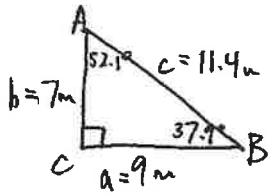
Pre-Calc 11 - Final Exam Review - Booklet (1) KEY

Ch. 7 Trig

(1a) $\angle C = 90°$ $a = 9m$ $b = 7m$

$c = 11.4m$ $\angle A = 52.1°$ $\angle B = 37.9°$



① $\tan A = \frac{9}{7}$

$\angle A = 52.1°$

② $\angle B = 180 - 90 - 52.1°$

$= 37.9°$

③ side c

$a^2 + b^2 = c^2$

$9^2 + 7^2 = c^2$

$81 + 49 = c^2$

$130 = c^2$

$c = 11.4m$

(1b) oblique ΔABC

$\angle A = 76.2°$ $a = 84.8m$

$\angle B = 38.2°$ $b = 54m$

$\angle C = 65.6°$ $c = 79.5m$

① sin law to find side c

$\frac{\sin 38.2°}{54} = \frac{\sin 65.6°}{c}$

$c \sin 38.2° = 54 \sin 65.6°$

$0.6184c = 49.1769$

$c = 79.5m$

② To find $\angle A$.

$180 - 38.2 - 65.6$

$\angle A = 76.2°$

③ sin law to find side a

$\frac{\sin 76.2°}{a} = \frac{\sin 38.2°}{54}$

$a \sin 38.2° = 54 \sin 76.2$

$0.6184a = 52.44$

$a = 84.8m$

(1c) oblique ΔGHK

$\angle G = 67°$ $g = 8.8cm$

$\angle H = 30.7°$ $h = 4.9cm$

$\angle K = 82.3°$ $k = 9.5cm$

① use cos law to find side g:

$g^2 = h^2 + k^2 - 2hk \cos G$

$g^2 = 4.9^2 + 9.5^2 - 2(4.9)(9.5)\cos 67°$

$g^2 = 24.01 + 90.25 - 36.38$

$g = 8.8cm$

② use sin law to find $\angle H$

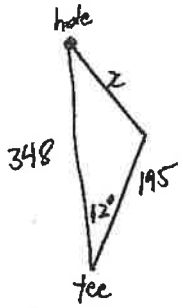
$\frac{\sin 67°}{8.825} = \frac{\sin H}{4.9}$

$\angle H = 30.7°$

③ $\angle K = 180 - 67° - 30.7°$

$= 82.3°$

(2)



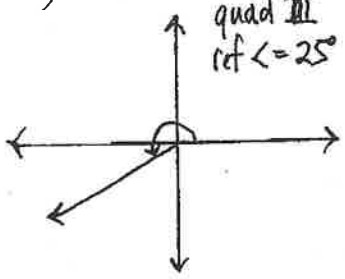
cos law

$x^2 = 348^2 + 195^2 - 2(348)(195)\cos 12°$

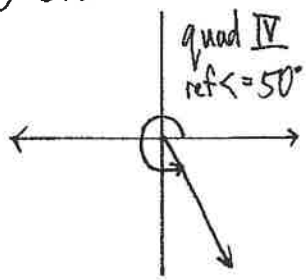
$x = 162$

The ball is 162 yards from the hole

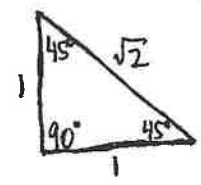
(3) a) 205°

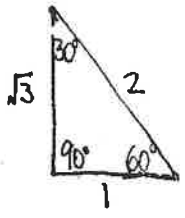


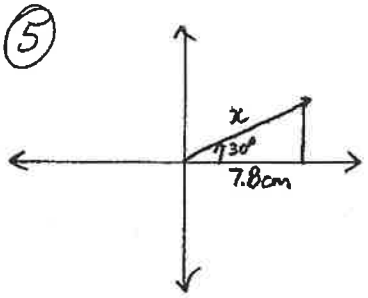
(b) 310°



(4)





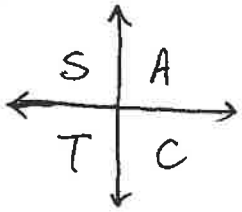
(5) 

$\cos 30° = \frac{7.8}{x}$

$x = \frac{7.8}{\cos 30°}$

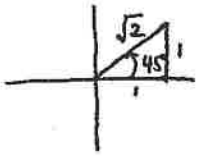
$x = 9.0\text{cm}$

The terminal arm is 9.0cm.

