5.1 Defining Polynomials

Variable

A variable is a letter whose value is an unknown real number.

Examples: $a, b, c, \ldots, x, y, z$

Term

A term is a number, or product of a number and variable(s) raised to a power.

Examples: $5,\ 2x,\ -3x^2,\ -2xy$

Coefficient

The coefficient of a term is the numerical factor.

Examples:

Term	Coefficient
$5x$	5
$\frac{x^2}{3}$	$\frac{1}{3}$
$-1.3xy$	-1.3
$-y$	-1
-3	-3

*Note: If a term is a number only, it is called a constant term or simply a **constant**. A letter that can be any one of various numbers is called a **variable**. For example in the term $2x^3$, 2 is the coefficient, x is the variable, and 3 is the exponent.*

Monomial

A monomial is an expression of the type ax^n, where a is a real number constant, and n is a non-negative integer.

Examples of monomials: $\frac{3}{4}y^2,\ -4,\ 2x$

Examples of non-monomials: $\frac{1}{x},\ \sqrt{x},\ x^{\frac{2}{3}}$

Polynomial

A polynomial is a monomial, or a combination of sums and/or differences of monomials.

Examples: $3x,\ 2x-1,\ 4x^2+y+2$

Degree and Leading Term of a Polynomial

The **degree** of a term is the sum of the exponents of its variables.

The **leading term** of a polynomial is the term of highest degree. The degree of a polynomial is the same as the degree of its leading term.

Consider the following polynomials:

$5x^2 - 3x + 4$

Term	$5x^2$	$-3x$	4
Degree	2	1	0
Leading Term	$5x^2$		
Degree of Polynomial	2		

$-5 + xy$

Term	-5	xy
Degree	0	2
Leading Term	xy	
Degree of Polynomial	2	

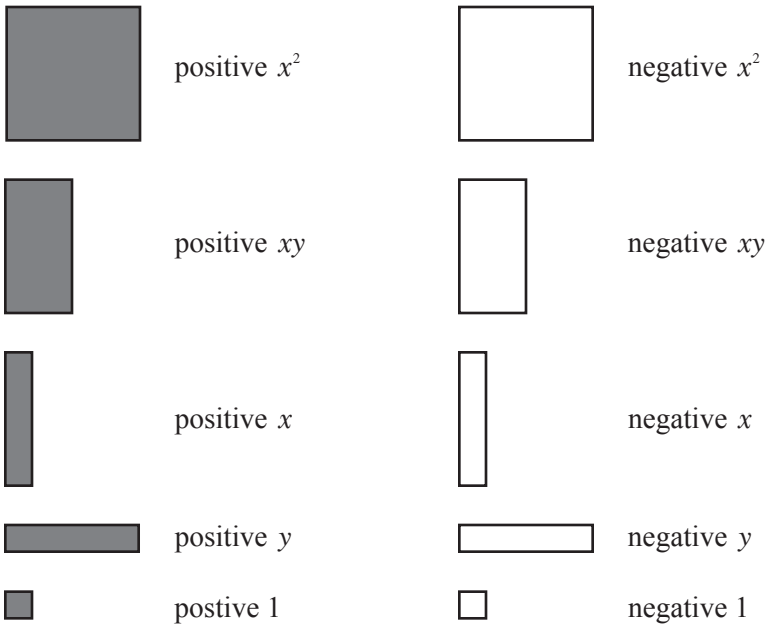
The following are names for certain kinds of polynomials

Type	Definition	Examples					
Monomial	A polynomial of one term	3	$-2x$	$5x^2$	$-4xy$	0	2^2x^2
Binomial	A polynomial of two terms	$2x - y$		$3x - 1$		$2xy - x^2$	
Trinomial	A polynomial of three terms	$x^2 - 3x + 4$			$5y^2 - 2ay - a^2$		

Note: *A polynomial is usually written in descending order of powers. For example, the polynomial* $2x + 4 - 3x^2$ *is written as* $-3x^2 + 2x + 4$.

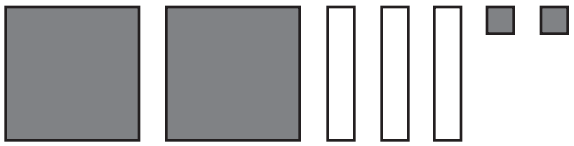
A polynomial with more than one variable is written in alphabetic order. For example, the polynomial $xy + y^2 - x^2$ *is written as* $-x^2 + xy + y^2$, *since all terms are of degree two, and x is before y in alphabetic order.*

Algebra Tiles



Example 1 Express the polynomial $2x^2 - 3x + 2$ using algebra tiles.

►*Solution*:



Example 2 Express the polynomial $-x^2 + 2x - 3$ using algebra tiles.

►*Solution*:

Example 3 Express the polynomial $-2xy + 3x - 2y + 1$ using algebra tiles.

►*Solution*:

Like Terms

Terms with the same variable raised to exactly the same powers are called **like terms**. Terms that are not like terms are called **unlike terms**.

Like Terms	Unlike Terms
$5x, 3x$	$5x^2, 3x$
$2x^2, -x^2$	$3x^2, 3y^2$
$2xy, -4yx$	$2xy, -3yz$
xy^2, y^2x	xy^2, x^2y

Like terms can be added or subtracted, but unlike terms cannot be added or subtracted.

Note: xy is the same as yx

Example 4 Simplify $4x^2 + 3x^2 - 2x^2$

►*Solution*: $4x^2 + 3x^2 - 2x^2 = 5x^2$

Example 5 Simplify $4x + 3x^2 - 2x^2$

►*Solution*: $4x + (3x^2 - 2x^2) = x^2 + 4x$ ($4x$ is a unlike term to $3x^2$ and $2x^2$)

Example 6 Simplify $2xy^2 + 3x^2y - 4x^2y + xy^2$

►*Solution*: $(3x^2y - 4x^2y) + (2xy^2 + xy^2) = -x^2y + 3xy^2$

5.1 Exercise Set

1. Match the description on the right with the expression on the left. (Some may have two descriptions)

a) $\frac{2}{x}$ _____ *i*) monomial

b) $2 - x$ _____ *ii*) binomial

c) xy^2 _____ *iii*) trinomial

d) $-5xy$ _____ *iv*) polynomial with leading coefficient of -4

e) $6y - 4x^2$ _____ *v*) non-polynomial

f) $x^2 - 3x + 5$ _____ *vi*) polynomial of degree higher than 2

g) $x^2 - xy + y + 3$ _____ *vii*) polynomial written in descending power of x

h) $xy + y - x$ _____

2. Determine if the expression is a polynomial.

a) x^0 y / n **b)** $\frac{1}{x}$ y / n

c) $2x^2 + 3$ y / n **d)** $2xy + y - 3$ y / n

e) $\sqrt{x} + 2x$ y / n **f)** $\frac{1}{3}$ y / n

g) $\sqrt{2}\,x^2 + 3x - 1$ y / n **h)** $x^2 + x + x^0$ y / n

i) $x^{-2} + x^{-1} + x$ y / n **j)** $2^{-3}x^2$ y / n

3. Determine the degree of each polynomial.

a) $3x - x^2 + 2$ _____ **b)** $2^3 + x^2$ _____

c) -5 _____ **d)** 3^2 _____

e) $2^3x - xy$ _____ **f)** $-y + 3x^2$ _____

g) $7x^2 + 2^5$ _____ **h)** $2 - x + 5x^2$ _____

i) x^0 _____ **j)** $\sqrt{2}\,x^2 + 3x$ _____

4. Determine the coefficient of each monomial.

a) $2x$ _____ **b)** $-8x^3$ _____

c) $1.9y^5$ _____ **d)** $\frac{2}{3}x^{11}$ _____

e) $-\frac{7}{2}x^6$ _____ **f)** $3.7x^0$ _____

g) -8 _____ **h)** 0 _____

5. Arrange in descending order of powers.

a) $xy - x^2 + y^2$ _____ **b)** $2x - xy + x^2$ _____

c) $2^2 + x + x^2$ _____ **d)** $5^0 - x^2 + x$ _____

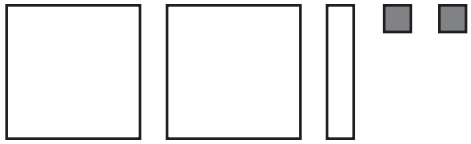
e) $y^2 - 2xy + x^2$ _____ **f)** $xy - 8x^2 - y$ _____

