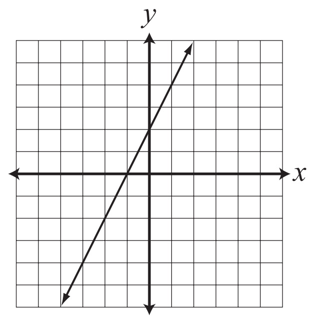
This is also true, therefore $y = 2x + 4$ matches the graph.

Example 2 Which one of the graphs matches the equation $x - 2y = -4$?

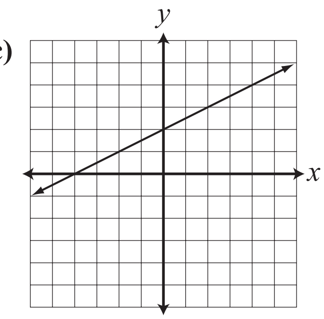
a)



b)



c)



► *Solution*: Choose any two points on the graph. Using a point with x or y equal to zero makes the calculations easier, so test the following values:

a) (0, 2):
$$\begin{aligned} x - 2y &= -4 \\ 0 - 2(2) &= -4 \\ -4 &= -4 \end{aligned}$$
(1, 0):
$$\begin{aligned} x - 2y &= -4 \\ 1 - 2(0) &= -4 \\ 1 &\neq -4 \end{aligned}$$

b) (0, 2):
$$\begin{aligned} x - 2y &= -4 \\ 0 - 2(2) &= -4 \\ -4 &= -4 \end{aligned}$$
(− 1, 0):
$$\begin{aligned} x - 2y &= -4 \\ -1 - 2(0) &= -4 \\ -1 &\neq -4 \end{aligned}$$

c) (0, 2):
$$\begin{aligned} x - 2y &= -4 \\ 0 - 2(2) &= -4 \\ -4 &= -4 \end{aligned}$$
(− 4, 0):
$$\begin{aligned} x - 2y &= -4 \\ -4 - 2(0) &= -4 \\ -4 &= -4 \end{aligned}$$

Graph **c)** has two true results. To verify this we test a third point on the graph: (4, 4).

$$\begin{aligned} x - 2y &= -4 \\ 4 - 2(4) &= -4 \\ -4 &= -4 \end{aligned}$$

This is also true, therefore graph **c)** is $x - 2y = -4$.

Summary

1. Select two points on the graph to make calculations easier. Select x-intercept point $(a, 0)$ and y-intercept $(0, b)$ if these have integer x and y values.

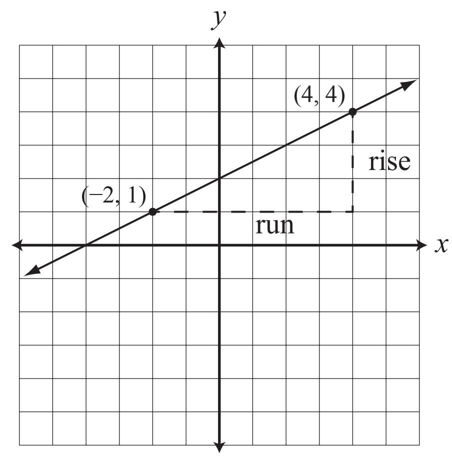
2. After choosing the answer which includes those two points, pick a third point to make sure no errors were made in your calculations.

3. To be a solution, all test points must be true.

Determining the Equation of a Graph

It is possible to find the slope of a line from a graph. Consider $y = mx + b$. In slope intercept form we know that the y-intercept is (0, b), but what does the constant m tell us about the graph? The constant m tells us the **slope** of the line. Some graphs slope upward as we go from left to right, and others slope downward from left to right. Others can be vertical or horizontal. Some slopes are steeper than others.

Consider the graph to the right. On the graph select two points with integer x and y values. We picked out (− 2, 1) and (4, 4). There are others. We call the change in y the **rise**, and the change in x the **run**. The ratio of rise over run is called the **slope**. Notice that the slope is the same for any two points on the line.



$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{the change in } y}{\text{the change in } x}$$

In our example, slope $= \frac{\text{rise}}{\text{run}} = \frac{3}{6} = \frac{1}{2}$

The graph intercepts the y-axis at 2. Therefore the equation of the line is $y = \frac{1}{2}x + 2$

Alternatively $y = \frac{1}{2}x + b$ or $y = \frac{1}{2}x + b$

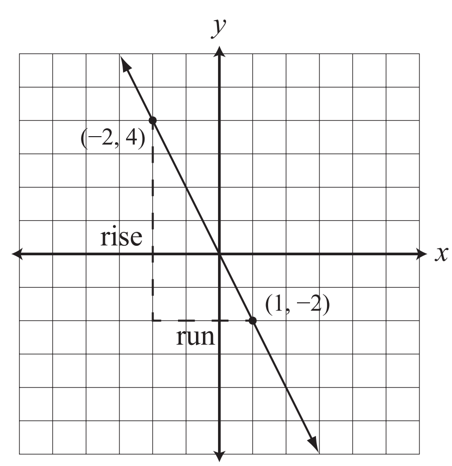
$$1 = \frac{1}{2}(-2) + b \qquad 4 = \frac{1}{2}(4) + b$$

$$1 = -1 + b \qquad 4 = 2 + b$$

$$b = 2 \qquad b = 2$$

Therefore the equation of the line is $y = \frac{1}{2}x + 2$.

As mentioned previously, not all graphs move up as we go from left to right. Consider two points (− 2, 4), (1, − 2) on the graph to the right. In this example the rise is **negative**, so our slope is negative.



$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{-6}{3} = -2$$

The graph intercepts the y-axis at 0. Therefore the equation of the line is $y = -2x$.

Alternatively $y = -2x + b$ or $y = -2x + b$

$$4 = -2(-2) + b \qquad -2 = -2(1) + b$$

$$4 = 4 + b \qquad -2 = -2 + b$$

$$b = 0 \qquad b = 0$$

Therefore the equation of the line is $y = -2x$.