(6) 

The letters explain which trigonometric ratios are a positive value in each respective quadrant.
quad I ⇒ A ⇒ All trig ratios (sin, cos, tan) are positive
quad II ⇒ S ⇒ Sin is the only trig ratio that's pos (cos and tan are neg)
quad III ⇒ T ⇒ tan only tan is pos.
quad IV ⇒ C ⇒ cos only cos is pos

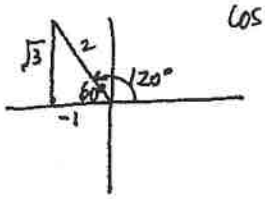
(7) (a) tan 45°



$\tan 45° = \frac{1}{1}$

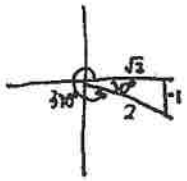
$\tan 45° = 1$

(b) cos 120°

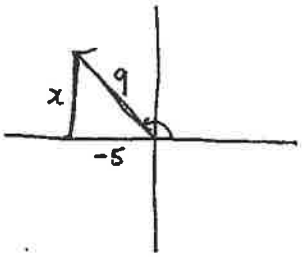


$\cos 120° = \frac{-1}{2}$

(c) sin 330°



$\sin 330° = \frac{-1}{2}$

(8) 

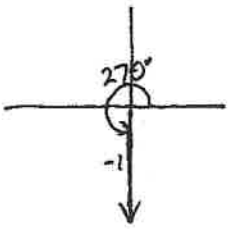
$\tan\theta = -\frac{2\sqrt{14}}{5}$

$x^2 + (-5)^2 = 9^2$

$81 - 25 = x^2$

$x = \sqrt{56}$

$x = 2\sqrt{14}$

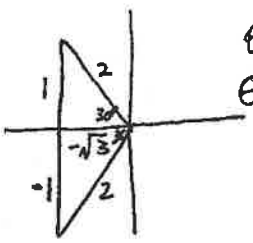
(9) 

$\sin 270° = \frac{-1}{1} = -1$

$\cos 270° = \frac{0}{1} = 0$

$\tan 270° = \frac{-1}{0} = \text{undef}$

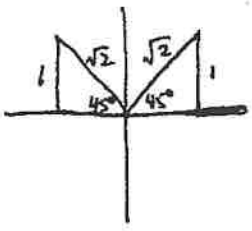
(10) (a) $\cos\theta = \frac{-\sqrt{3}}{2}$ quads II and III



$\theta = 150°$

$\theta = 210°$

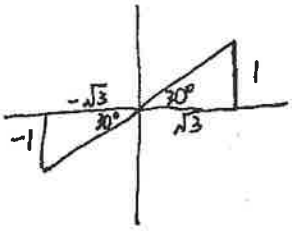
(b) $\sin\theta = \frac{1}{\sqrt{2}}$ quads I and II



$\theta = 45°$

$\theta = 135°$

(c) $\tan\theta = \frac{1}{\sqrt{3}}$ quads I and III

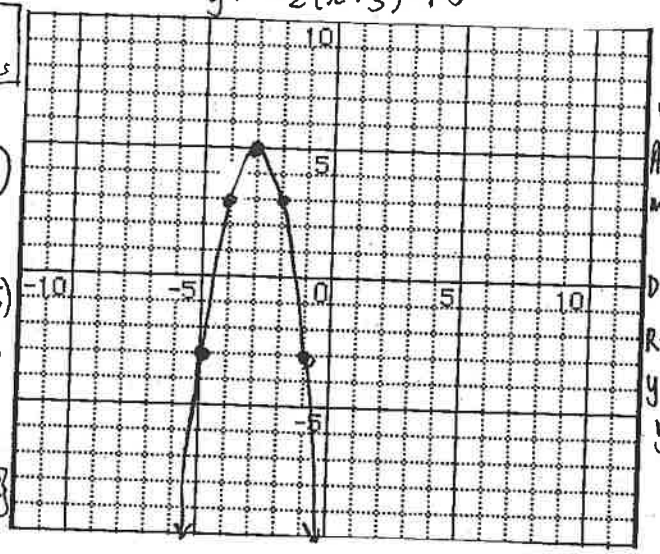


$\theta = 30°$

$\theta = 210°$

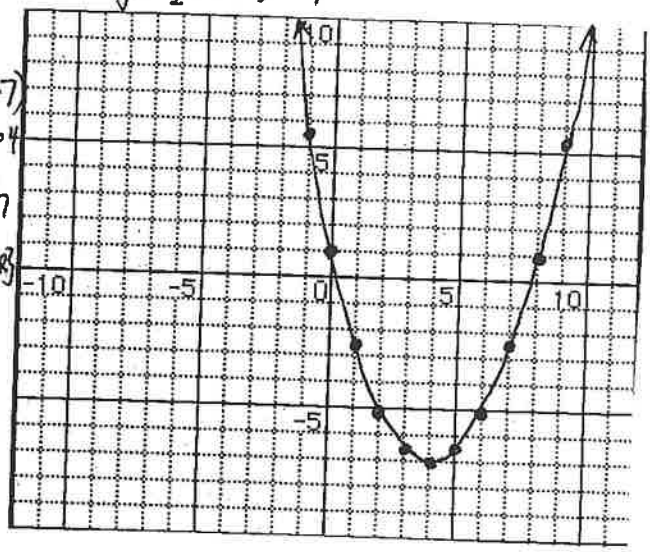
Ch 2 Quadratics

(11) $y = -2(x+3)^2+5$



(a) Vertex $(-3,5)$
A of S $x=-3$
max @ $y=5$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \leq 5, y \in \mathbb{R}\}$