6. Write a polynomial expression for the algebraic tile models.

a)

_____

b)



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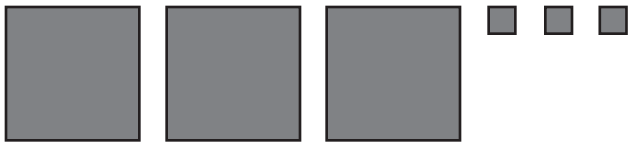
c)



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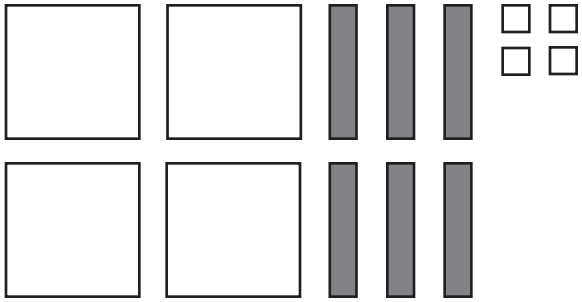
d)



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7. Draw algebra tiles to model the polynomials.

a) $-2x^2 + 2$

b) $2x^2 - 3x$

c) $-2x^2 - x + 3$

d) $-4x + 1$

e) $x^2 + 4x - 2$

f) $3x^2 - x + 2$

g) $-2x^2 + 2x - 1$

h) $xy + x - y$

i) $2x^2 - xy + 3x - 2y$

j) $-x^2 + xy + 2$

8. Simplify.

a) $2x - 3 - x + 2$

b) $-3 + 3x - 2 - x$

c) $2x^2 + x - x^2 - x$

d) $3x^2 - y^2 + 3y^2 - x^2$

e) $-4x^2 - 3x + 4 + 2x$

f) $-5x + 2y + 4y - x$

g) $4x - 3y + 6x - 5x$

h) $-2x^2 - 5 + 2x + x^2 - 3 - x$

i) $3xy - 4yx + 2xy - yx$

j) $-5xy - 2yx - y + 2x - 6yx$

k) $13x - 7 - 4x + 2$

l) $22 + 5x + 7y - x + y - 28$

m) $\frac{2}{3}x + \frac{1}{3}x$

n) $2y + \frac{3}{4}y + y$

o) $\frac{1}{2}x + x - 5x$

p) $-8 + 11x - 5y + 7x - 7y + 6$

q) $8x - 5x + 4 + 3y - 2x - 7$

r) $-3x^2 - 4x + x^2 - 2x + 4$

s) $-2xy + 3x + 5xy - 2y - x - y$

t) $4xy - 2x - 5xy + 4x - 7$

9. Write a polynomial expression in simplified form for the algebra tiles.

a) 

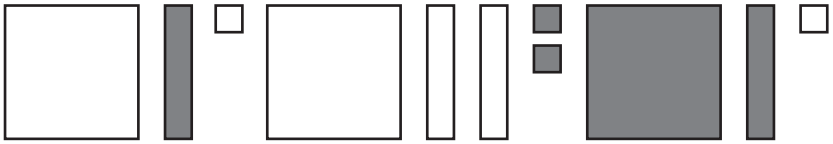
_____

b) 

_____

c) 

_____

d) 

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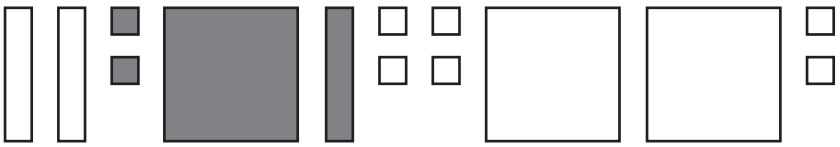
e) 

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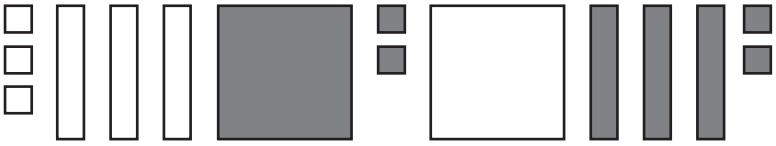
f) 

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10. Simplify each polynomial by collecting like terms.

a) $-a+\frac{3}{4}+15a^2-a-\frac{1}{2}-3a^2$

b) $3+2b^2-5b-2b+4b^2$

c) $-6c^2+c-5c+7c^2+1$

d) $-2d-2d-2d+d^2-5d^2$

e) $2e-\frac{5}{6}+4e^2+e+\frac{1}{3}-2e$

f) $-3fg+4gf-4f^2+g^2$

g) $\frac{1}{4}h^2-5+\frac{1}{2}h^2-2h-9$

h) $\frac{1}{3}i^2+2i-\frac{1}{6}i^2+4-9$

i) $\frac{1}{4}j^2-j-\frac{1}{6}j+\frac{3}{8}j^2+\frac{5}{16}j^2$

j) $\frac{1}{5}k^2+\frac{1}{5}-2k+\frac{1}{10}-\frac{3}{15}k^2+2k-\frac{3}{10}$

k) $\frac{1}{4}x^2+\frac{3}{4}x+\frac{1}{6}x^2-\frac{5}{6}x$

l) $\frac{2}{5}x^2-\frac{1}{3}x+\frac{4}{3}x^2+\frac{1}{5}x$

m) $2.8x^2-x+1.4x^2-3.2x+1$

n) $-4.9x-3.2y-1.3x+4.2y+1$

o) $-3.2x+1.7y-1.2x-2.8y-5$

p) $2.9x+2.3y-1.9x-1.8y$

q) $\frac{11}{4}x+\frac{2}{3}y-\frac{3}{5}x-\frac{1}{3}y+10$

r) $\frac{11}{2}x+\frac{9}{5}y-\frac{2}{3}x-\frac{3}{10}y-37$

5.2 Adding and Subtracting Polynomials

Adding Polynomials Horizontally

To add polynomials, we use a plus sign and add like terms. Arranging like terms together may save some computational errors.

Example 1 Add $(-2x^2-4)+(3x^2-2x+2)$

►*Solution*: $(-2x^2-4)+(3x^2-2x+2)$
$-2x^2-4+3x^2-2x+2$
$-2x^2+3x^2-2x-4+2$
x^2-2x-2

Adding Polynomials Vertically

To add polynomials vertically, place the polynomials with like terms in the same columns. Leave a blank space if a term in one of the polynomials does not have a match in the other.

Example 2 Add $(-2x^2-4)+(3x^2-2x+2)$

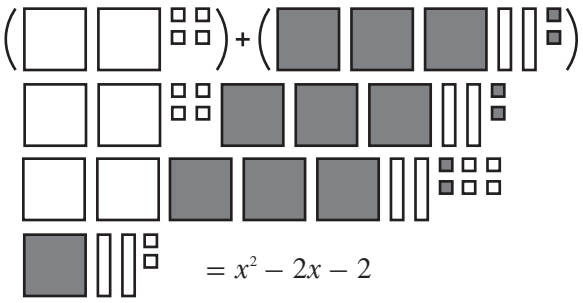
►*Solution*:
$$\begin{array}{rr} -2x^2 & -4 \\ 3x^2-2x+2 & \\ \hline x^2-2x-2 & \end{array}$$

Adding Polynomials with Algebra Tiles

Adding polynomials with algebra tiles is similar to adding polynomials horizontally.