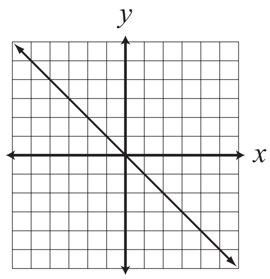
4.4 Exercise Set

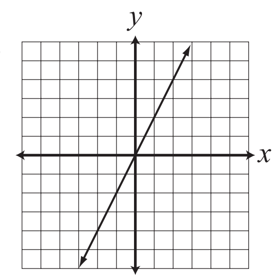
1. Match the equation with its linear graph.

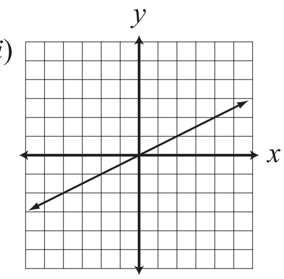
a) $y = 2x$ ______

$y = \frac{1}{2}x$ ______

$y = -x$ ______

i)


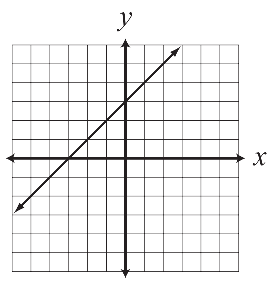
ii)


iii)


b) $y = x$ ______

$y = x + 3$ ______

$y = x - 3$ ______

i)


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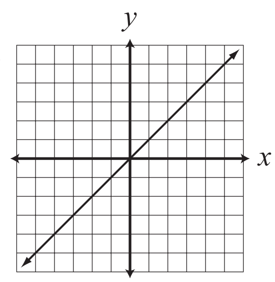

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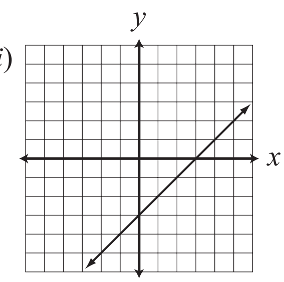
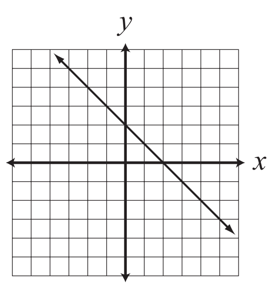
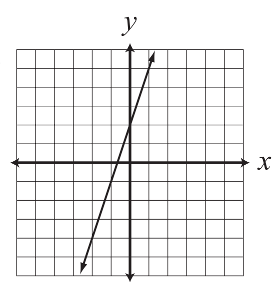


c) $y = 3x + 2$ ______

$y = -x + 2$ ______

$y = \frac{1}{3}x + 2$ ______

i)


ii)


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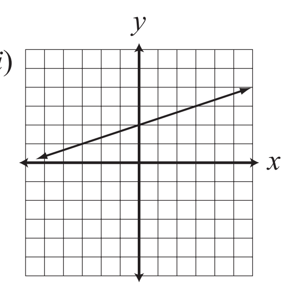
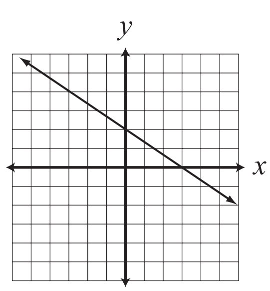
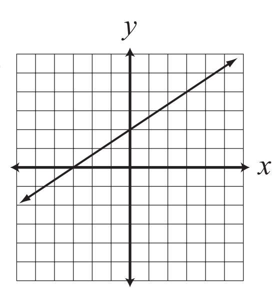


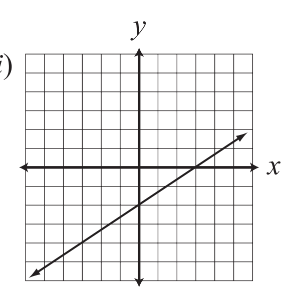
d) $2x - 3y = 6$ ______

$2x + 3y = 6$ ______

$2x - 3y = -6$ ______

i)


ii)


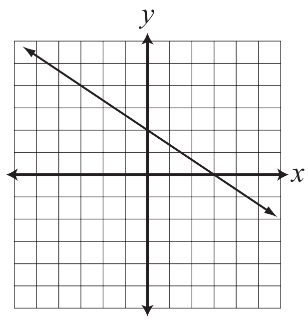
iii)