(b) $y = \frac{1}{2}(x-4)^2-7$



V $(4,-7)$
A of S $x=4$
min @ $y=-7$
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq -7, y \in \mathbb{R}\}$

(12) $y = \pm a(x \pm p)^2 \pm q$

$-29 = a(7-4)^2-2$

$-29 = a(3)^2-2$

$-29 = 9a-2$

$-27 = 9a$

$a = -3$

Answer: $y = -3(x-4)^2-2$

(13) (a) $y = x^2-4x-1$

$b=-4$ $\frac{-4}{2}=-2$ save $(-2)^2=4$ use

$y = x^2-4x+4-4-1$

$y = (x^2-4x+4)-5$

$y = (x-2)^2-5$

vertex $(2,-5)$ a of s: $x=2$

min @ $y=-5$ D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq -5, y \in \mathbb{R}\}$

(6) $y = (3x^2-12x)+11$

$y = 3(x^2-4x)+11$

$b=-4$ $\frac{-4}{2}=-2$ save $(-2)^2=4$ use

$y = 3(x^2-4x+4-4)+11$

$y = 3(x^2-4x+4)-12+11$

$y = 3(x-2)^2-1$ vertex $(2,-1)$

a of s: $x=2$ / min @ $y=-1$

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq -1, y \in \mathbb{R}\}$

(13c) $y = -3x^2+5x$

$y = -3(x^2-\frac{5}{3}x)$

$b=\frac{-5}{3}$ $\frac{b}{2}=\frac{-5}{6}$, $(\frac{-5}{6})^2=\frac{25}{36}$

save × use

$y = -3(x^2-\frac{5}{3}x+\frac{25}{36}-\frac{25}{36})$

$y = -3(x^2-\frac{5}{3}x+\frac{25}{36})+\frac{75}{36}$

$y = -3(x-\frac{5}{6})^2+\frac{75}{36}\div 3$

$y = -3(x-\frac{5}{6})^2+\frac{25}{12}$

vertex $(\frac{5}{6}, \frac{25}{12})$ a of s: $x=\frac{5}{6}$

max @ $y=\frac{25}{12}$

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \leq \frac{25}{12}, y \in \mathbb{R}\}$

(14) (a) $t=0$

$h = -16(0)^2+384(0)+50$

$h = 50$m

The ball is 50m above the ground when thrown.

(b) $h = (-16t^2+384t)+50$

$h = -16(t^2-24t)+50$

$b=-24$ $\frac{-24}{2}=-12$, $(-12)^2=144$

$h = -16(t^2-24t+144-144)+50$

$h = -16(t^2-24t+144)+2304+50$

$h = -16(t-12)^2+2354$

The ball reaches 2354m high.

(c) It takes the ball 12 seconds to reach the highest point

* Quite the throw ☺

(d) $h=0$

$0 = -16(t-12)^2+2354$

$-2354 = -16(t-12)^2$

$147.125 = (t-12)^2$

$12.13 = t-12$

$t = 24.13$s

It takes 24.13s for the ball to hit the ground.