Example 3 Add $(-2x^2-4)+(3x^2-2x+2)$

►*Solution*:



$= x^2-2x-2$

Subtracting Polynomials Horizontally

To subtract a polynomial, we change the sign of every term. This is the same as multiplying by -1. For example $-(-2x^2+4x+1)$ is written as $2x^2-4x-1$ without brackets.

Example 4 Subtract $(2x^2-4x-3)-(-x^2+2x-1)$

►*Solution*: $(2x^2-4x-3)-(-x^2+2x-1)$

$2x^2-4x-3+x^2-2x+1$

$2x^2+x^2-4x-2x-3+1$

$3x^2-6x-2$

Subtracting Polynomials Vertically

To subtract a polynomial vertically, place the polynomials with like terms in the same columns in descending order of powers. Multiply the negative sign outside of the parenthesis into the polynomial, and add the polynomials. Leave a blank space if a term in one of the polynomials does not have a match in the other.

Example 5 Subtract $(2x^2-4x-3)-(-x^2+2x-1)$

►*Solution*:

$$\begin{array}{r} 2x^2-4x-3 \\ \underline{-(-x^2+2x-1)} \end{array} \quad \rightarrow \quad \begin{array}{r} 2x^2-4x-3 \\ \underline{+\ \ x^2-2x+1} \end{array}$$

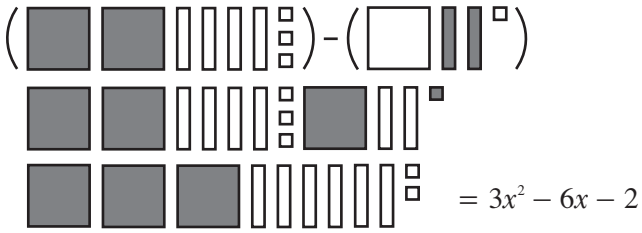
$$\begin{array}{r} 2x^2-4x-3 \\ \underline{+\ \ x^2-2x+1} \\ 3x^2-6x-2 \end{array}$$

Subtracting Polynomials with Algebra Tiles

To subtract a polynomial with algebra tiles, the signs of the polynomial tiles with a negative in front of the parentheses need to be changed before combining the tiles.

Example 6 Subtract $(2x^2-4x-3)-(-x^2+2x-1)$

►*Solution*:



$= 3x^2-6x-2$

5.2 Exercise Set

1. Add.

a) $(2x-3)+(-4x+1)$

b) $(-3x^2+2x)+(x^2-3x-2)$

c) $(x^2-2)+(x^2+2)$

d) $(2x-3)+(-x^2-3x+1)$

e) $(5-x^2+2x)+(-3x-2x^2+1)$

f) $(-3x+2-4x^2)+(-4+2x^2)$

g) $(-x^2+2-3x)+(-4x^2+x-5)$

h) $(-3+4x^2-2x)+(5x-2x^2+4)$

i) $(4x-2x^2)+(-5+x^2)$

j) $(2-3x^2+4x)+(-x^2-2x)$

k) $(-3xy-x^2-2y^2)+(-2xy+x^2-y^2)$

l) $(x^2-y^2-4xy)+(3xy-3x^2+2y^2)$

m) $(-2xy+x^2-3y^2)+(-y^2-xy+2x^2)$

n) $(3xy-x+y)+(-3y+2x-xy)$

o) $(3x-2xy+2y)+(xy-3y)+(-3y-x)$

p) $(-2y+3x+xy)+(2xy-x-y)+(-x-4xy)$

2. Add.

a) $-3x+1$
$\underline{\quad 2x-3}$

b) $-3x^2+4x$
$\underline{\quad x^2-3x-2}$

c) x^2-3
$\underline{x^2+3}$

d) $-x^2-2x+3$
$\underline{\qquad 4x-1}$

e) $\quad x^2+4x-3$
$\underline{-2x^2-2x-1}$

f) $-2x^2\qquad -3$
$\underline{\quad 4x^2-3x+1}$

g) $\quad -x^2-4x+2$
$\underline{-4x^2+\quad x+5}$

h) $\quad 3x^2-2x-5$
$\underline{-2x^2+5x-3}$

i) $\quad 3-2x^2+5x$
$\underline{-x^2+2x\qquad}$

j) $-2xy-3x^2+4y^2$
$\underline{\quad 4xy+\quad x^2-\quad y^2}$

k) $\quad 2x^2+\quad y^2-4xy$
$\underline{-3xy-3x^2+\ 2y^2}$

l) $4xy+x-y$
$\underline{2x-xy+y}$

m) $2x^2-3x+2$
$-x\ +\ 4\ -2x^2$
$\underline{\quad 3\ -\ x^2+x}$

n) $\quad xy-\quad x\ \ +y$
$-3x+\ \ y\ \ +2xy$
$\underline{-\ \ y-5xy-2x}$

3. Add by algebra tiles.

a) $(-2x^2 + x - 3) + (x^2 - 2x - 1)$

b) $(-3x - x^2 + 4) + (-2 + 2x^2 - 4x)$

c) $(-4x + 2) + (-x^2 + x - 3)$

d) $(x^2 - 3x) + (2x^2 + x - 2)$

e) $(-x^2 + 4x + 3) + (-2x^2 - 2x - 1)$

f) $(-x^2 - x) + (2x^2 + 3) + (-2x + 2)$

g) $(x^2 - 2) + (2x + 3) + (-2x^2 - x)$

h) $(xy - y + 2x) + (-2xy + 2y - x)$

4. Subtract.

a) $(4x + 2) - (-2x + 3)$

b) $(-5x + 2) - (x^2 + x - 3)$

c) $(-x^2 + 5 - 4x) - (6 - 2x)$

d) $(-2x^2 + 3x - 1) - (-x^2 + x + 3)$

e) $(-2x + x^2 - 4) - (3 - 2x^2 - 4x)$

f) $(-4x^2 + 2x) - (-2x - 3x^2 + 2)$

g) $(-3x + 2x^2) - (-3x + 2x^2 - 1)$

h) $(5x + 4y - 9z) - (-11x + 9y - 5z)$

i) $(2x^2 + 34y) - (7x^2 + 12y + 15z)$

j) $(-5x - 2y + 3z) - (-2x + 9y)$

k) $(-3x - 6y) - (9x - 4y - 25z)$

l) $(-2x - 3y) - (4x + 2y) - (x - 3y)$

m) $(4x - 2y) - (2x + 3y) - (-2x + y)$

n) $(-2xy + y - 3x) - (-y - 2x + xy)$

o) $(3xy + x - 2y) - (y - x - xy)$

p) $(x^2 - 3xy + y^2) - (-2xy + x^2 - y^2)$

5. Subtract.

a) $5x + 2$
$-\ 3x + 1$

b) $-x^2 + 2x - 4$
$-\ 5x + 1$

c) $-\ 3x + 4$
$-x^2 + x - 2$

d) $-3x^2 + 2$
$x^2 - 3x$

e) $-2x^2 - x + 1$
$-4x^2 + 3x - 2$

f) $-3x^2 - 2x$
$4x - 3$

g) $3x^2 - 4x + 5$
$-x^2 - 3x + 3$

h) $-x^2 - 4$
$-\ 3x + 2$

i) $x^2 - 3xy - y^2$
$x^2 + 3xy - y^2$

j) $2x^2 - 5 + 3x$
$-2x + x^2 - 3$

k) $-x^2 - xy + y^2$
$-2xy + 3y^2 - 4x^2$

l) $-3xy + x - 4y$
$-y - xy + 2x$

m) $4x - 3xy + 2y$
$-2y + xy - 4x$

n) $-4y + 2xy$
$2x - 3y - 4xy$

6. Subtract by algebra tiles.

a) $(-2x+3)-(-4x-1)$

b) $(-x^2+2)-(x-3)$

c) $(x^2-4-3x)-(2x-1+2x^2)$

d) $(-3x^2-2+x)-(-x^2+2x-1)$

e) $(3x^2-x+1)-(2x-x^2)$

f) $(-2x+x^2-1)-(2x^2-3)$

g) $(-2x^2+3x)-(x^2-x+1)$

h) $(2xy-y+2x)-(xy+x-y)$

7. Simplify the combined operations.

a) $(2x-3)-(4x+2)+(3x-4)$

b) $(x^2-1)+(2x^2+3)+(3x^2-4x)$

c) $-(2-x^2+x)+(x-3x^2+4)-(2x^2-1)$

d) $(-2x-1)-(x^2-3x+2)-(-3-x^2)$

e) $(-3x^2+4x-6)-(-2x^2-4)+(5x-3)$

f) $(-4+x^2+x)-(-6-x+3x^2)-(-x^2+x)$

g) $(-y^2-7y+y)+(-2y^2+5y-2)-(-6y^2+y)$

h) $(2xy-x-y)-(-3xy+y)+(-x+2y)$

i) $(-x-xy-y)-(2xy-y)-(x-y)$

j) $(-3x+2y-xy)+(-2xy-y)-(x-y)$

k) $(-3x+4y)+(6x-5y)-(2x+11y-5z)$

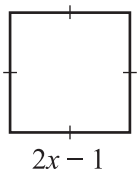
l) $(-2x^2+3xy-1)-(-2xy+x^2-1)+(x^2-5xy)$

m) $(-2xy+9z)+(4x^2-11z)-(6x^2+8xy-11z)$

n) $(3x^2+5y-6z)-(-4x^2-7y+2z)-(x^2-3y-4z)$

8. Determine the perimeter of each figure. Simplify. (All lengths are in cm)

a)



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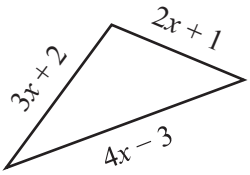

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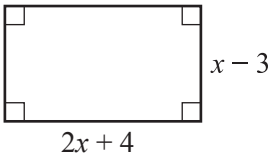
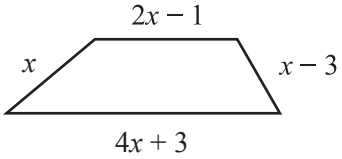
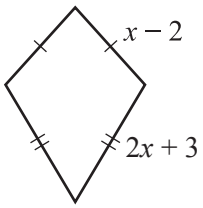


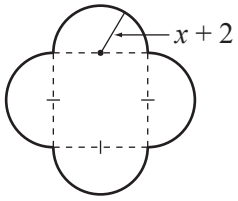
d)



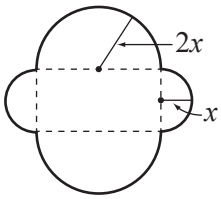
e)



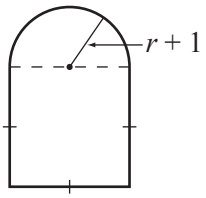
f)



g)

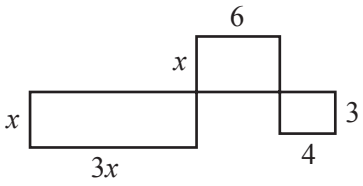


h)

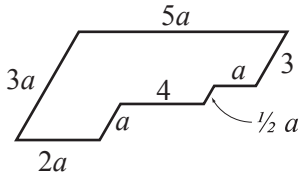


9. Determine the perimeter. Simplify. (All lengths are in cm)

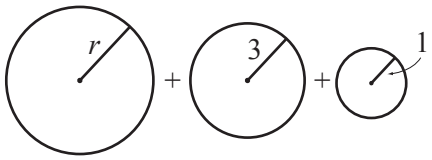
a)



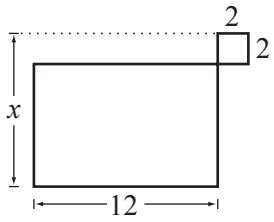
b)



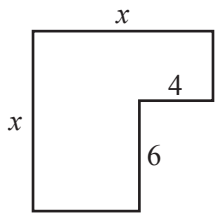
c)



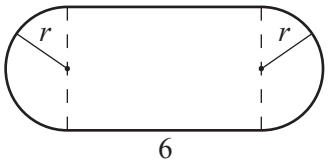
d)



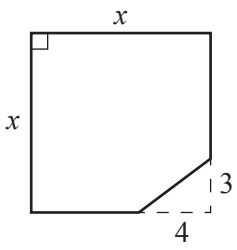
e)



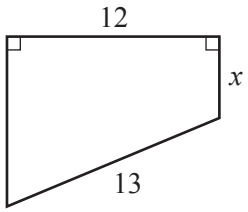
f)



g)

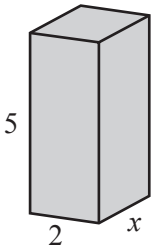


h)

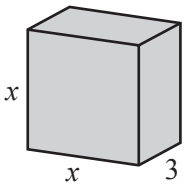


10. Find the surface area of each figure. (All lengths are in cm)

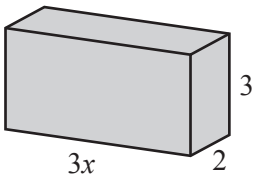
a)



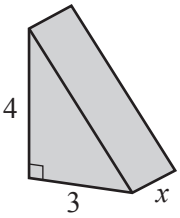
b)



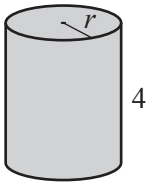
c)



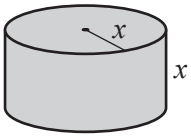
d)



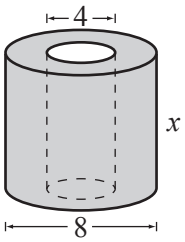
e)



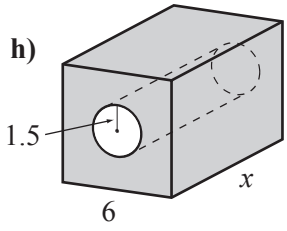
f)



g)



h)



5.3 Multiplying Polynomials

Multiplying Monomials

To multiply two monomials, multiply the numerical factors, then multiply the variable factors.

Example 1 Multiply $(2x^2)(-3x)$

►*Solution*: $(2x^2)(-3x) = (2)(-3)(x^2)(x) = -6x^{2+1} = -6x^3$

To find the product of a monomial and a polynomial with more than one term, we use the distributive property. Recall $2(3+4) = 2\times 3 + 2\times 4 = 6 + 8 = 14$. The general form of the distributive property is given by $a(b+c) = ab + ac$. This suggests the following rule:

Multiplying Polynomials by Monomials

To multiply a polynomial with more than one term by a monomial, use the distributive property to remove parentheses, then simplify.