2. Which equation matches the graph?

a) $2x + 3y = 6$

$2x - 3y = 6$

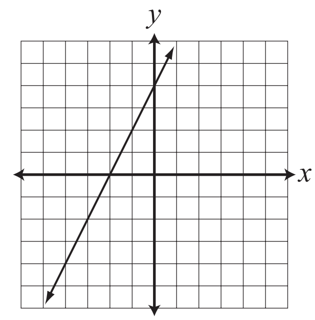
$-2x + 3y = 6$



b) $2x + y = 4$

$2x - y = 4$

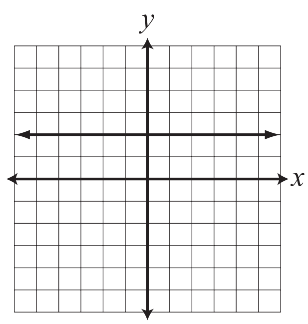
$2x - y = -4$



c) $x = 2$

$y = 2$

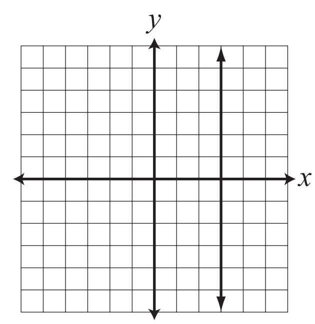
$x + y = 2$



d) $x = 3$

$y = 3$

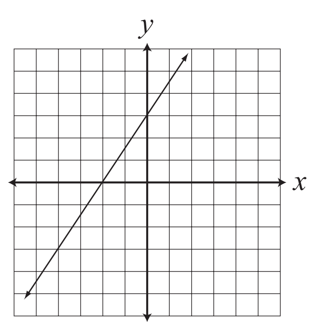
$x + y = 3$



e) $3x - 2y = 6$

$3x + 2y = 6$

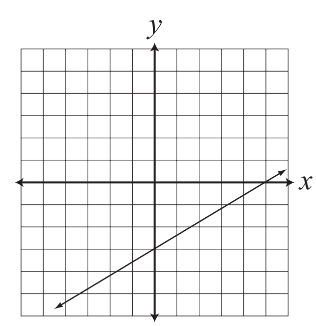
$3x - 2y = -6$



f) $3x + 5y = 15$

$3x - 5y = 15$

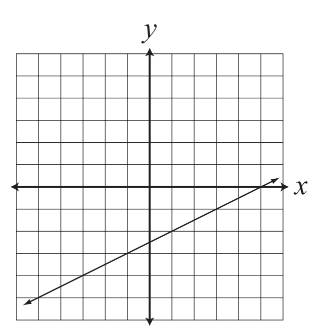
$3x - 5y = -15$



g) $x - 2y = 5$

$x + 2y = 5$

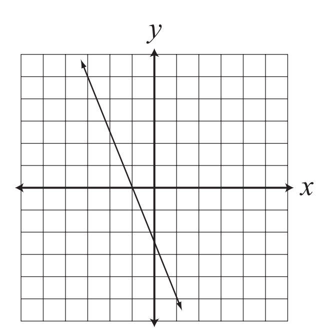
$x + 2y = -5$



h) $5x + 2y = 5$

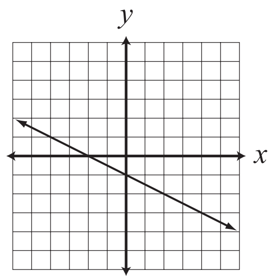
$5x - 2y = 5$

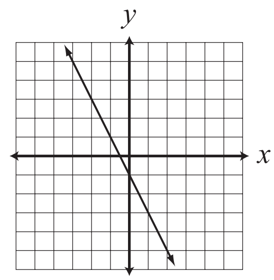
$5x + 2y = -5$

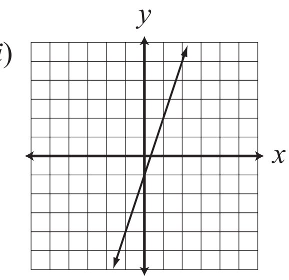


3. Which graph matches the equation?

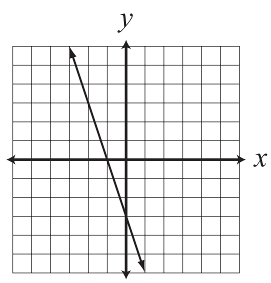
a) $y = -2x - 1$

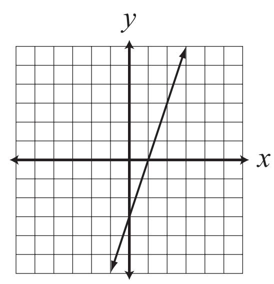
i)


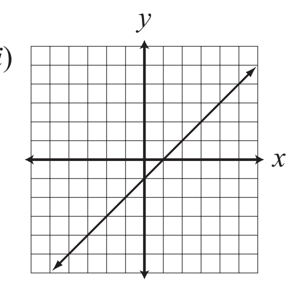
ii)


iii)


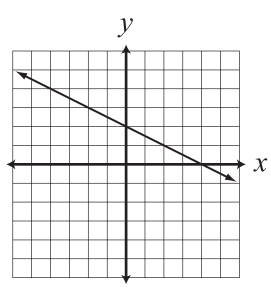
b) $3x - y = 3$

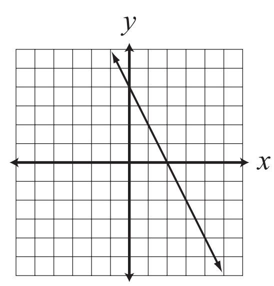
i)


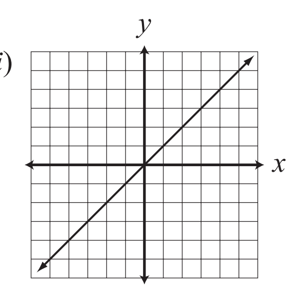
ii)


iii)


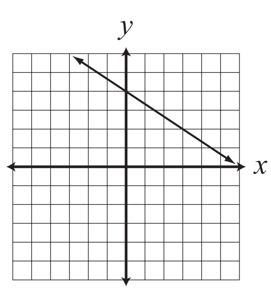
c) $x + 2y = 4$

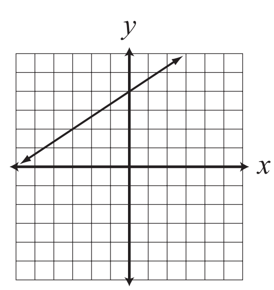
i)


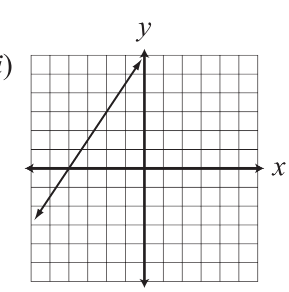
ii)


iii)


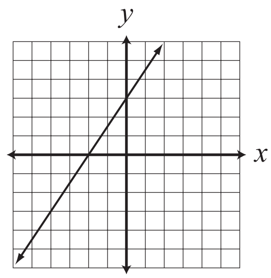
d) $2x - 3y = -12$

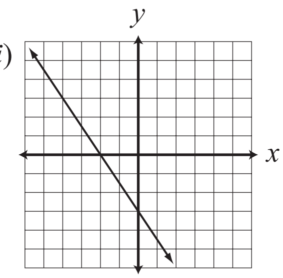
i)


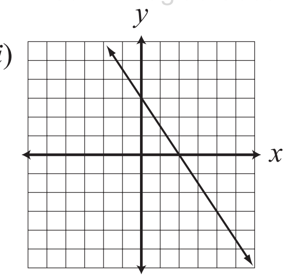
ii)


iii)