(15) ① $A = xy$

② $2x + y = 800$

$y = -2x + 800$

① $A = x(-2x + 800)$

$A = -2x^2 + 800x$

$A = -2(x^2 - 400x)$

$b = -400 \quad \frac{-400}{2} = -200$ save

$(-200)^2 = 40\,000$ use

$A = -2(x^2 - 400x + 40000 - 40000)$

$A = -2(x^2 - 400x + 40000) + 80\,000$

$A = -2(x-200)^2 + 80\,000$

$(200, 80\,000)$

x dim. ↑ max area

$2(200) + y = 800$

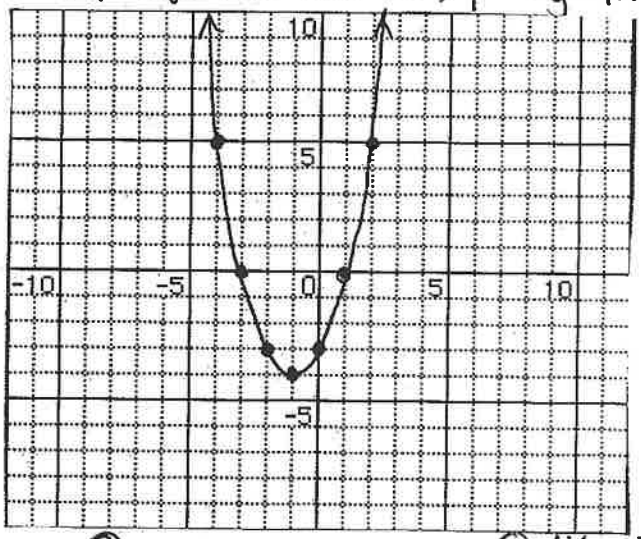
$y = 400$

The max area is $80\,000 m^2$ and the dimen. to do this are $200m \times 400$

Ch 3 - Quadratic Eqns

(16) $x^2 + 2x - 3 = 0$

x	y
-2	-3
-1	-4
0	-3
1	0
2	5
-3	0
-4	5



$x = -3, 1$

(17) (a) $x^2 + 2x - 35 = 0$

$(x+7)(x-5) = 0$

$x = -7, 5$

(b) $2x^2 + 11x + 5 = 0$

$2x^2 + 10x + x + 5 = 0$

$2x(x+5) + 1(x+5) = 0$

$(x+5)(2x+1) = 0$

$x = -5, -\frac{1}{2}$

(c) $2x^2 - 32 = 0$

$2(x^2 - 16) = 0$

$2(x+4)(x-4) = 0$

$x = \pm 4$

(d) $2x(x-2) + x(x+1) = 0$

$2x^2 - 4x + x^2 + x = 0$

$3x^2 - 3x = 0$

$3x(x-1) = 0$

$x = 0, 1$

(e) $4(x+3)^2 + 10(x+3) + 4 = 0$

let $p = x+3$

$4p^2 + 10p + 4 = 0$

$4p^2 + 8p + 2p + 4 = 0$

$4p(p+2) + 2(p+2) = 0$

$(p+2)(4p+2) = 0$

$p = -2, -\frac{1}{2}$

$x+3 = -2$ | $x+3 = -\frac{1}{2}$

$x = -5$ | $x = -3\frac{1}{2}$

(f) $\frac{1}{2}x^2 - 2x - 30 = 0$

$\frac{1}{2}(x^2 - 4x - 60) = 0$

$\frac{1}{2}(x-10)(x+6) = 0$

$x = 10, -6$

(18) $-\frac{2}{3}, \frac{4}{5}$

$(3x+2)(5x-4) = 0$

$15x^2 + 10x - 12x - 8 = 0$

$15x^2 - 2x - 8 = 0$

(19) Let w = width
Let length = $w+102$

$w(w+102) = 2440$

$w^2 + 102w - 2440 = 0$

$(w+122)(w-20) = 0$

$w = -122 \quad w = 20$

Reject $w = -122$ as it's negative

so... $w = 20$

length $= 20 + 102$
$= 122$

The dimensions of the long jump board are 20cm x 122cm.

(20) (a) $x^2 - 6x - 27 = 0$

$x^2 - 6x = 27$

$b = -6 \quad \frac{-6}{2} = -3$ save $\quad (-3)^2 = 9$ use

$x^2 - 6x + 9 - 9 = 27$

$x^2 - 6x + 9 = 36$

$(x-3)^2 = 36$

$\sqrt{(x-3)^2} = \pm\sqrt{36}$

$x - 3 = \pm 6$

$x - 3 = 6 \quad x - 3 = -6$

$x = 9 \quad x = -3$

(b) $x^2 + 8x = -11$

$b = 8 \quad \frac{8}{2} = 4, \; 4^2 = 16$

$x^2 + 8x + 16 - 16 = -11$

$x^2 + 8x + 16 = 5$

$(x+4)^2 = 5$

$x + 4 = \pm\sqrt{5}$

$x = \pm\sqrt{5} - 4$

(c) $3x^2 + 4x = 2$

$3(x^2 + \frac{4}{3}x) = 2$

$x^2 + \frac{4}{3}x = \frac{2}{3}$

$b = \frac{4}{3} \quad \frac{b}{2} = \frac{4}{6} = \frac{2}{3}, \; (\frac{2}{3})^2 = \frac{4}{9}$

$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{2}{3}_{\times 3} + \frac{4}{9}$

$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{6}{9} + \frac{4}{9}$

$(x + \frac{2}{3})^2 = \frac{10}{9}$

$x + \frac{2}{3} = \frac{\pm\sqrt{10}}{3}$

$x = \frac{\pm\sqrt{10} - 2}{3}$

(21) area of pic = $50cm^2$

area inside frame = $50(3) = 150cm^2$

$(10+2x)(5+2x) = 150$

$4x^2 + 30x + 50 = 150$

$2(2x^2 + 15x + 25) = 150$

$2x^2 + 15x + 25 = 75$

$2(x^2 + \frac{15}{2}x) = 50$

$x^2 + \frac{15}{2}x = 25$

$b = \frac{15}{2} \quad \frac{b}{2} = \frac{15}{4}$

$(\frac{15}{4})^2 = \frac{225}{16}$

$x^2 + \frac{15}{2}x + \frac{225}{16} - \frac{225}{16} = \frac{25}{1}_{\times 16}$