Example 2 Multiply $-4(3-x)$

►*Solution*: $-4(3-x) = -4\times 3 + (-4)\times(-x) = -12 + 4x$

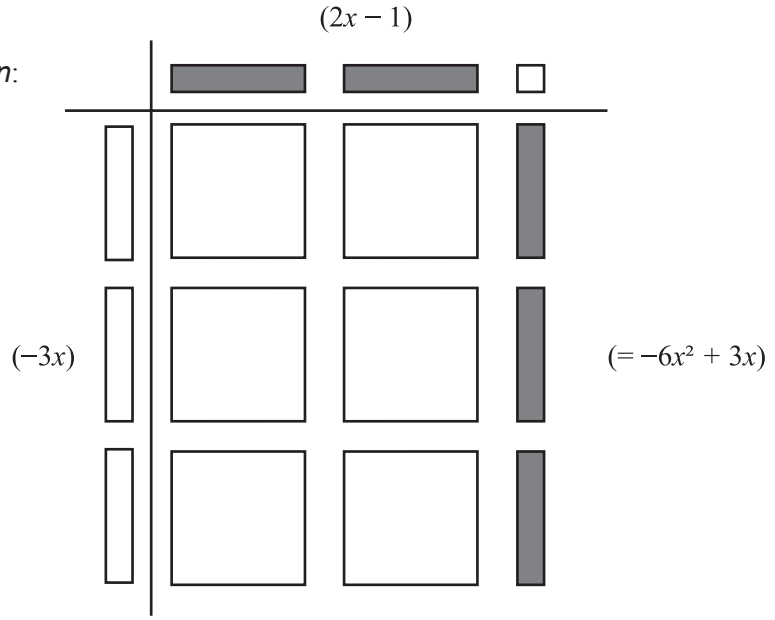
Example 3 Multiply $-4x(x^2 - 3x + 1)$

►*Solution*: $-4x(x^2 - 3x + 1) = (-4x)(x^2) + (-4x)(-3x) + (-4x)(1) = -4x^3 + 12x^2 - 4x$

Multiplying Polynomials Using Algebra Tiles

Example 4 Multiply $-3x(2x-1)$ using algebra tiles.

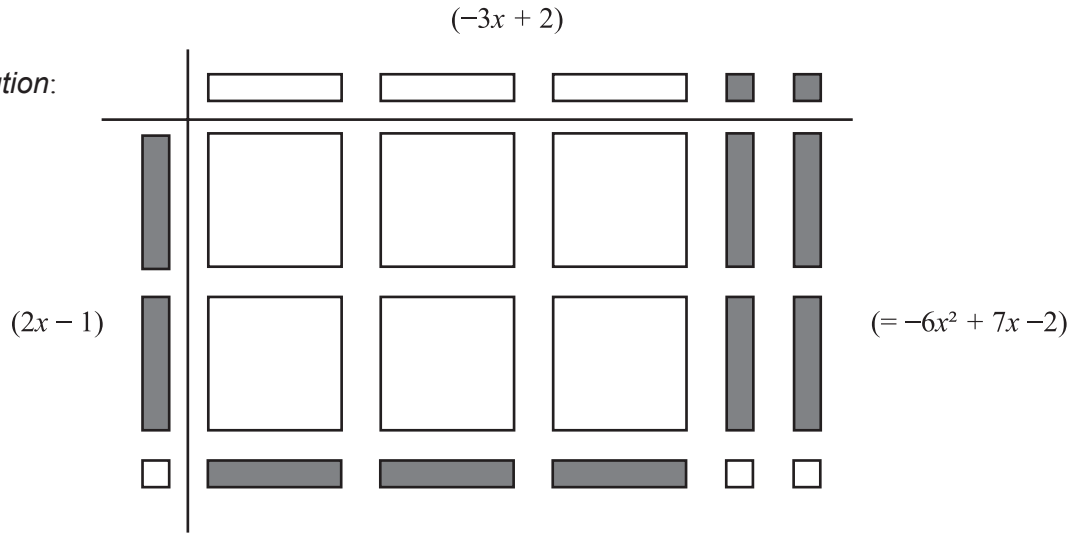
► *Solution*:



Therefore $-3x(2x-1) = -6x^2 + 3x$.

Example 5 Multiply $(-3x+2)(2x-1)$ using algebra tiles.

► *Solution*:



Therefore $(-3x+2)(2x-1) = -6x^2 + 7x - 2$.

5.3 Exercise Set

1. Find each product.

a) -5×3 _____ **b)** $-5\times 3x$ _____

c) $-5x\times 3x$ _____ **d)** $-5x\times 3x^2$ _____

e) $-5x^2\times 3x^2$ _____ **f)** $-4\times(-7)$ _____

g) $-4\times(-7x)$ _____ **h)** $-4x\times(-7x)$ _____

i) $-4x\times(-7x^2)$ _____ **j)** $-4x^2\times(-7x^2)$ _____

2. Find each product.

a) $2(x+3)$ _____ **b)** $2(x-3)$ _____

c) $-2(x+3)$ _____ **d)** $-2(-x-3)$ _____

e) $2x(x+3)$ _____ **f)** $2x(x-3)$ _____

g) $-2x(x+3)$ _____ **h)** $-2x(x-3)$ _____

i) $2x(x^2+3)$ _____ **j)** $2x(x^2-3)$ _____

k) $-2x(x^2+3)$ _____ **l)** $-2x(x^2-3)$ _____

m) $2x^2(x^2+3)$ _____ **n)** $2x^2(x^2-3)$ _____

o) $-2x^2(x^2+3)$ _____ **p)** $-2x^2(x^2-3)$ _____

q) $2x^2(x^2+3x)$ _____ **r)** $2x^2(x^2-3x)$ _____

s) $-2x^2(x^2+3x)$ _____ **t)** $-2x^2(x^2-3x)$ _____

3. Find each product.

a) $3(x-4)$ _____ **b)** $-3(a-4)$ _____

c) $4(y+5)$ _____ **d)** $-5(b^2-2)$ _____

e) $3x(x-2)$ _____ **f)** $-2y(y-1)$ _____

g) $-2c(4c^2-c)$ _____ **h)** $xy(x+y)$ _____

i) $m^2(m-1)$ _____ **j)** $p^2(-2p+4)$ _____

k) $-ab(a^2+b^2)$ _____ **l)** $-d^2(d^2-e)$ _____

m) $3xy(-2x+5y)$ _____ **n)** $-2n^2(3n-4)$ _____

o) $2z^2(z^2-3z)$ _____ **p)** $4tp(-2t^2+3p)$ _____

q) $2x^2(-x^2+3x)$ _____ **r)** $2x^2(-x^2-3x)$ _____

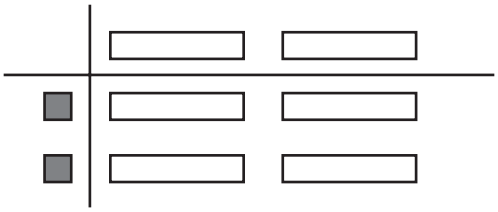
s) $-2x^2(-x^2+3x)$ _____ **t)** $-2x^2(-x^2-3x)$ _____

u) $2(x^2-3y+4z)$ _____ **v)** $2x(x+2y-3z)$ _____

w) $2x^2(-x^2+4y+3z)$ _____ **x)** $-2x^2(3x^2-2y^2+4z^2)$ _____

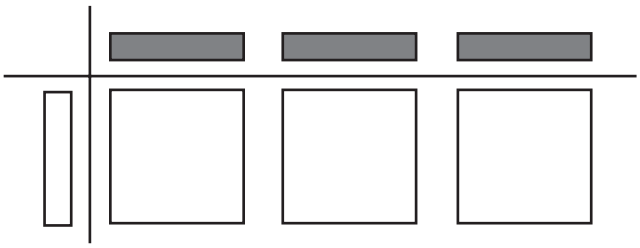
4. Determine the algebra tile multiplication indicated.

a)