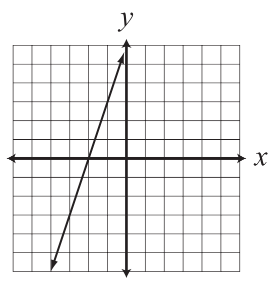
3. **e)** $3x + 2y = -6$

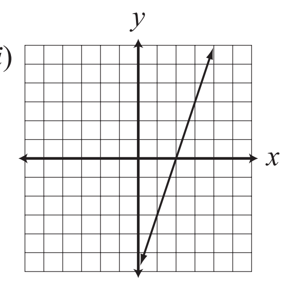
i) 

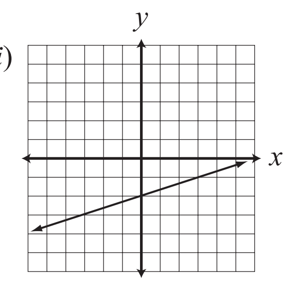
ii) 

iii) 

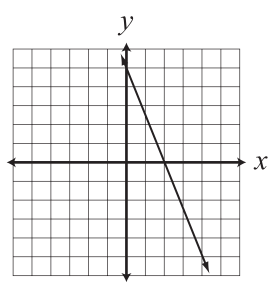
f) $x - 3y = 6$

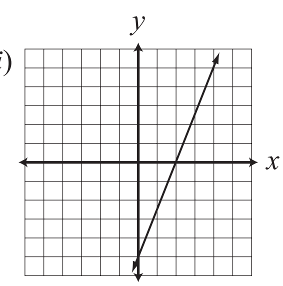
i) 

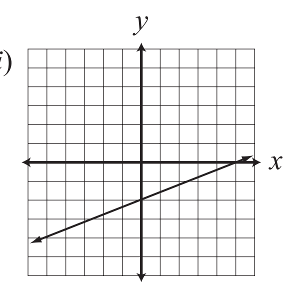
ii) 

iii) 

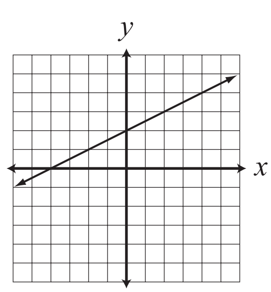
g) $2x - 5y = 10$

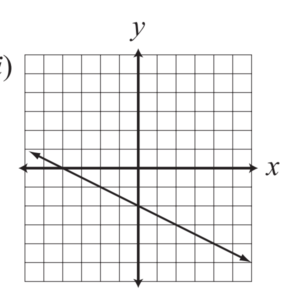
i) 

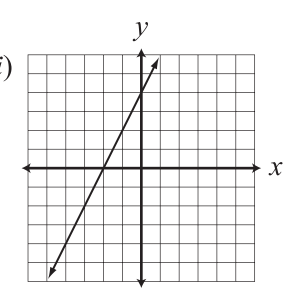
ii) 

iii) 

h) $3x - 6y = -12$

i) 

ii) 

iii) 

4. Match each equation with its graph.

a) $y = x + 3$ ______

b) $y = 3x$ ______

c) $y = -2$ ______

d) $x = 4$ ______

e) $y = -\frac{1}{3}x$ ______

f) $2x - 3y = 6$ ______

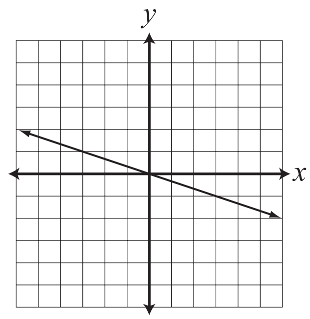
g) $y = -2x - 3$ ______

h) $3x + 2y = 6$ ______

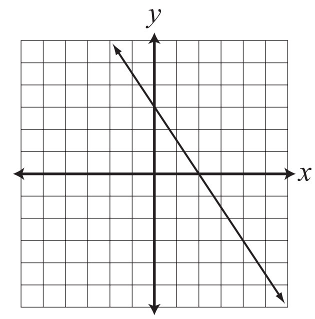
i) $2x - y = 2$ ______

j) $2x - y = -4$ ______

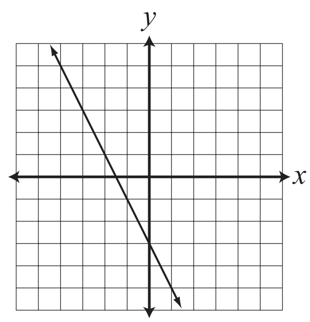
i)



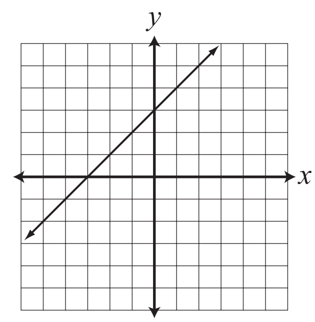
ii)



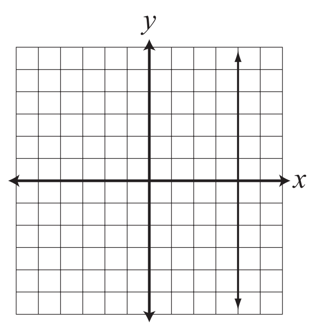
iii)



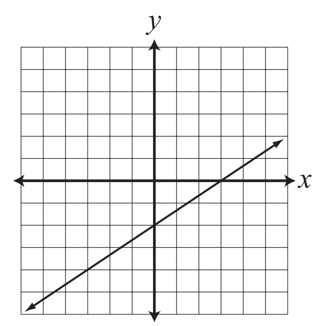
iv)



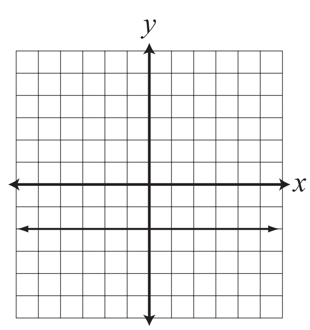
v)