$x^2 + \frac{15}{2}x + \frac{225}{16} = \frac{400}{16} + \frac{225}{16}$

$(x + \frac{15}{4})^2 = \frac{625}{16}$

$x + \frac{15}{4} = \pm\frac{25}{4}$

$x + \frac{15}{4} = \frac{25}{4} \quad x + \frac{15}{4} = -\frac{25}{4}$

$x = \frac{10}{4} = \frac{5}{2} \quad x = -\frac{40}{4}$

$x = -10$ Reject (neg)

$x = 2.5$

The width between the picture and the frame is 2.5cm.

(22) The discriminant is the radicand in the quadratic formula (b^2-4ac)
- If $b^2-4ac>0$, there will be two solutions
- If $b^2-4ac=0$, there will be one solution.
- If $b^2-4ac<0$, there will be no solutions.

(23) (a) $5x^2-8x-4=0$

$a=5, b=-8, c=-4$

$x=\frac{8\pm\sqrt{(-8)^2-4(5)(-4)}}{2(5)}$

$x=\frac{8\pm\sqrt{64+80}}{10}$

$x=\frac{8\pm\sqrt{144}}{10}$

$x=\frac{8\pm12}{10}$

$x=\frac{20}{10}$ | $x=\frac{-4}{10}$

$x=2$ | $x=\frac{-2}{5}$

(b) $3x^2-4x-2=0$

$a=3, b=-4, c=-2$

$x=\frac{4\pm\sqrt{(-4)^2-4(3)(-2)}}{2(3)}$

$x=\frac{4\pm\sqrt{16+24}}{6}$

$x=\frac{4\pm\sqrt{40}}{6}$

$x=\frac{4\pm2\sqrt{10}}{6}$

$x=\frac{2\pm\sqrt{10}}{3}$

(24) Area now $=9(13)=117m^2$

new area $=(117)(2)=234m^2$

$(9+x)(13+x)=234$

$x^2+22x+117=234$

$x^2+22x-117=0$

$a=1, b=22, c=-117$

$x=\frac{-22\pm\sqrt{22^2-4(1)(-117)}}{2(1)}$

$x=\frac{-22\pm\sqrt{484+468}}{2}$

$x=\frac{-22\pm\sqrt{952}}{2}$

$x=\frac{-22\pm30.8545}{2}$

$x=4.43$ $\quad x=-26.43$ Reject ⇒ neg.

Josie will need to add 4.43 m to the length and width of her garden.

Ch 4 - Systems of Eqns

(25) Both systems can have 0, 1, or 2 solutions. A quad-quad system can also have infinite solutions.