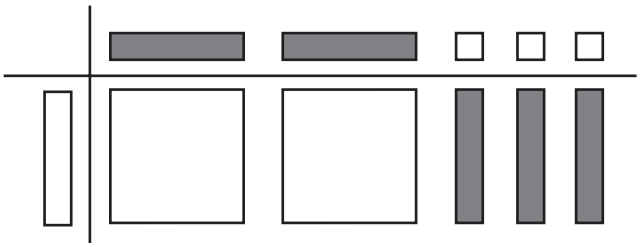
_____

b)



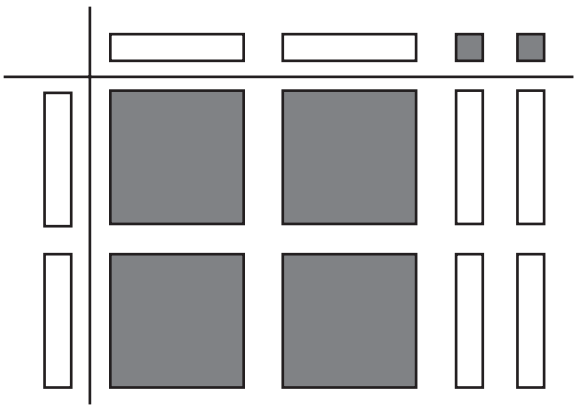
_____

c)



_____

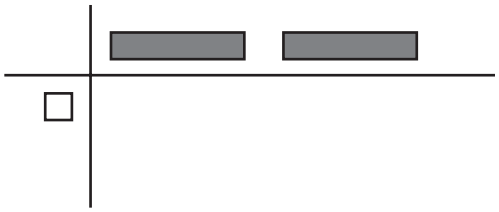
d)



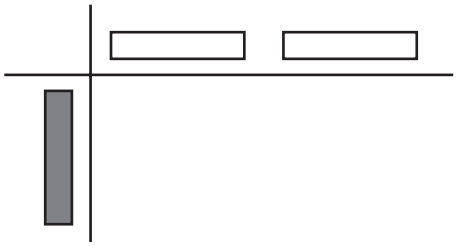
_____

5. Fill in the algebra tile operation

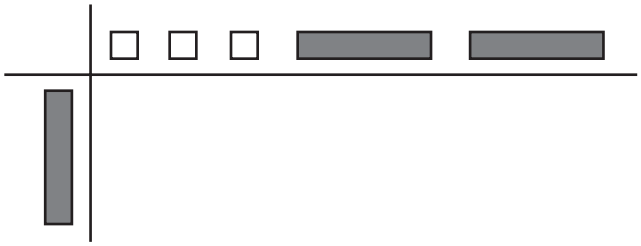
a)



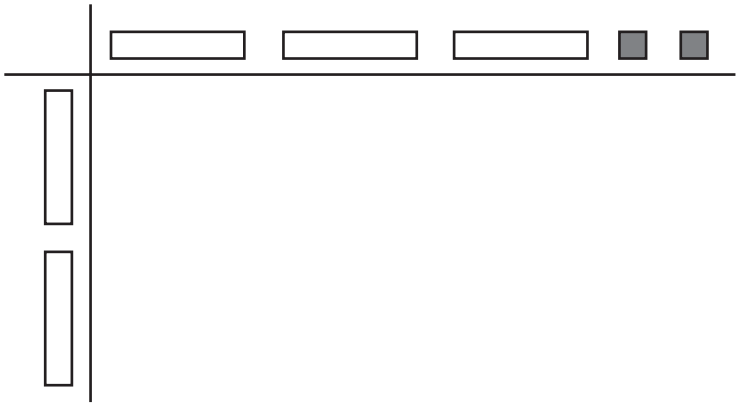
b)



c)



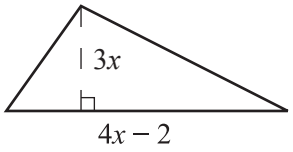
d)



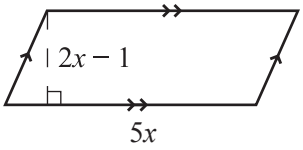
e)

6. Determine the area.

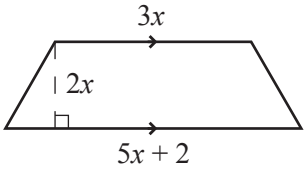
a)



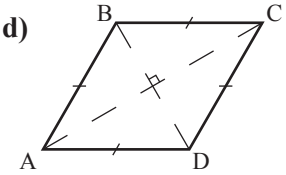
b)



c)



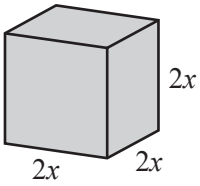
d)



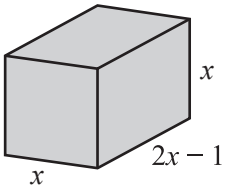
$AC = 12x$
$BD = 8x - 2$

7. Determine the volume.

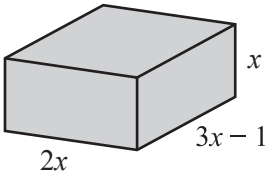
a)



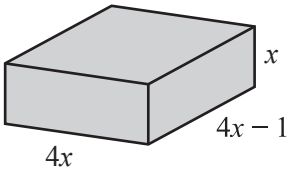
b)



c)



d)



8. Explain the error, then write the correct solution.

a) $-2(x-3) = -2x-6$

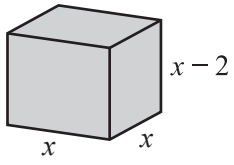
b) $5(2x-4) = 10x-15$

c) $-2x(x+7) = -2x^2-14$

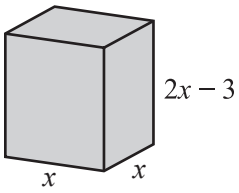
d) $8(-3x-1) = -24x+8$

9. Determine the surface area of the open top box.

a)

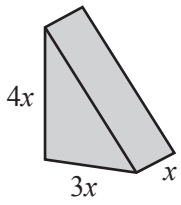


b)

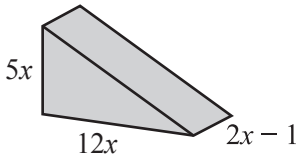


10. Determine the surface area of the right triangular prism.

a)

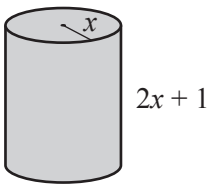


b)

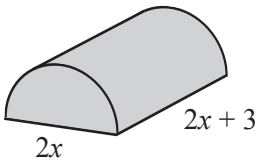


11. Determine the surface area.

a)



b)



5.4 Dividing Polynomials

Dividing a Polynomial by a Constant

To divide a polynomial by a constant, divide each term of the polynomial by the constant.

$$\frac{a+b+\cdots+z}{c}=\frac{a}{c}+\frac{b}{c}+\cdots+\frac{z}{c},\ c\neq 0$$

Example 1 Divide $\frac{9x^2-3x+6}{3}$.