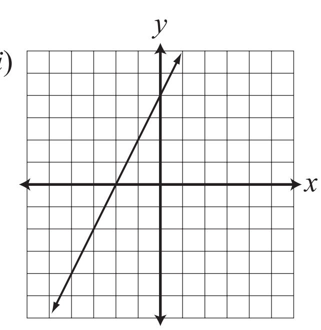
vi)



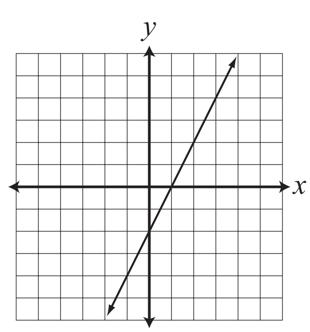
vii)



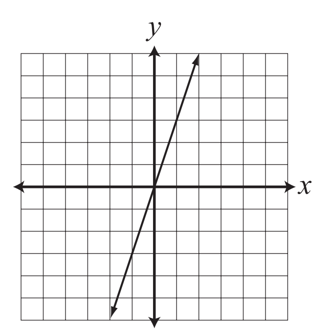
viii)



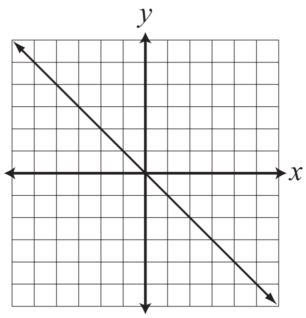
ix)

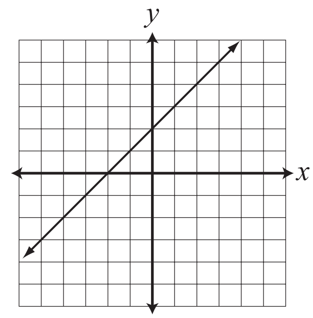


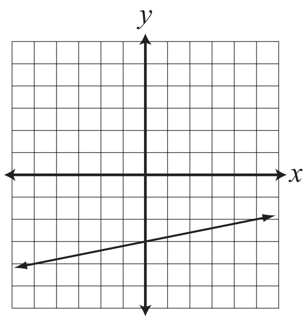
x)

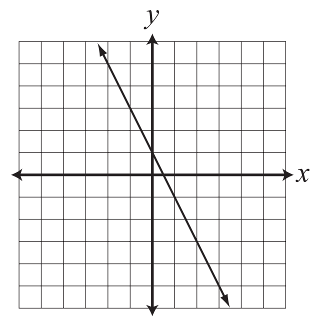


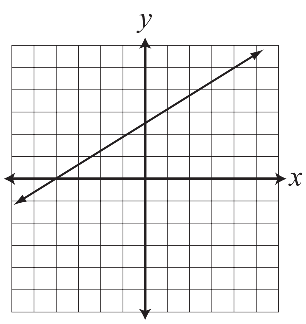
5. Determine the y-intercept for each graph, if it exists.

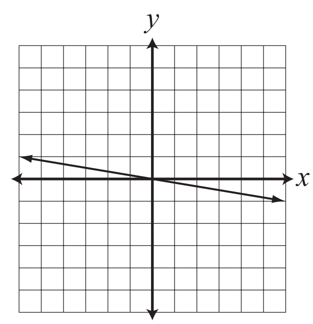
a)


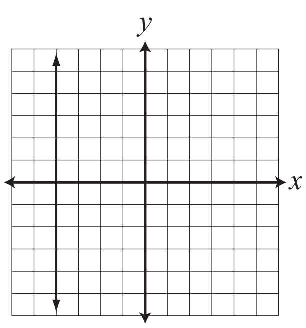
b)


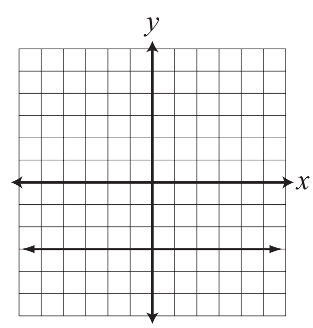
c)


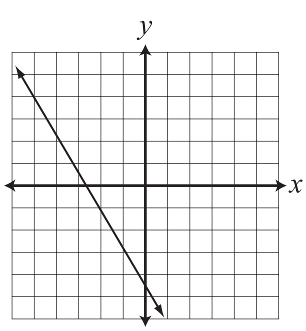
d)


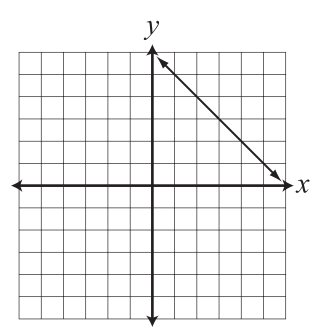
e)


f)


g)


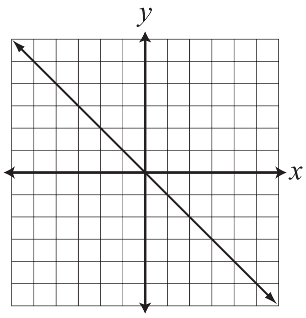
h)


i)


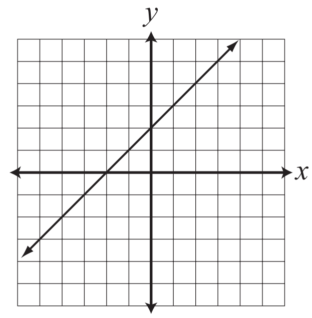
j)


6. Determine the slope for each graph, if it exists.

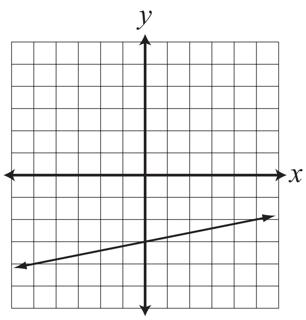
a)



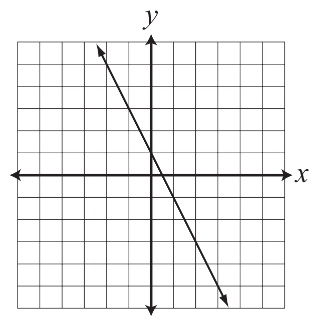
b)