(26) ① $y=x^2-4x+8$

$b=-4 \quad \frac{-4}{2}=-2, (-2)^2=4$

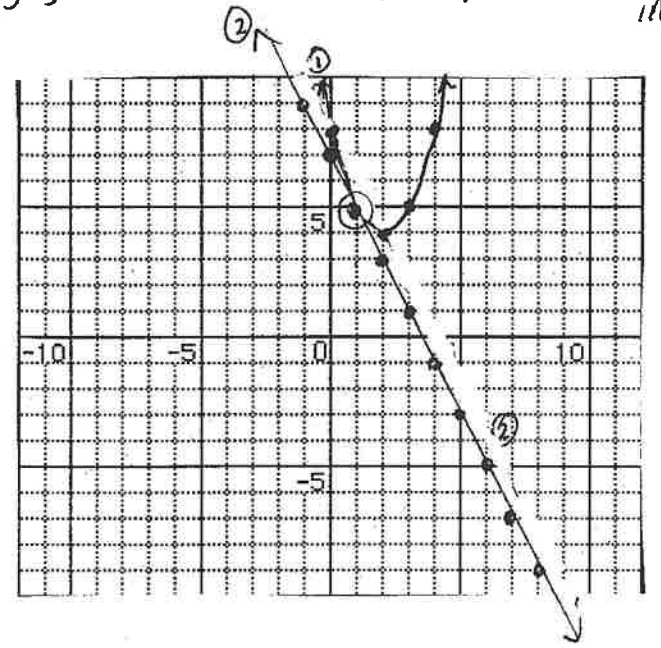
$y=x^2-4x+4-4+8$

① $y=(x-2)^2+4$

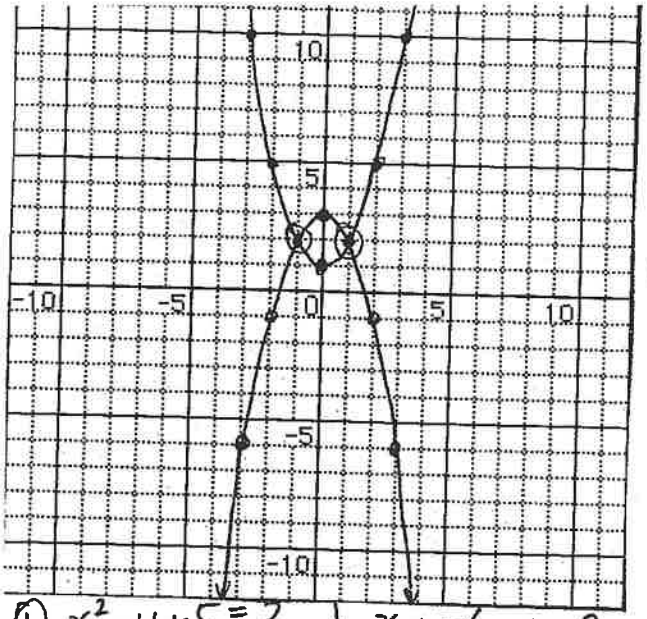
② $2x+y=7$

$y=-2x+7$

Solution : $(1,5)$



(27)



① $y = -x^2 + 3$
vertex $(0, 3)$
opens down
regular count

② $y = x^2 + 1$
Vertex $(0, 1)$
Opens up
regular count

Solutions:
$(-1, 2)$ & $(1, 2)$

(28)

① $x^2 - y + 5 = 3$
② $4x = 14 - y$

① $y = x^2 + 2$
② $y = -4x + 14$

$x^2 + 2 = -4x + 14$
$x^2 + 4x - 12 = 0$
$(x+6)(x-2) = 0$

$x = -6, \; x = 2$

$4(-6) = 14 - y$ $\quad$ $4(2) = 14 - y$
$-24 = 14 - y$ $\quad$ $8 = 14 - y$
$y = 38$ $\quad$ $y = 6$

$(-6, 38)$ & $(2, 6)$

(29)

① $3x^2 - x - y = 2$
$-$ ② $6x^2 + 4x - y = 4$
$-3x^2 - 5x = -2$
$3x^2 + 5x - 2 = 0$
$3x^2 + 6x - x - 2 = 0$
$3x(x+2) - 1(x+2) = 0$
$(x+2)(3x-1) = 0$
$x = -2, \; \frac{1}{3}$

$x = -2$
$3(-2)^2 - (-2) - y = 2$
$3(4) + 2 - 2 = y$
$y = 12$
$(-2, 12)$

$x = \frac{1}{3}$
$3(\frac{1}{3})^2 - (\frac{1}{3}) - y = 2$
$3(\frac{1}{9}) - \frac{1}{3} - y = 2$
$\frac{1}{3} - \frac{1}{3} - 2 = y$
$y = -2$
$(\frac{1}{3}, -2)$

(30)

① $y = \frac{1}{6}x$

$y = -0.002x^2 + 0.5x$
$\frac{1}{6}x = -0.002x^2 + \frac{1}{2 \times 3}x$
$-0.002x^2 + \frac{1}{3}x = 0$
$a = -0.002, \; b = \frac{1}{3}, \; c = 0$

$x = \frac{-\frac{1}{3} \pm \sqrt{(\frac{1}{3})^2 - 4(-0.002)(0)}}{2(-0.002)}$

$x = \frac{-\frac{1}{3} \pm \sqrt{\frac{1}{9}}}{-0.004}$ $\quad$ $x = \frac{-\frac{1}{3} \pm \frac{1}{3}}{-0.004}$

$x = \frac{-\frac{2}{3}}{-0.004}, \; 0$

$x = 167$
$y = \frac{1}{6}(166.667)$
$y = 28$
$(167, 28)$

The ball went 167 yds horizontally and 28 yds vertically.