►*Solution*: $\frac{9x^2-3x+6}{3}=\frac{9x^2}{3}-\frac{3x}{3}+\frac{6}{3}=3x^2-x+2$

Example 2 Divide $\frac{16x^2-12x+8}{-4}$

►*Solution*: $\frac{16x^2-12x+8}{-4}=\frac{16x^2}{-4}-\frac{12x}{-4}+\frac{8}{-4}=-4x^2+3x-2$

Dividing a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

$$\frac{a+b+\cdots+z}{c}=\frac{a}{c}+\frac{b}{c}+\cdots+\frac{z}{c},\ c\neq 0$$

Example 3 Divide $\frac{-9x^2+6xy}{3x}$

►*Solution*: $\frac{-9x^2+6xy}{3x}=-\frac{9x^{\not 2}}{3\not x}+\frac{6\not xy}{3\not x}=-3x+2y$

Example 4 Divide $\frac{-25x^2y+10xy^2-5xy}{-5xy}$

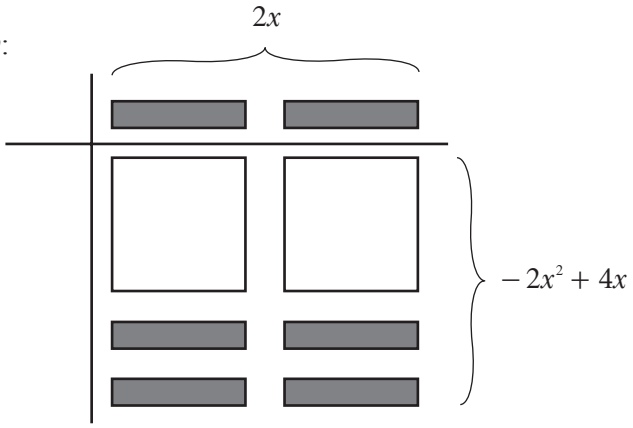
►*Solution*: $\frac{-25x^2y+10xy^2-5xy}{-5xy}=\frac{-25x^2y}{-5xy}+\frac{10xy^2}{-5xy}-\frac{5xy}{-5xy}=5x-2y+1$

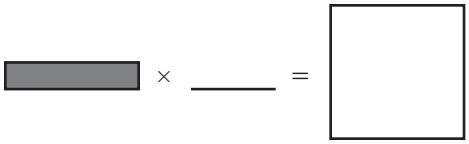
Using Algebra Tiles for Division

When dividing by a monomial with algebra tiles, the process of multiplication is reversed. The solution will be found on the left column of the table.

Example 5 Divide $\frac{-2x^2+4x}{2x}$ with algebra tiles.

► *Solution*:





x $\quad -x^2$

Therefore the unknown value is  because $x \cdot (-x) = -x^2$.

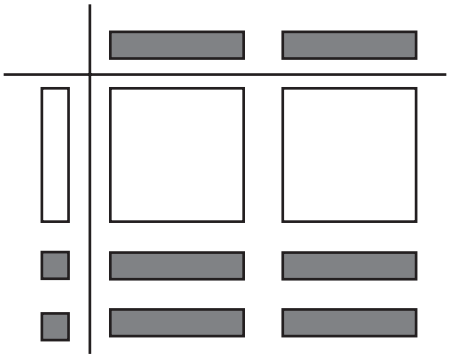
$-x$

x $\quad 2x$

Therefore the unknown value is  because $x \cdot 2 = 2x$.

2

Therefore the algebra tile diagram must have:



or $\frac{-2x^2+4x}{2x} = -x + 2$

5.4 Exercise Set

1. Divide.

a) $\frac{2x+4}{2}$ _____ **b)** $\frac{2x-4}{2}$ _____

c) $\frac{-2x+4}{2}$ _____ **d)** $\frac{-2x-4}{2}$ _____

e) $\frac{6x+12y}{3}$ _____ **f)** $\frac{6x-12y}{3}$ _____

g) $\frac{-6x+12y}{3}$ _____ **h)** $\frac{-6x-12y}{3}$ _____

i) $\frac{-10x+25y}{-5}$ _____ **j)** $\frac{10x-25y}{-5}$ _____

k) $\frac{-10x-25y}{-5}$ _____ **l)** $\frac{-10x+5}{5}$ _____

2. Divide.

a) $\frac{5x^2-10x-15}{5}$ _____ **b)** $\frac{5x^2-10x+15}{5}$ _____

c) $\frac{-5x^2+10x-15}{5}$ _____ **d)** $\frac{-5x^2-10x-15}{5}$ _____

e) $\frac{5x^2-10x+15}{-5}$ _____ **f)** $\frac{-5x^2-10x+15}{-5}$ _____

g) $\frac{5x^2-10xy+15y^2}{-5}$ _____ **h)** $\frac{-5x^2+10xy-15y^2}{-5}$ _____

i) $\frac{-5x^2+10x+5}{-5}$ _____ **j)** $\frac{-5x^2+10x-5}{-5}$ _____

3. Divide.

a) $\frac{6x^2 + x}{x}$ _____ **b)** $\frac{6x^2 - x}{x}$ _____

c) $\frac{-6x^2 + x}{x}$ _____ **d)** $\frac{-6x^2 - x}{x}$ _____

e) $\frac{-6x^2 + x}{-x}$ _____ **f)** $\frac{6x^2 - x}{-x}$ _____

g) $\frac{9x^2 + 15xy}{3x}$ _____ **h)** $\frac{9x^2 - 15xy}{3x}$ _____

i) $\frac{-9x^2 + 15xy}{3x}$ _____ **j)** $\frac{-9x^2 - 15xy}{3x}$ _____

k) $\frac{-9x^2 + 15xy}{-3x}$ _____ **l)** $\frac{9x^2 + 15xy}{-3x}$ _____

m) $\frac{9x^3 - 6x^2 + 3x}{3x}$ _____ **n)** $\frac{-9x^3 + 6x^2 - 3x}{3x}$ _____

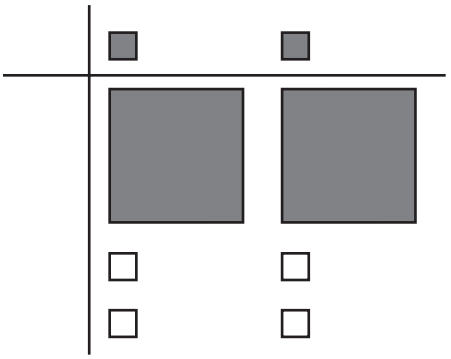
o) $\frac{9x^3 - 6x^2 - 3x}{3x}$ _____ **p)** $\frac{-9x^3 - 6x^2 + 3x}{3x}$ _____

q) $\frac{9x^3 - 6x^2 + 3x}{-3x}$ _____ **r)** $\frac{-9x^3 + 6x^2 - 3x}{-3x}$ _____

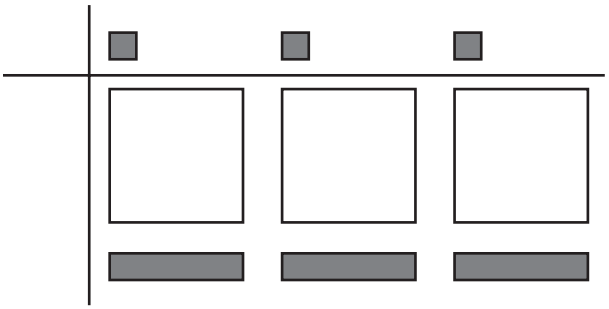
s) $\frac{9x^3 - 6x^2 - 3x}{-3x}$ _____ **t)** $\frac{-9x^3 - 6x^2 + 3x}{-3x}$ _____