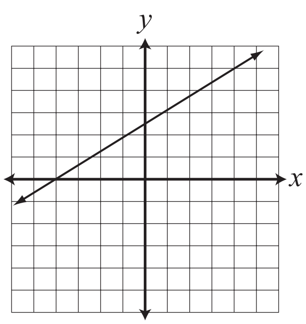
c)



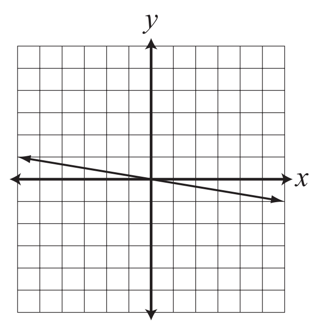
d)



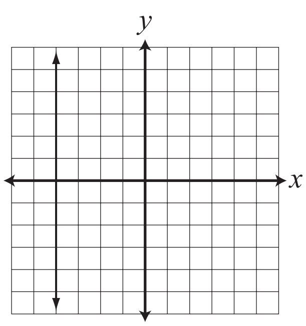
e)



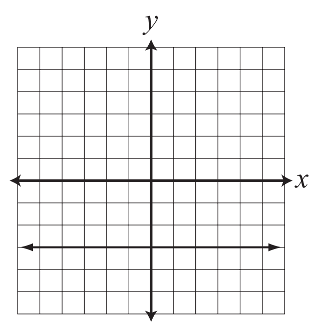
f)



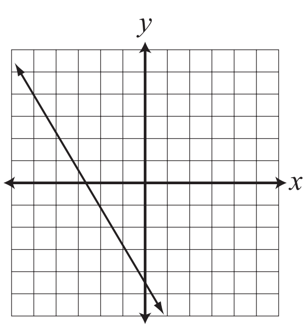
g)



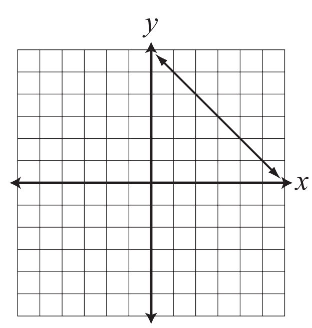
h)



i)

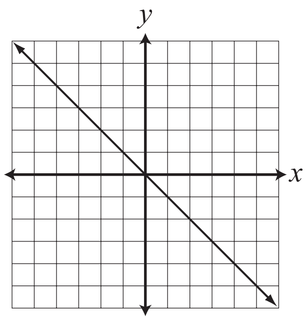


j)

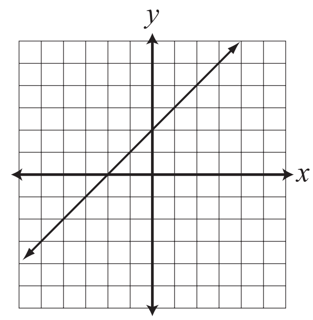


7. Determine the equation for each graph, in slope intercept form.

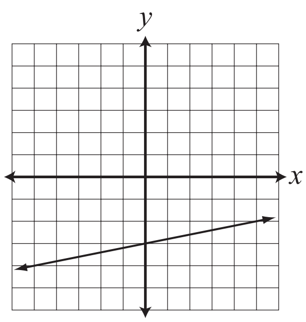
a)



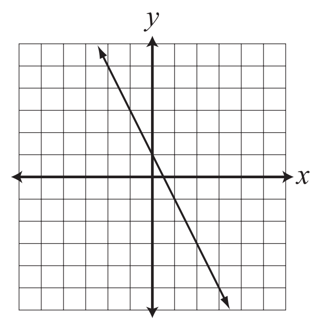
b)



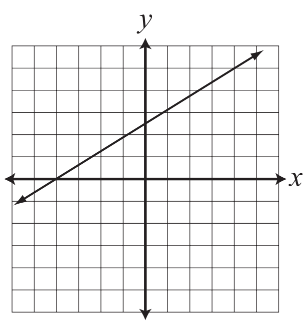
c)



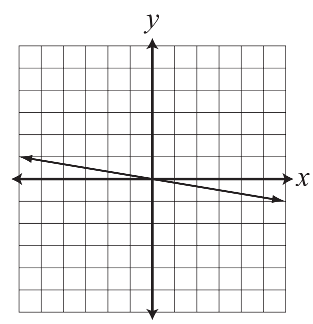
d)



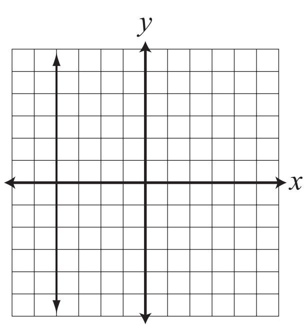
e)



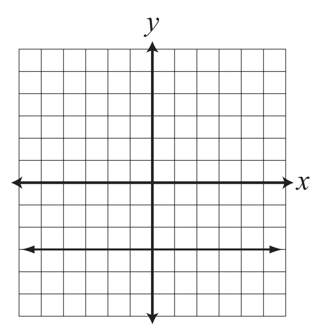
f)



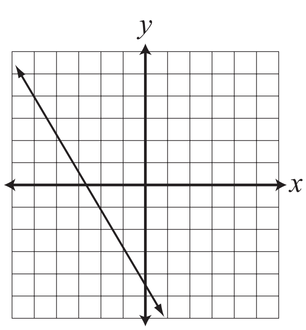
g)



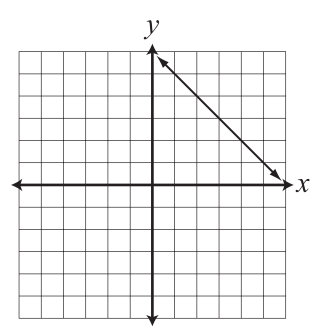
h)



i)



j)



8. Tony earns a weekly salary of $750 plus 10% of his sales volume. Write a linear equation describing Tony's weekly earnings for x dollars in sales.