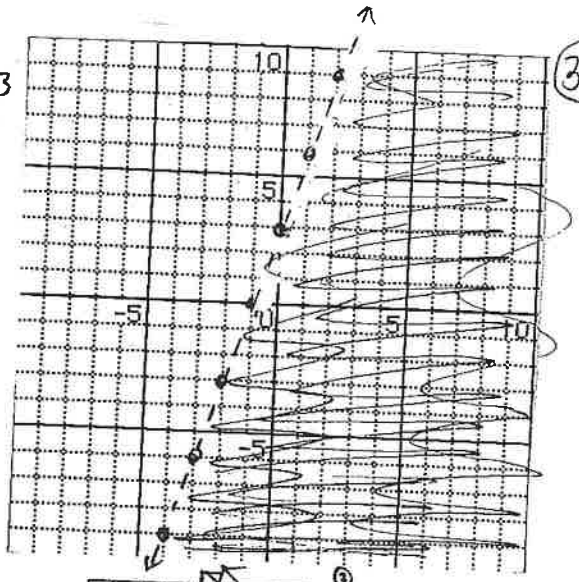
(31) (a) $3x - y > -3$

$3x > y - 3$

$3x + 3 > y$

$y < 3x + 3$

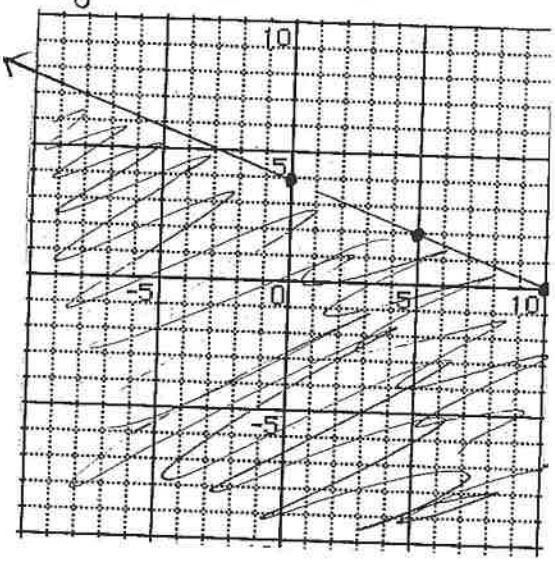
- dashed line
- shade below



(31b) $2x + 5y \leq 20$

$5y \leq -2x + 20$

$y \leq -\frac{2}{5}x + 4$



(32) Let x = # of cheese

Let y = # of pepperoni

① $x + y \geq 6$

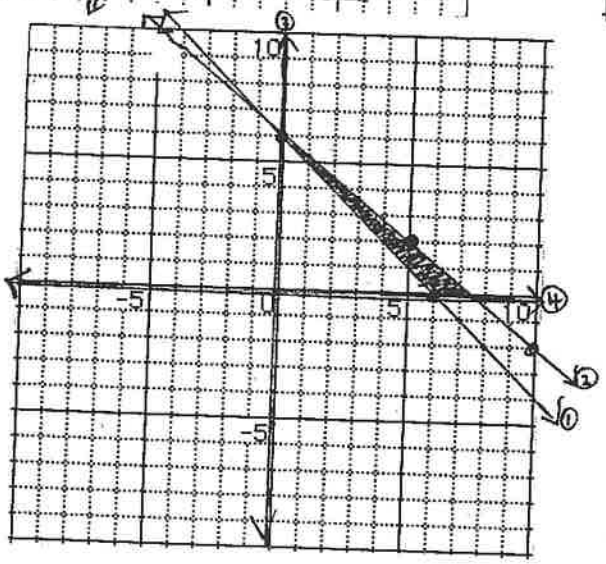
② $12x + 15y \leq 90$

③ $x \geq 0$ ④ $y \geq 0$

① $y \geq -x + 6$

② $y \leq -\frac{4}{5}x + 6$

ex - Can buy 6 cheese and 1 pepperoni (6, 1) or 3 and 3 (3, 3) etc.



(33) (a) $x^2 - x \leq 12$

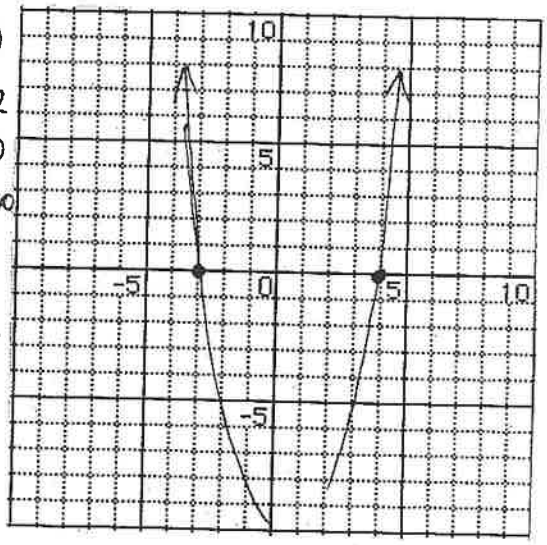
$x^2 - x - 12 \leq 0$

$(x-4)(x+3) \leq 0$

$x = 4, -3$

opens up

Where is the parabola on or below the x-axis?