4. Use algebra tiles to perform each division.

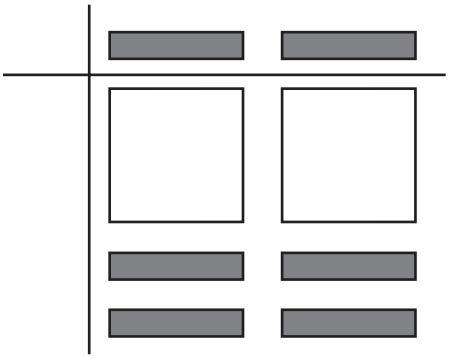
a) $\frac{2x^2 - 4}{2}$



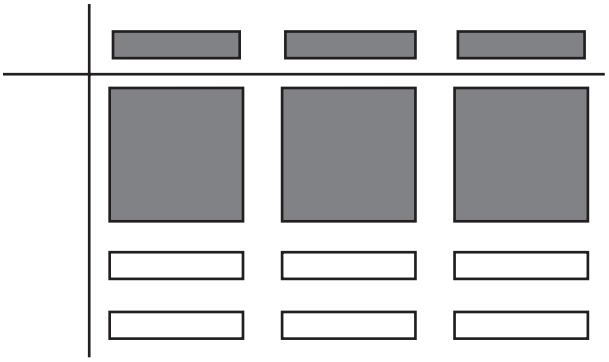
b) $\frac{-3x^2 + 3x}{3}$



c) $\frac{-2x^2 + 4x}{2x}$

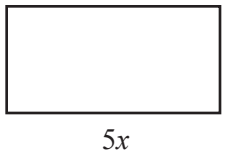


d) $\frac{3x^2 - 6x}{3x}$



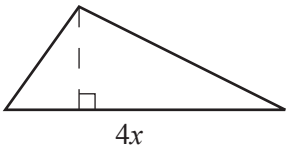
5. Determine the height.

a)



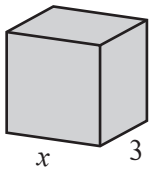
Area $= 10x^2 - 15x$

b)



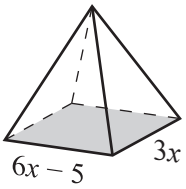
Area $= 12x^2 - 6x$

c)



Volume $= 18x^2 - 27x$

d)



Volume $= 24x^2 - 20x$

6. The perimeter of a square is $(8x^2 + 4x - 16)$ meters. Find the length of each side.

7. The area of a parallelogram is $(12x^2 - 6x)$ square metres. If its base is $3x$ meters, what is its height?

8. The volume of a rectangular solid shaped swimming pool is $(96x^2 - 24x)$ square metres. If its width is $3x$ m, and its height is 2 m, determine its length.

9. The volume of a pyramid with a rectangular base is $(12x^2 - 4x)$ square metres. If the width of the base is 6 m and the length is $(3x - 1)$ m, determine the height.

10. Perform the combined operations.

a) $2x(x+3)+\frac{6x^2-4x+8}{2}$

b) $-\frac{(3x^2-6x+9)}{3}-4x(2-x)$

c) $\frac{-2x^2(3-x)}{2x}+\frac{6x(2x-4)}{2}$

d) $\frac{x(-6x^2+9x)}{-3x}-\frac{6-4x}{2}$

e) $x(2x-3y+1)-\frac{4x^2+8xy-12x}{4}$

f) $\frac{-(4x^2-2x)}{2x}-3(1-x)$

g) $\frac{(8x^2-6x)}{2x}+\frac{4x(1-x)}{x}$

h) $\frac{(6x-9x^2)}{3x}-\frac{(8x^2-12x)}{4}$

i) $\frac{-x(6x^2-4x+2)}{2x}-\frac{(6x-4x^2+8)}{2}$

j) $\frac{(2x-3x^2+5)}{-1}-2x(2-3x)$

k) $\frac{-3x(8x-4x^2+6)}{-2}+\frac{4x(6-8x)}{2}$

l) $\frac{-5x(3-6x)}{3}-\frac{4x(10x-5)}{5}$

5.5 Chapter Review

Section 5.1

1. Determine the degree of the polynomials.

a) 5^3 _____ **b)** 2^2y^2 _____

c) $3x^3 + 2x^2y^2$ _____ **d)** $\sqrt{3}\,x^3 - 3x^2 + 5$ _____

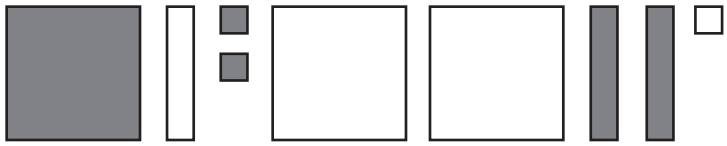
2. Simplify each polynomial.

a) $2x^2 - y^2 + 3x^2 + 2y^2$ **b)** $4x^2 - 3y + 2x - y$

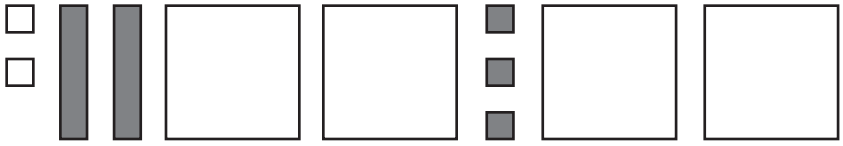
c) $-2xy^2 - x^2y + xy^2 - x^2y$ **d)** $-2xy - 3yx + 4xy + 5yx$

3. Write a polynomial expression in simplified form for the algebra tiles.

a)



b)



Section 5.2

4. Add.

a) $(3 - 2x^2 + 4x) + (-2x + x^2 - 5)$ **b)** $(2xy + x - y) + (-xy - 2x - y)$

c) $(-5 + 3x^2 + 7x) + (-x^2 + 4 - 3x)$ **d)** $(-4xy - x + 3y) + (-xy + 7y - 3x)$

5. Add using algebra tiles.

a) $(2x - x^2 + 3) + (-3x + 2x^2 - 1)$

b) $(-2xy + x + 2y) + (-xy - x - y)$

6. Subtract.

a) $(-3x + 2x^2 + 4) - (-2 - x^2 + 4x)$

b) $(2xy - x + 3y) - (y - x + xy)$

c) $(4x - 5x^2 - 1) - (5 - 3x^2 - 2x)$

d) $(3x - 2x^2 + 4) - (-2x^2 + 3x - 4)$

Section 5.3

7. Find the product.

a) $3(x^2 - 2x + 3)$

b) $-3x(-x^2 + 2x - 3)$

c) $3x^2(2x^2 - x + 1)$

d) $-2xy(-x^2 + 2xy + y^2)$

e) $2xy^2(x^2 - xy + y^2)$

f) $-2xyz(x - y + z)$

g) $3xy(2x - 3y + 2z)$

h) $-2xyz(-x^2 + y^2 - z^2)$

i) $-2xy(3x - y + z)$

j) $3xyz(-2x^2 - 3y^2 + 4z^2)$

k) $2x^2y(-x + 2y - 3z)$

l) $-3y^2z(-2x + y - 3z)$

Section 5.4

8. Find the quotient.

a) $\frac{3x^2 - 9x + 6}{3}$

b) $\frac{-4x^2 + 8x + 6}{-2}$

c) $\frac{10x^2 - 5xy}{5x}$

d) $\frac{-7x^2 + 21xy}{-7x}$

e) $\frac{-4x^2 + 8xy - 12x}{2x}$

f) $\frac{a^2bc - ab^2c + abc^2}{-abc}$

g) $\frac{-a^2b^2c + ab^2c^2 - a^2b^2c^2}{ab^2c}$

h) $\frac{-a^2bc^2 - a^2bc + abc^2}{-abc}$

9. Perform the combined operations.

a) $3x(2x - 3) - \frac{6x^2 + 4x - 8}{2}$

b) $\frac{-8x^2(2 - x)}{4x} + 3x(2 - x)$

c) $-2x(3x - 2y + 1) - \frac{8x - 4x^2 + 2xy}{2}$

d) $\frac{-6x^2(4 - 3x)}{2x} - 2x(3 - 2x)$

e) $\frac{-3x^2(2 - 5x)}{-x} + 3x(4x - 3)$

f) $\frac{-4x^2(3x - 3y + 2)}{2x} - \frac{6x^2(3 - 2x + y)}{-x}$