9. In a 16 team volleyball tournament, the number 1 ranked team plays the number 16 ranked team, and the number 2 ranked team plays the number 15 ranked team. If x is the ranking of the higher ranked team, and y is the ranking of the lower ranked team, what is the linear equation that describes the match up?

10. On a certain evening a 30 minute telephone call costs $1.50. The next evening a 8 minute telephone call costs $0.40. If the cost of an evening phone call is linear, find an equation to represent the cost of an evening phone call.

11. Water freezes at 0°C, and boils at 100°C. Water also freezes at 32°F, and boils at 212°F. Find a linear equation relating degrees Fahrenheit to degrees Celsius.

12. A piece of string 60 cm long has 5 pieces of length x cut from it.

a) Write a linear equation that models the length of the string that is remaining.

b) Complete the table for the linear equation.

x	0	3	6	9	12
S					

c) Determine the value of S when x is 12, and interpret the result.

d) Determine the value of x when $S = 5$, and interpret the result.

4.5 Chapter Review

Section 4.1

1. Determine the common difference in the linear pattern.

a) $-7, -4, -1, \ldots$ ________ **b)** $-1, -4, -7, \ldots$ ________

c) $2\frac{1}{2}, 2\frac{3}{4}, 3, \ldots$ ________ **d)** $6.5, 5.3, 4.1, \ldots$ ________

2. Determine the 100th term of the linear pattern.

a) $12, 9, 6, 3, \ldots$ ________ **b)** $-7, -3, 1, 5, \ldots$ ________

3. Determine a general equation of the *n*th term in linear pattern.

a) $12, 9, 6, 3, \ldots$ ________ **b)** $-7, -3, 1, 5, \ldots$ ________

4. The cost of printing a math workbook is a setup cost plus a cost for each book printed. If 1000 books printed cost \$14 300, and 5000 books printed cost \$64 300, what is the setup cost for printing the math workbooks?

5. The value of a new computer network for an office is \$12 000. If the depreciation of the computers, with respect to time, in dollars per year is 15%, in how many years will the computer network have no value?

Section 4.2

6. Determine the missing ordered pair values for the given equations.

a) $y = \frac{2}{3}x - 4$

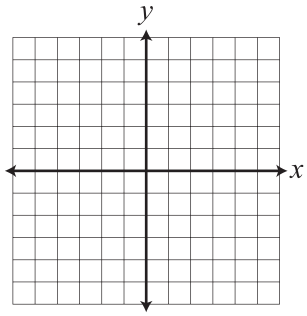
x	y
0	
	0
-3	

b) $y = -\frac{4}{3}x + 2$

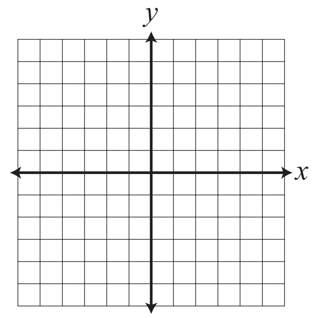
x	y
0	
	0
	6

7. Graph the equation and identify the y-intercept

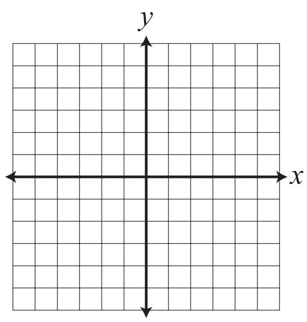
a) $y = -\frac{3}{4}x + 2$



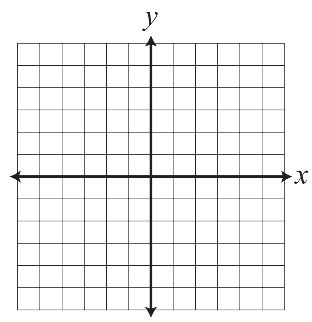
b) $y = \frac{5}{3}x - 1$



c) $y = -\frac{5}{2}x + 1$

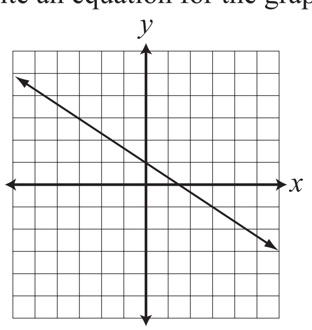


d) $y = \frac{1}{3}x + 1$

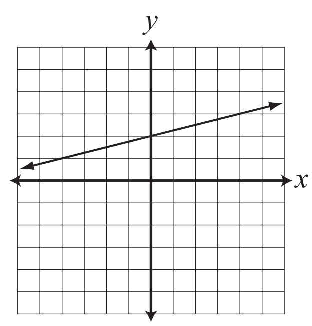


8. Write an equation for the graph.

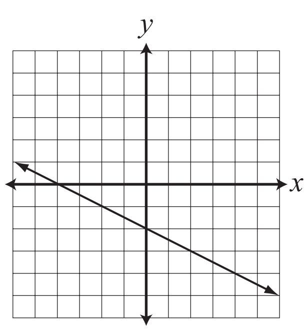
a)



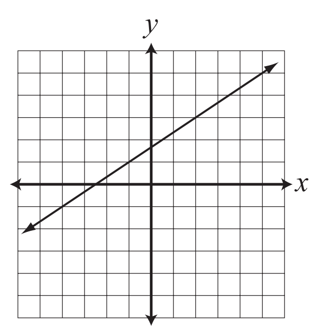
b)



c)



d)

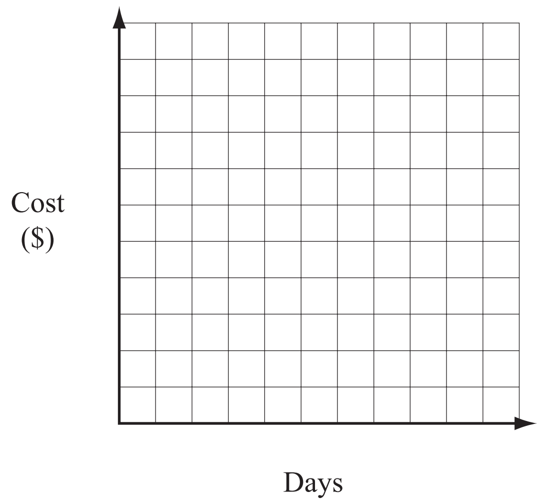


9. The cost, C, in dollars for renting a riding lawn mower is $C = 65 + 40d$, where d is the number of days renting.

a) Graph this equation from 1 to 20 days.

b) Find the cost of renting the lawn mower for 16 days.