Solution: $-3 \leq x \leq 4$

(33b) $2x^2 - 5x - 3 > 0$

$ac = -6 \quad -6, 1$

$2x^2 - 6x + x - 3 > 0$

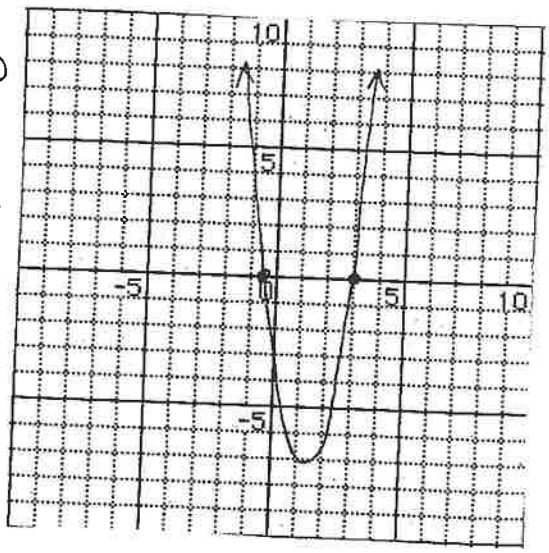
$2x(x-3) + 1(x-3) > 0$

$(x-3)(2x+1) > 0$

$x = 3, -\frac{1}{2}$

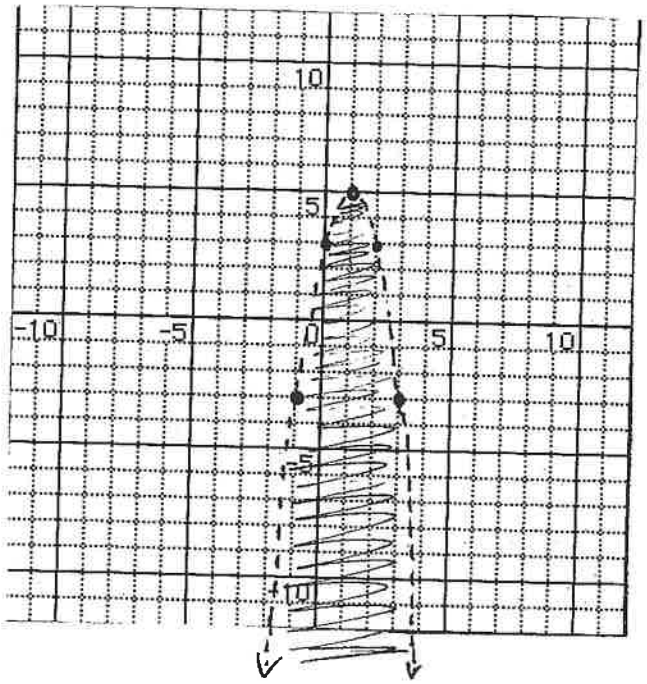
- opens up

Where is the parabola ABOVE the x-axis?



Solution: $x < -\frac{1}{2}, \; x > 3$

(34)



$y < -2x^2 + 4x + 3$

$y < -2(x^2 - 2x) + 3$

$b = -2 \quad \frac{-2}{2} = -1, \quad (-1)^2 = 1$

$y < -2(x^2 - 2x + 1 - 1) + 3$

$y < -2(x^2 - 2x + 1) + 2 + 3$

$y < -2(x-1)^2 + 5$

- shade below
- dashed line

vertex (1, 5)
opens down
double down count

(35)

$2x^2 - 13x + 53 < 60$

$2x^2 - 13x - 7 < 0$

$2x^2 - 14x + x - 7 < 0$

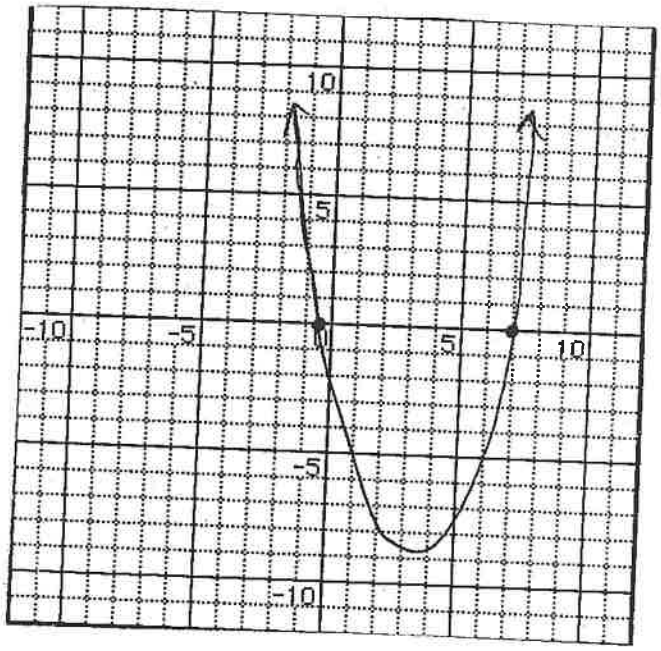
$2x(x-7) + 1(x-7) < 0$

$(x-7)(2x+1) < 0$

$x = 7, -\frac{1}{2}$

Where is the parabola below the x axis?

$-\frac{1}{2} < x < 7$



x = units of math texts and thus cannot be negative.

To keep the cost below $60 per book, less than 7 books must be produced per hour.