Name: Key

June 2019

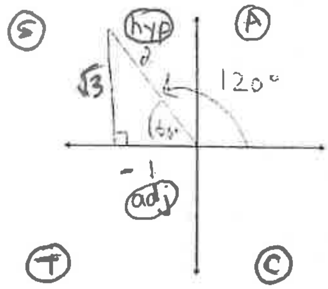
Pre-Calculus 11 → Practice Final

/46

Show all of your work.

SOHCAHTOA

1a) Sketch 120° in standard position (0.5 marks). b) State the reference angle (0.5 marks). c) Determine the exact value of cos120 ° (1 mark)



$\cos 60 = \frac{1}{2}$

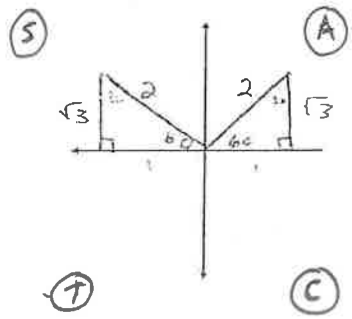
$\cos 120 = \frac{-1}{2}$ $\frac{adj}{hyp}$

ANSWERS:

b) REF ANGLE: 60°

c) $\frac{-1}{2}$

2) Solve for θ (2 marks): $\sin\theta = \frac{\sqrt{3}}{2}, 0° \leq \theta \leq 360°$



- $\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{3}}{2}$
- sin is (+) in QI and QII

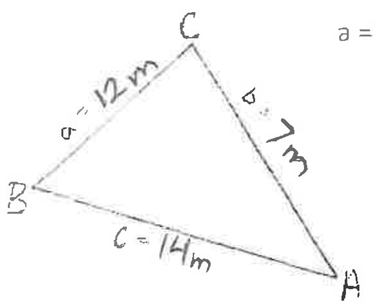
$\theta_R = 60$

$\theta = 180 - 60 = 120$

ANSWER(S):

60°, 120°

3) Sketch and solve the triangle to the nearest tenth (3 marks):



a = 12m, b = 7m, c = 14m

$c^2 = a^2 + b^2 - 2ab\cos C$

$14^2 = 12^2 + 7^2 - 2(12)(7)\cos C$

$196 - 193 = 144 + 49 - 193 - 168\cos C$

$\frac{3}{-168} = \frac{-168\cos C}{-168}$

$\cos C = -0.017857$

$C = \cos^{-1}(-0.017857)$

$\boxed{C = 91.0°}$

$\angle B = 180 - 59 - 91 = 30°$

$\frac{\sin C}{c} = \frac{\sin A}{a} \rightarrow \frac{\sin 91}{14} = \frac{\sin A}{12}$

$\sin A = \frac{12 \times \sin 91}{14}$

$\sin A = 0.857006$

$\angle A = \sin^{-1}(0.857006)$

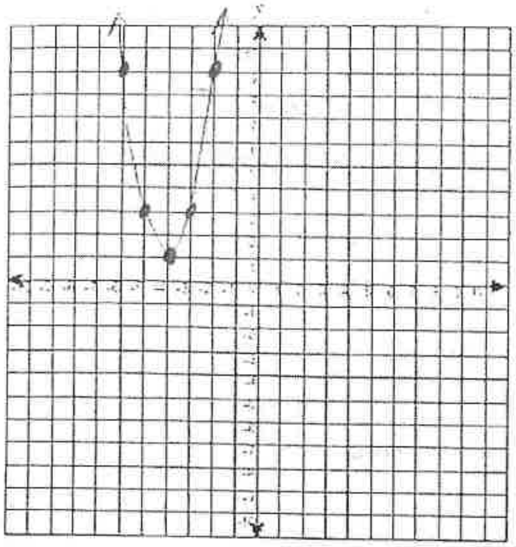
$\angle A = 59.0°$

<A= 59.0°

<B = 30°

<C = 91.0°

4a) Graph $y = 2x^2 + 16x + 33$ (2 marks) by first completing the square (2 marks), and state the b) axis of symmetry equation (0.5 marks), and c) the range (0.5 marks).



$y = (2x^2 + 16x) + 33$

$= 2(x^2 + 8x) + 33$

$= \underbrace{2(x^2 + 8x + 16 - 16)}\, + 33$

$= 2(x^2 + 8x + 16) - 32 + 33$

$y = 2(x+4)^2 + 1$

vertex $(-4, 1)$

$b = 8$

$\frac{b}{2} = 4$ save

$(\frac{b}{2})^2 = 16$ use

count: $a = +2$

over 1 up 2 (1×2)

over 2 up 8 (4×2)

over 3 up 18 (9×2)

ANSWERS:

b) $x = -4$

c) $y \geq 1$

5) The school play charges \$10 for admission, and on average, 80 people attend the show. For each \$1 increase, attendance drops by 5 people. What ticket price will maximize the school's revenue and what will that maximum revenue be?