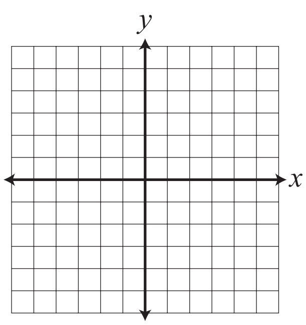
c) If the lawn mower is worth $2265, in how many days would the rental charge equal the value of the lawn mower?



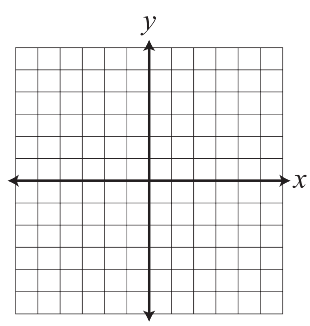
Section 4.3

10. Graph the equations.

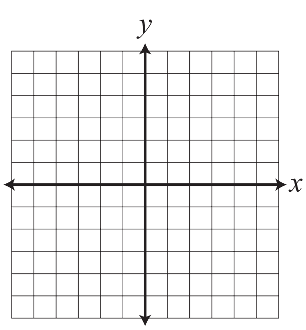
a) $2x + \frac{2}{3}y = 2$



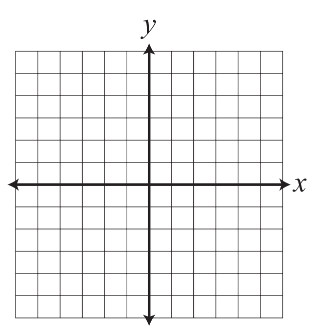
b) $\frac{1}{3}x - 0.2y = 1$



c) $2x - \frac{3}{2}y = -6$



d) $0.25x + \frac{1}{5}y = -1$



Section 4.4

11. Match each equation with its graph.

a) $y = -x + 2$ ______

b) $y = \frac{1}{2}x$ ______

c) $y = 2$ ______

d) $x = -3$ ______

e) $y = -2x$ ______

f) $3x - 2y = 6$ ______

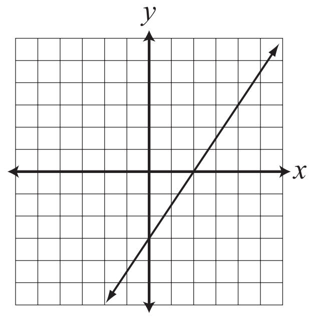
g) $2x + 3y = 6$ ______

h) $x - 2y = 6$ ______

i) $3x + 2y = -6$ ______

j) $2x + 3y = -6$ ______

i)



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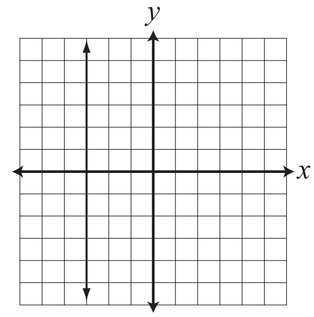

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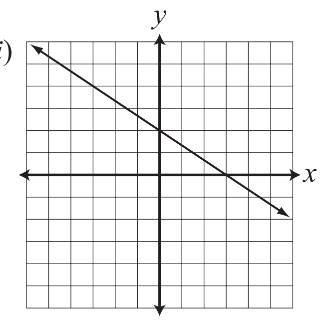

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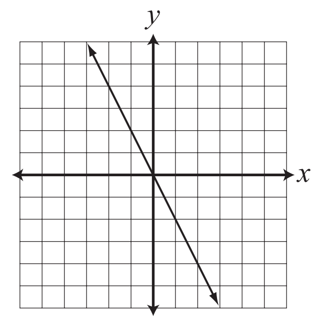

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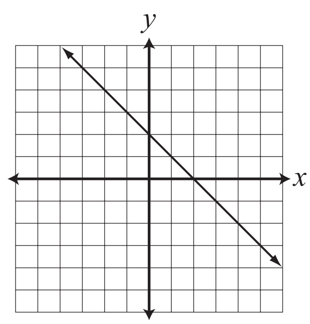

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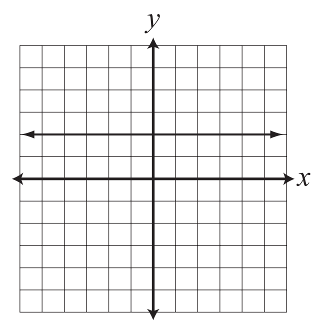

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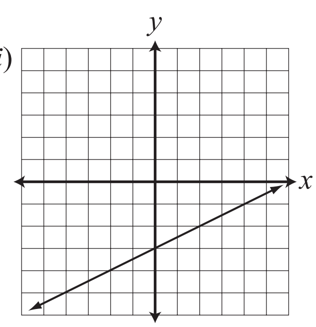

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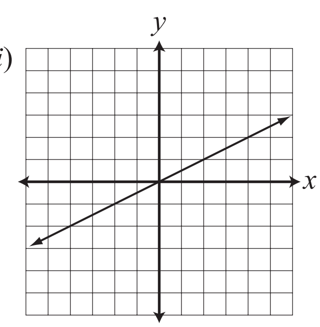

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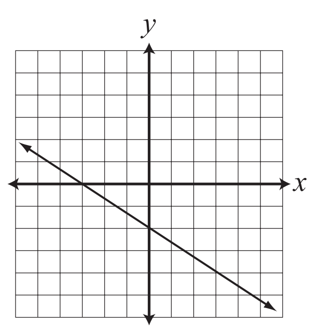

x)