$P = 10 + x$

$N = 80 - 5x$

let x be the number of \$1 price increases

$R = (10 + x)(80 - 5x)$

$= 800 + 80x - 50x - 5x^2$

$= -5x^2 + 30x + 800$

$= -5(x^2 - 6x) + 800$

$= -5(x^2 - 6x + 9 - 9) + 800$

$= -5(x^2 - 6x + 9) + 45 + 800$

$b = -6$

$\frac{b}{2} = -3$

$(\frac{b}{2})^2 = 9$

$\rightarrow R = -5(x-3)^2 + 845$

vertex: $(3, 845)$

$\uparrow$ x $\quad \uparrow$ R

$P = 10 + (3) = 13$

SENTENCE ANSWER:

The maximum Revenue will be \$845 with a ticket price of \$13.00

6) Solve by factoring (2 marks): $-10x = -3x^2 + 8$

$+3x^2 - 8 \quad +3x^2 - 8$

decomp!

$3x^2 - 10x - 8 = 0$

$(3)(-8)$

$-12 \times 2 = -24$

$-12 + 2 = -10$

$\underbrace{3x^2 - 12x}\underbrace{+2x - 8} = 0$

$3x(x-4) + 2(x-4) = 0$

$(x-4)(3x+2) = 0$

$x = 4 \quad x = \frac{-2}{3}$

ANSWER(S): $x = 4, \frac{-2}{3}$

7) Solve using the quadratic formula and leave answer in exact values (2 marks):

$6 - 2x^2 = 3x$

$-6 + 2x^2 \quad +2x^2 - 6$

need in $ax^2 + bx + c = 0$ form!

$2x^2 + 3x - 6 = 0$

$a = 2, b = 3, c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{(3)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 48}}{4} = \frac{-3 \pm \sqrt{57}}{4}$$

ANSWER(S): $x = \frac{-3 \pm \sqrt{57}}{4}$

8) Solve algebraically (3 marks): $x^2 - y = 10$ and $2x - 3y = -10$

substitution!

$x^2 - y = 10$

$-10 + y \quad -10 + y$

$y = (x^2 - 10)$

$2x - 3y = -10$

$2x - 3(x^2 - 10) = -10$

$2x - 3x^2 + 30 = -10$

$+10 \quad +10$

$(-3x^2 + 2x + 40 = 0) \times -1$

$3x^2 - 2x - 40 = 0$

decomp!

$(3)(-40)$

$-12 \times 10 = -120$

$-12 + 10 = -2$

$3x^2 - 12x + 10x - 40 = 0$

$3x(x-4) + 10(x-4) = 0$

$(x-4)(3x+10) = 0$

"right over left, switch sign"

$\boxed{x = 4, \ x = \frac{-10}{3}}$

$y = x^2 - 10$

$x = 4$:

$y = (4)^2 - 10$

$= 16 - 10$

$= 6$

$(4, 6)$

$x = \frac{-10}{3}$:

$y = \left(\frac{-10}{3}\right)^2 - 10$

$= \frac{100}{9} - 10$

$= \frac{100}{9} - \frac{90}{9}$

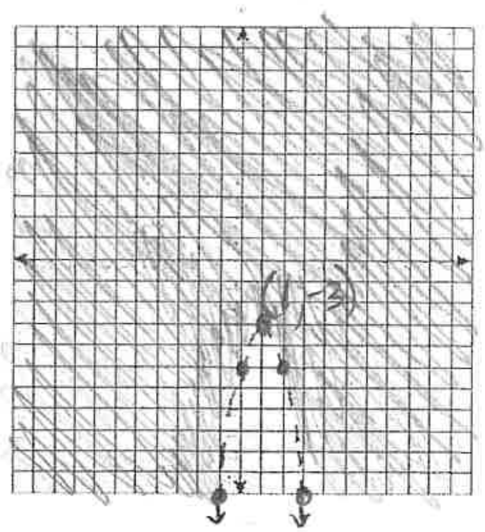
$= \frac{10}{9}$

$\left(\frac{-10}{3}, \frac{10}{9}\right)$

ANSWER(S): $(4, 6), \left(\frac{-10}{3}, \frac{10}{9}\right)$

dotted line.
shade above

9) Solve the quadratic inequality by graphing (2 marks): $y > -2(x-1)^2 - 3$



vertex $(1, -3)$

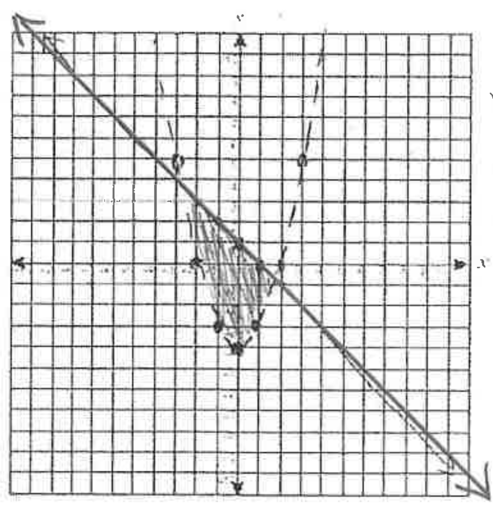
$a = -2$

over 1 down 2

over 2 down 8

over 3 down 18

10) Solve the inequality by graphing (3 marks): $y > x^2 - 4$ and $y \leq -x + 1$



$y > x^2 - 4$

vertex $(0, -4)$

$a = 1$

over 1 up 1

over 2 up 4

over 3 up 9

dotted line

shade above

$y \leq -x + 1$

y int: 1

slope: -1

solid line

shade below

11) Simplify the expression (1.5 marks). State any restrictions on the variable (0.5 marks).

$$3x\sqrt[4]{16x} - 5\sqrt[4]{x^5} + \frac{x\sqrt[4]{81x}}{3}$$

$$3x(2)\sqrt[4]{x} - 5x\sqrt[4]{x} + \frac{x(3)\sqrt[4]{x}}{3}$$

$$6x\sqrt[4]{x} - 5x\sqrt[4]{x} + x\sqrt[4]{x}$$

$$2x\sqrt[4]{x}$$

ANSWER:

$2x\sqrt[4]{x}$

Restriction:

$x \geq 0$

12) Simplify by rationalizing the denominator (2 marks). State any restrictions on the variable (0.5 marks)

$$\frac{(\sqrt{2}+\sqrt{3})\cdot(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})\cdot(\sqrt{2}+\sqrt{3})}$$

F.O.I.L.!

$$\frac{(\sqrt{2})(\sqrt{2})+(\sqrt{2})(\sqrt{3})+\sqrt{3}(\sqrt{2})+(\sqrt{3})(\sqrt{3})}{(\sqrt{2})(\sqrt{2})+(\sqrt{2})(\sqrt{3})-(\sqrt{3})(\sqrt{2})-(\sqrt{3})(\sqrt{3})}$$

$$\frac{2+\sqrt{6}+\sqrt{6}+3}{2-3} = \frac{5+2\sqrt{6}}{-1} \text{ or } -5-2\sqrt{6}$$

ANSWER: $-5-2\sqrt{6}$

Restriction: —

13) Solve. State any restrictions and check for extraneous roots (3 marks).

$$(\sqrt{4-3x})^2 = (x+8)^2$$

$$4-3x = (x+8)(x+8)$$

$$4-3x = x^2+16x+64$$

$$-4+3x \qquad +3x-4$$

$$0 = x^2+19x+60$$

$$0 = (x+15)(x+4)$$

$$x=-15 \qquad x=-4$$

CHECK: $x = -15$	CHECK: $x = -4$
$\sqrt{4-3(-15)} = (-15)+8$	$\sqrt{4-3(-4)} = (-4)+8$
$\sqrt{4+45} = -7$	$\sqrt{4+12} = 4$
$\sqrt{49} = -7$	$\sqrt{16} = 4$
$7 \neq -7$	$4 = 4$
positive only square root because it was given in the original equation	
reject $x=-15$	$x = -4$

ANSWER(S): $x = -4$

Restriction: $x \leq \frac{4}{3}$

$$4-3x \geq 0$$

$$\frac{4}{3} \geq \frac{3x}{3}$$

$$x \leq \frac{4}{3}$$

TURN OVER

14) Solve (3 marks). $\frac{x+4}{x} + \frac{16}{x^2-4x} = \frac{-3}{x-4}$

→ an equation, so eliminate fractions by multiplying each term by L.C.D.!

LCD: $x(x-4)$

$\cancel{x}(x-4) \cdot \frac{x+4}{\cancel{x}} + \frac{16}{\cancel{x(x-4)}} \cancel{x(x-4)} = \frac{-3}{\cancel{x-4}} x\cancel{(x-4)}$

identify non-permissables

$x \neq 0, 4$

FOIL!

$(x-4)(x+4) + 16 = -3x$

$x^2 - \cancel{4x} + \cancel{4x} - 16 + 16 = -3x$

$x^2 = -3x$

$+3x \quad +3x$

Factor out GCF... x!

$x^2 + 3x = 0$

$x(x+3) = 0$

$x = 0, -3$

Non permissible value

so $x = -3$

ANSWER(S):

$x = -3$

Non-permissible(s):

$x \neq 0, 4$